SHAZAM

REFERENCE MANUAL

VERSION 11

Diana Whistler
Kenneth J. White
David Bates
Madeleine Golding

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"Teaching econometrics from a robust point of view with a focus on finite sample properties puts extraordinary demands upon the Monte Carlo capabilities of statistical software. I've moved from a main frame computer to SHAZAM on a PC when the 386 chip showed up and never looked back. SHAZAM's fabulous site license allows us to provide our students at zero marginal cost with a full copy of the software I use for research purposes."

Professor David Levy
Economics Department, George Mason University
Virginia, United States

SHAZAM has been used at our university for many years. SHAZAM has a wide variety of ready-to-use routines but is also a very fast and general programming language that can be used to create custom and niche procedures. The program's versatility and reasonable price have been very important to us."

Professor Jim Schmidt
Applied Economics, College of Business Administration, University of Nebraska-Lincoln
Nebraska, United States

"SHAZAM has earned a well-deserved reputation as an excellent general-purpose econometrics package and the new version promises to take it to the next level. Moreover, intuitive commands, common sense defaults, and extensive on-line help make it ideal for teaching. Customer support is responsive and generous."

Professor Simon Power
Department of Economics, University of Carleton
Ottawa, Canada

"Since the 1980s I have used SHAZAM as a critical resource for research and teaching. SHAZAM's user-friendly nature is extremely helpful for students and anyone not familiar with programming languages. It still requires users to understand the techniques they are actually applying. In the past I have particularly appreciated applying non-linear techniques when estimating demand models and, as my research has shifted to Economic Statistics, I use it now for those particular applications. SHAZAM has innovated in new econometric procedures ahead of competing software and, interestingly, at a most competitive price. The SHAZAM documentation and support is clear and advanced."

Professor Achille Vernizzi
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Professor Mamoru Obayashi
School of Commerce, Senshu University
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"SHAZAM is the ideal econometrics software to teach with. Students can learn the basics very quickly and as they progress they will, like their teachers, appreciate SHAZAM's scope and programming flexibility."

Professor Bruno Larue
Agricultural Econometrics Department , University of Laval
Canada

"SHAZAM 11 covers the latest in estimators, user interface, graphics…Professionals and practitioners really have a tool for doing econometrics and statistics."

Torbjørn Lorentzen
Scientist, Bjerknes Centre for Climate Research, University of Bergen
Norway

"I have been using SHAZAM for over 25 years and it is my favorite statistical software package. It is the first program I try whenever I am looking at a new data set or working on a new research project."

Dr. Dennis Olson
Department of Finance, American University of Sharjah
United Arab Emirates

"SHAZAM is an excellent, comprehensive package of econometrics computer programs full of useful estimation and test methods available to applied econometricians."

Professor Toshinobu Matsuda
Agricultural Economics Department, Tottori University
Japan

"I am a longtime SHAZAM user, having used it for doctoral research and beyond. Over the years I have frequently appreciated using SHAZAM's simple-yet-elegant and sophisticated user interface and intuitive commands in my research. Data sets are easy to upload and readily compatible with Excel. It is particularly brilliant for Panel Data with Cross Section dummy variables.

SHAZAM has proved a wonderful teaching tool: the online samples are very useful and instructive and students understand quickly using this package.

SHAZAM's user-friendly support is great."

Dr. Jo Voola
Faculty of Engineering, Computer Science and Mathematics
University of Western Australia
Australia

"...sound research cannot be produced merely by feeding data to a computer and saying SHAZAM."

Peter Kennedy
A Guide to Econometrics

"A casual reader may wonder whether the names of some of these programs, particularly ORACLE and SHAZAM, reflect in any way the Delphic nature of econometric predictions."

Ivor Francis
Statistical Software: A Comparative Review

"...detailed implementation in concrete computer programs or systems. Hence, names familiar to many such as...GREMLIN, TROLL, AUTOREG, SHAZAM..."

Richard E. Quandt
Handbook of Econometrics

"The easiest solution to an inconclusive bounds test is to use a program such as SHAZAM..."

Judge, Hill, Griffiths, Lütkepohl and Lee
Introduction to the Theory and Practice of Econometrics, 2nd Edition

"In instances where d falls within the inconclusive region the exact critical value d* can be found numerically, providing appropriate computer software [e.g., SHAZAM, White(1978)] is available."

Judge, Griffiths, Hill, Lütkepohl and Lee
The Theory and Practice of Econometrics, 2nd Edition

"Beach and MacKinnon devised an iterative procedure for maximizing Equation (8-77), which has now been incorporated in White's SHAZAM program..."

Jack Johnston
Econometric Methods, 3rd ed.

"Some Computer Programs (e.g. SHAZAM, White (1978)) allow for the estimation of ρ and provide an unconditional covariance matrix."

Fomby, Hill and Johnson
Advanced Econometric Methods

Senator: "Do you realize that I got a bill passed today that's going to put a million people to work? You know how I did it? I said one word."

Wife: "SHAZAM?"

Senator: "No. Subcommittee."

From the movie:
The Seduction of Joe Tynan

CONTENTS

1. INTRODUCTION

"I think there is a world market for about five computers."
Thomas J. Watson
Chairman of the Board - IBM, 1943

SHAZAM is a comprehensive software package for econometricians, statisticians, biometricians, engineers, sociometricians, psychometricians, politicometricians and others who use statistical techniques. The primary strength of SHAZAM is for the estimation and testing of many types of econometric and statistical models. The SHAZAM command language has great flexibility and provides capabilities for programming procedures. This Reference Manual provides detailed descriptions of the SHAZAM commands available to perform all SHAZAM analytical techniques. It contains numerous theoretical explanations, practical examples and sample code.

SHAZAM includes features for:

- data transformations, handling missing observations, matrix manipulation, evaluation of derivatives and integrals, data sorting, computation of cumulative distribution functions for a variety of probability distributions;
- descriptive statistics, calculation of price indexes, moving averages, exponential smoothing, seasonal adjustment,
- financial time series,
- ARIMA (Box-Jenkins) time series models,
- cointegration and unit root testing, Dickey-Fuller and Phillips-Perron unit root tests,
- nonparametric density estimation;
- OLS estimation, restricted least squares, weighted least squares, ridge regression, distributed lag models, estimation with autoregressive or moving average errors, estimation with heteroskedastic errors, stepwise regression
- generalized least squares
- ARCH and GARCH models,
- Box-Cox regressions,
- probit models, logit models, tobit models,
- estimation using regression quantiles (including MAD estimation),

- regression with non-normal errors (including exponential regression, beta regression and Poisson regression),
- regression with time varying coefficients,
- nonparametric methods,
- generalized entropy methods,
- fuzzy set models;
- linear and nonlinear hypothesis testing,
- calculation of confidence intervals and ellipse plots, computation of the Newey-West autocorrelation consistent covariance matrix,
- regression diagnostic tests (including tests for heteroskedasticity, CUSUM tests, RESET specification error tests), computation of p-values for many test statistics (including the p-value for the Durbin-Watson test),
- nonlinear least squares,
- simulated annealing
- estimation of systems of linear and nonlinear equations by SURE, 2SLS and 3SLS,
- generalized method of moments (GMM) estimation,
- panel data and pooled time-series cross-section methods;
- principal components and factor analysis, principal components regression,
- minimizing and maximizing nonlinear functions,
- solving nonlinear simultaneous equations.
- linear programming
- quadratic programming
- forecasting;

SHAZAM has thousands of users in more than 90 countries and is one of the most popular econometric software packages in the world. It can be found at the Northernmost (University of Tromso, Norway) and Southernmost (University of Otago, New Zealand) Universities in the world and in locations from Antarctica to Greenland.

Users should always be aware of the version of SHAZAM being used. This manual describes Version 11 of SHAZAM. Some options and commands described in this manual were not available in earlier versions so users should be certain an old version of SHAZAM is not used with this manual. See the chapter *NEW FEATURES IN SHAZAM* for further details.

SHAZAM *AWARDS*

A list of articles that cite SHAZAM is available at the SHAZAM website.

SHAZAM *Hall Of Fame*

SHAZAM users who have cited SHAZAM in at least 5 refereed printed articles during their careers will be granted membership into the SHAZAM Hall of Fame. There are 3 categories:

Regressor	- Cite SHAZAM in 5 articles
Executive Regressor	- Cite SHAZAM in 10 articles
Lifetime Achievement Award	- Cite SHAZAM in 15 or more articles

Members of the SHAZAM Hall of Fame are entitled to discounts or free upgrades on various SHAZAM products.

2. A CHILD'S GUIDE TO ANALYSIS WITH SHAZAM

"Difficult? A child could do it."

Arthur S. Goldberger (1930-2009)
Professor of Economics, 1971

This chapter should be read by all users as it provides step-by-step basic information on how to run SHAZAM techniques, illustrated using the Ordinary Least Squares (OLS) regression technique.

SHAZAM Data

A simple data set to analyze is the textile demand data set from Theil [1971, p. 102]. There are 17 years of observations on the variables *YEAR*, *CONSUME*, *INCOME* and *PRICE*. The data can be entered in a file, say **MYDATA**, and a listing of the file is:

```
1923  99.2  96.7 101.0
1924  99.0  98.1 100.1
1925 100.0 100.0 100.0
1926 111.6 104.9  90.6
1927 122.2 104.9  86.5
1928 117.6 109.5  89.7
1929 121.1 110.8  90.6
1930 136.0 112.3  82.8
1931 154.2 109.3  70.1
1932 153.6 105.3  65.4
1933 158.5 101.7  61.3
1934 140.6  95.4  62.5
1935 136.2  96.4  63.6
1936 168.0  97.6  52.6
1937 154.3 102.4  59.7
1938 149.0 101.6  59.5
1939 165.5 103.8  61.3
```

Setting the Sample Size

The first SHAZAM command is usually **SAMPLE** to specify the sample size. The sample size is the number of observations in the data set. The general format of the **SAMPLE** command is:

SAMPLE *beg end*

where *beg* is the first observation to be used (usually 1), and *end* is the last observation. If your data has 17 observations your **SAMPLE** command will look like:

```
sample 1 17
```

This **SAMPLE** command sets the sample size to 17 observations and the data will be placed in observations 1 through 17 (because *beg* is specified as 1).

Data Input

Having set the sample size, you are ready to enter your data with the **READ** command. In general, the **READ** command will look like:

READ(*filename*) *vars* / **OPTIONS**

where *filename* is the name of the data file (this is optional), *vars* is a list of variable names for the data and *options* is a list of desired options. Variable names may be up to 8 characters long and must consist only of letters or numbers and start with a letter. If the data on *YEAR*, *CONSUME*, *INCOME* and *PRICE* is stored in the file named **MYDATA** the **READ** command would be:

```
read(mydata) year consume income price
```

Alternatively, it may be convenient to enter the data directly in a SHAZAM command file following the **READ** command so no data filename is used. The SHAZAM command file would contain the lines:

```
read year consume income price
   1923  99.2  96.7 101.0
   1924  99.0  98.1 100.1
   1925 100.0 100.0 100.0
   1926 111.6 104.9  90.6
   1927 122.2 104.9  86.5
   1928 117.6 109.5  89.7
   1929 121.1 110.8  90.6
   1930 136.0 112.3  82.8
   1931 154.2 109.3  70.1
   1932 153.6 105.3  65.4
   1933 158.5 101.7  61.3
   1934 140.6  95.4  62.5
   1935 136.2  96.4  63.6
   1936 168.0  97.6  52.6
   1937 154.3 102.4  59.7
   1938 149.0 101.6  59.5
   1939 165.5 103.8  61.3
```

Normally, data is typed observation by observation where the observations for all variables begin on a new line. You may use more than one line per observation. If your data is set up variable by variable instead of observation by observation you will have to use the **BYVAR** option on your **READ** command. For details on the **BYVAR** option see the chapter *DATA INPUT AND OUTPUT*. All variables entered on a **READ** command must have an equal number of observations. In the above example, the first column read will be stored in variable *YEAR*, the next in *CONSUME*, and so on.

NOTE: Be sure that the number of variables you give on the **READ** command matches the number of variables in your data.

The STAT Command

The **STAT** command will print some useful descriptive statistics including the means, standard deviations, variances, minimums and maximums for the variables listed. The format of the **STAT** command is:

STAT *vars*

where *vars* is a list of variables. There are many options available on the **STAT** command which are described in the chapter *DESCRIPTIVE STATISTICS*.

The OLS Command

The **OLS** command will run an ordinary least squares regression. The format of the **OLS** command is:

OLS *depvar indeps* / **OPTIONS**

where *depvar* is the name of the dependent variable, *indeps* are the names of the independent variables and *options* is a list of options desired. A very simple **OLS** command might look like this:

```
ols consume income price
```

This will run a regression of the variable *CONSUME* on variables *INCOME* and *PRICE*. A *CONSTANT* is automatically included in the regression. No options were requested. There are many available options which are described in the chapter *ORDINARY LEAST SQUARES*. Some common ones are:

ANOVA	Prints the **AN**alysis **O**f **VA**riance tables
GF	Prints **G**oodness-of-**F**it statistics testing for normality of residuals
LIST	**LIST**s residuals plus residual summary statistics
NOCONSTANT	Suppresses the intercept
PCOV	**P**rints a **COV**ariance matrix of coefficients
RSTAT	Prints **R**esidual summary **STAT**istics (Durbin-Watson etc.)

Users should note that the **LIST** option will substantially increase the amount of output, particularly if the sample size is large. An example of an **OLS** command with options is:

```
ols consume income price / rstat pcov
```

This example runs an **OLS** regression of *CONSUME* on *INCOME*, *PRICE* and a *CONSTANT* and prints residual summary statistics and a covariance matrix of coefficients along with the standard output. Do not forget to separate the options from the list of variables by a slash (/). The options may be listed in any order.

The SHAZAM Command File

The command file for a complete SHAZAM run might look like:

```
* i hope this works!
sample 1 17
read year consume income price
   1923  99.2  96.7 101.0
   1924  99.0  98.1 100.1
   1925 100.0 100.0 100.0
   1926 111.6 104.9  90.6
   1927 122.2 104.9  86.5
   1928 117.6 109.5  89.7
   1929 121.1 110.8  90.6
   1930 136.0 112.3  82.8
   1931 154.2 109.3  70.1
   1932 153.6 105.3  65.4
   1933 158.5 101.7  61.3
   1934 140.6  95.4  62.5
   1935 136.2  96.4  63.6
   1936 168.0  97.6  52.6
   1937 154.3 102.4  59.7
   1938 149.0 101.6  59.5
   1939 165.5 103.8  61.3
stat consume income price
ols consume income price / rstat pcov
ols consume income
stop
```

NOTE: The line in the example beginning with an asterisk (*) is a SHAZAM *comment*. You can insert these at various places in the run, except within the data. Comment lines are printed on the output and sometimes help document the output. Comment lines must begin with an asterisk (*) in column 1. The rest of the line may contain anything.

In this simple example there are 4 input variables *YEAR, CONSUME, INCOME* and *PRICE* and 17 observations. The **SAMPLE** command sets the sample size to 17 observations. The **STAT** command is used to get descriptive statistics of the variables. Two **OLS** regressions are run. Both regressions use *CONSUME* as the dependent variable.

The **STOP** command is a signal to SHAZAM that it has reached the end of the commands.

SHAZAM output from the STAT Command

The output for the **STAT** command is shown below. Note that SHAZAM commands typed by the user appear in SHAZAM output following the symbol |_.

```
|_STAT CONSUME INCOME PRICE
 NAME      N   MEAN      ST. DEV     VARIANCE     MINIMUM      MAXIMUM
 CONSUME  17   134.51     23.577      555.89       99.000       168.00
 INCOME   17   102.98     5.3010      28.100       95.400       112.30
 PRICE    17   76.312     16.866      284.47       52.600       101.00
```

For a variable X with N observations denoted by X_t, t = 1, . . . , N the statistics listed by the **STAT** command are calculated as:

MEAN $\quad \bar{X} = \frac{1}{N}\sum_{t=1}^{N} X_t$

VARIANCE $\quad \hat{\sigma}_X^2 = \frac{1}{N-1}\sum_{t=1}^{N}(X_t - \bar{X})^2 = \frac{1}{N-1}\left[\sum_{t=1}^{N} X_t^2 - N\bar{X}^2\right]$

ST. DEV. $\quad \sqrt{\hat{\sigma}_X^2}$ (Standard Deviation)

MINIMUM $\quad$ the smallest value of X_t

MAXIMUM $\quad$ the largest value of X_t

SHAZAM output from the OLS Command

The output for the first **OLS** command is:

```
| OLS CONSUME INCOME PRICE / RSTAT PCOV
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =    .9513     R-SQUARE ADJUSTED =    .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE     -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT   130.71      27.09       4.824      .000  .790      .0000      .9718

VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
INCOME     .71115E-01
PRICE     -.39974E-02   .70248E-02
CONSTANT  -7.0185      -.12441       734.10
             INCOME       PRICE        CONSTANT

DURBIN-WATSON = 2.0185    VON NEUMANN RATIO = 2.1447    RHO =  -.18239
RESIDUAL SUM =  -.12434E-12  RESIDUAL VARIANCE =   30.951
SUM OF ABSOLUTE ERRORS=   72.787
R-SQUARE BETWEEN OBSERVED AND PREDICTED =  .9513
RUNS TEST:    7 RUNS,    9 POSITIVE,    8 NEGATIVE, NORMAL STATISTIC = -1.2423
```

The calculations used for the **OLS** output are now described. The linear regression model can be written as:

$$Y_t = \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \ldots + \beta_K X_{Kt} + \varepsilon_t \quad \text{for } t = 1, \ldots, N$$

where there are N observations and Y_t is observation t on the dependent variable, X_{kt} is observation t on the k^{th} explanatory variable for k = 1, . . ., K, β_k are parameters to estimate and ε_t is a random error that is assumed to have zero mean and variance σ^2.

In matrix notation this can be stated as:

$$Y = X\beta + \varepsilon$$

where β is a K x 1 vector of parameters, $Y = [Y_1 \ Y_2 \ldots \ Y_N]'$ and X is the N x K matrix:

$$X = \begin{bmatrix} X_{11} & X_{21} & . & X_{K1} \\ X_{12} & X_{22} & . & X_{K2} \\ . & . & . & . \\ X_{1N} & X_{2N} & . & X_{KN} \end{bmatrix}$$

If a constant term is included in the model then one of the columns of the X matrix will be a column where every element is a 1. SHAZAM automatically makes the last column of the X matrix the one that corresponds to the constant term.

The **OLS** estimated coefficients are calculated as: $\hat{\beta} = (X'X)^{-1}X'Y$

The N x 1 vector of **OLS** residuals denoted by **e** are obtained as: $e = Y - X\hat{\beta}$

and the predicted values are: $\hat{Y} = X\hat{\beta}$

With $\overline{Y}$ the mean of the dependent variable the deviations from the sample mean are denoted by:

$$y_t = Y_t - \overline{Y} \qquad \text{and} \qquad \hat{y}_t = \hat{Y}_t - \overline{Y} \qquad \text{for } t = 1, \ldots, N$$

The output from the **OLS** command includes:

R-SQUARE

$$R^2 = 1 - \frac{\sum_{t=1}^{N} e_t^2}{\sum_{t=1}^{N} (Y_t - \overline{Y})^2} = 1 - \frac{e'e}{y'y}$$

R-SQUARE ADJUSTED

$$\overline{R}^2 = 1 - \left(1 - R^2\right)\frac{N-1}{N-K}$$

VARIANCE OF THE ESTIMATE-SIGMA**2

$$\hat{\sigma}^2 = \frac{1}{N-K}\sum_{t=1}^{N} e_t^2 = \frac{e'e}{N-K}$$

STANDARD ERROR OF THE ESTIMATE-SIGMA

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

SUM OF SQUARED ERRORS-SSE

$$\sum_{t=1}^{N} e_t^2 = e'e$$

MEAN OF DEPENDENT VARIABLE $$\bar{Y} = \frac{1}{N}\sum_{t=1}^{N} Y_t$$

With the assumption that the errors are normally distributed the log-likelihood function for the linear regression model can be expressed as:

$$-\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)$$

On the SHAZAM output the LOG OF THE LIKELIHOOD FUNCTION is evaluated as:

$$-\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\tilde{\sigma}^2) - \frac{N}{2} \qquad \text{where} \qquad \tilde{\sigma}^2 = \frac{e'e}{N}$$

The above uses the result: $(Y - X\hat{\beta})'(Y - X\hat{\beta})/\tilde{\sigma}^2 = e'e/\tilde{\sigma}^2 = N$

When the **NOCONSTANT** option is used on the **OLS** command the SHAZAM output reports the RAW MOMENT R-SQUARE calculated as:

$$1 - \frac{e'e}{Y'Y}$$

The variance-covariance matrix of the **OLS** parameter estimates (printed with the **PCOV** option) is estimated as:

$$V(\hat{\beta}) = \hat{\sigma}^2 (X'X)^{-1}$$

Denote $V(\hat{\beta}_k)$ as the k^{th} diagonal element of the $V(\hat{\beta})$ matrix. For the **OLS** estimated coefficients the SHAZAM output reports:

STANDARD ERROR (of $\hat{\beta}_k$) $$SE_k = \sqrt{V(\hat{\beta}_k)} \qquad \text{for } k = 1, \ldots, K$$

T-RATIO $$t_k = \frac{\hat{\beta}_k}{SE_k}$$

PARTIAL CORRELATION $$\frac{t_k}{\sqrt{t_k^2 + N - K}}$$

STANDARDIZED COEFFICIENT $\hat{\beta}_k \left(\frac{\hat{\sigma}_{X_k}}{\hat{\sigma}_Y} \right)$

ELASTICITY AT MEANS $E_k = \hat{\beta}_k \left(\frac{\overline{X}_k}{\overline{Y}} \right)$

The p-values reported on the **OLS** output are the tail probabilities for a two-tail test of the null hypothesis H_0: $\beta_k = 0$. The p-value is the exact level of significance of a test statistic. If the p-value is less than a selected level of significance (say 0.05) then there is evidence to reject H_0. For a discussion of p-values see any good econometrics textbook.

Residual Statistics

If the **RSTAT**, **LIST**, or **MAX** options are used the SHAZAM output also includes a number of residual statistics calculated as:

DURBIN WATSON	$DW = \frac{\sum_{t=2}^{N} (e_t - e_{t-1})^2}{\sum_{t=1}^{N} e_t^2}$	VON NEUMANN RATIO	$DW \left(\frac{N}{N-1} \right)$
RHO	$\hat{\rho} = \frac{\sum_{t=2}^{N} e_t e_{t-1}}{\sum_{t=2}^{N} e_{t-1}^2}$	RESIDUAL SUM	$\sum_{t=1}^{N} e_t$
RESIDUAL VARIANCE	$\hat{\sigma}^2 = \frac{1}{N-K} \sum_{t=1}^{N} e_t^2$	SUM OF ABSOLUTE ERRORS	$\sum_{t=1}^{N} \lvert e_t \rvert$

The R-SQUARE BETWEEN OBSERVED AND PREDICTED is calculated as: $R^2 = \frac{\left(\sum \hat{y}_t y_t \right)^2}{\sum \hat{y}_t^2 \sum y_t^2}$

For discussion on goodness-of-fit measures see, for example, Judge, Griffiths, Hill, Lütkepohl and Lee [1985, pp. 29-31] and Kvalseth [1985].

The runs test (discussed in, for example, Gujarati [2003, p. 465]) gives a test for independent errors. A run is defined as an uninterrupted sequence of either positive or negative residuals. Let N_1 be the total number of positive residuals and N_2 be the total number of negative residuals (so that $N_1+N_2=N$). Let n be the total number of runs. The RUNS TEST reported on the SHAZAM output is computed as:

$$\frac{n - E(n)}{\sigma_n} \quad \text{where} \quad E(n) = \frac{2N_1N_2}{N_1 + N_2} + 1 \quad \text{and} \quad \sigma_n^2 = \frac{2N_1N_2(2N_1N_2 - N_1 - N_2)}{(N_1 + N_2)^2(N_1 + N_2 - 1)}$$

Under the null hypothesis of independence and assuming N_1>10 and N_2>10 the runs test statistic has an asymptotic standard normal distribution. If N_1 or N_2 are less than 10 then tables of critical values available in Gujarati [2003, pp. 974-5] can be consulted.

Model Selection Test Statistics and Analysis of Variance

When the **ANOVA** option is specified the SHAZAM output will also include a series of model selection test statistics as well as two analysis of variance tables. The output from the model selection tests will appear as:

```
 MODEL SELECTION TESTS - SEE JUDGE ET AL. (1985,P.242)
 AKAIKE (1969) FINAL PREDICTION ERROR - FPE =      36.413
    (FPE IS ALSO KNOWN AS AMEMIYA PREDICTION CRITERION - PC)
 AKAIKE (1973) INFORMATION CRITERION - LOG AIC =   3.5912
 SCHWARZ (1978) CRITERION - LOG SC =               3.7382
MODEL SELECTION TESTS - SEE RAMANATHAN (1998,P.165)
 CRAVEN-WAHBA (1979)
    GENERALIZED CROSS VALIDATION - GCV =           37.583
 HANNAN AND QUINN (1979) CRITERION =               36.811
 RICE (1984) CRITERION =                           39.392
 SHIBATA (1981) CRITERION =                        34.485
 SCHWARZ (1978) CRITERION - SC =                   42.023
 AKAIKE (1974) INFORMATION CRITERION - AIC =       36.277
```

With $\tilde{\sigma}^2 = \frac{e'e}{N}$ the calculations for the above statistics are:

AKAIKE (1969) FINAL PREDICTION ERROR - FPE	$\tilde{\sigma}^2\left(\frac{N + K}{N - K}\right)$
AKAIKE (1973) INFORMATION CRITERION - LOG AIC	$\ln \tilde{\sigma}^2 + \frac{2K}{N}$
SCHWARZ (1978) CRITERION - LOG SC	$\ln \tilde{\sigma}^2 + \frac{K \ln N}{N}$
CRAVEN-WAHBA (1979) GENERALIZED CROSS VALIDATION	$\tilde{\sigma}^2\left(1 - \frac{K}{N}\right)^{-2}$
HANNAN AND QUINN (1979) CRITERION	$\tilde{\sigma}^2 (\ln N)^{2K/N}$
RICE (1984) CRITERION	$\tilde{\sigma}^2\left(1 - \frac{2K}{N}\right)^{-1}$

SHIBATA (1981) CRITERION $\tilde{\sigma}^2\left(\frac{N+2K}{N}\right)$

SCHWARZ (1978) CRITERION - SC $\tilde{\sigma}^2 N^{K/N}$

AKAIKE (1974) INFORMATION CRITERION - AIC $\tilde{\sigma}^2 \exp\left(\frac{2K}{N}\right)$

The output for the analysis of variance tables is:

```
                    ANALYSIS OF VARIANCE - FROM MEAN
                      SS          DF            MS                F
REGRESSION         8460.9          2.         4230.5            136.683
ERROR              433.31         14.         30.951            P-VALUE
TOTAL              8894.3         16.         555.89               .000

                    ANALYSIS OF VARIANCE - FROM ZERO
                      SS          DF            MS                F
REGRESSION        .31602E+06       3.        .10534E+06        3403.474
ERROR              433.31         14.         30.951            P-VALUE
TOTAL             .31646E+06      17.         18615.               .000
```

The calculations for this output are:

ANALYSIS OF VARIANCE - FROM MEAN

	SS	DF	MS	F
REGRESSION	$y'y - e'e$	$K-1$	$\frac{y'y-e'e}{K-1}$	$\frac{N-K}{K-1}\left(\frac{y'y-e'e}{e'e}\right)$
ERROR	$e'e$	$N-K$	$\frac{e'e}{N-K}$	
TOTAL	$y'y$	$N-1$	$\frac{y'y}{N-1}$	

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	$Y'Y - e'e$	K	$\frac{Y'Y-e'e}{K}$	$\frac{N-K}{K}\left(\frac{Y'Y-e'e}{e'e}\right)$
ERROR	$e'e$	$N-K$	$\frac{e'e}{N-K}$	
TOTAL	$Y'Y$	N	$\frac{Y'Y}{N}$	

Tests for Normality of the Residuals

If the **GF** option is specified some tests for normality of the residuals are reported on the SHAZAM output as:

```
COEFFICIENT OF SKEWNESS =  -0.0343 WITH STANDARD DEVIATION OF 0.5497
COEFFICIENT OF EXCESS KURTOSIS =  -0.8701 WITH STANDARD DEVIATION OF 1.0632

JARQUE-BERA NORMALITY TEST- CHI-SQUARE(2 DF)=    0.6662 P-VALUE= 0.717

     GOODNESS OF FIT TEST FOR NORMALITY OF RESIDUALS -  6 GROUPS
OBSERVED  0.0  2.0  6.0  7.0  2.0  0.0
EXPECTED  0.4  2.3  5.8  5.8  2.3  0.4
CHI-SQUARE =    1.1126 WITH  1 DEGREES OF FREEDOM, P-VALUE= 0.292
```

For the residuals, the sample k^{th} central moments are:

$$m_k = \frac{1}{N}\sum_{t=1}^{N}(e_t - \bar{e})^k$$

Define: $S_k = \frac{1}{N}\sum_{t=1}^{N} e_t^k$ for $k = 1, 2, 3, 4$

The sample central moments can be computed as:

$$m_2 = S_2 - S_1^2 \quad , \quad m_3 = S_3 - 3S_1S_2 + 2S_1^3 \quad \text{and}$$

$$m_4 = S_4 - 4S_1S_3 + 6S_1^2S_2 - 3S_1^4$$

A measure of skewness is: $\gamma_1 = \frac{m_3}{m_2^{3/2}}$

and a measure of excess kurtosis is: $\gamma_2 = \frac{m_4}{m_2^2} - 3$

The above statistics are biased estimators of skewness and kurtosis that can be used for large N. Unbiased estimators of skewness and kurtosis are described in, for example, Smillie [1966]. The SHAZAM calculations incorporate these small sample adjustments as follows.

The COEFFICIENT OF SKEWNESS is calculated as: $g_1 = \dfrac{k_3}{k_2^{3/2}}$

and the COEFFICIENT OF EXCESS KURTOSIS is calculated as: $g_2 = \dfrac{k_4}{k_2^{2}}$

where the k-statistics are:

$$k_2 = \frac{N}{N-1} m_2 \quad , \qquad k_3 = \frac{N^2}{(N-1)(N-2)} m_3 \qquad \text{and}$$

$$k_4 = \frac{N^2}{(N-1)(N-2)(N-3)}\left[(N+1)m_4 - 3(N-1)m_2^2\right]$$

If the residuals are normally distributed then g_1 and g_2 have zero means and standard deviations:

$$\sigma_{g_1} = \sqrt{\frac{6N(N-1)}{(N-2)(N+1)(N+3)}} \qquad \text{and}$$

$$\sigma_{g_2} = \sqrt{\frac{24N(N-1)^2}{(N-3)(N-2)(N+3)(N+5)}}$$

The chi-square GOODNESS OF FIT TEST FOR NORMALITY OF RESIDUALS is computed by dividing the residuals into m groups. The number of groups is automatically set by SHAZAM based on the sample size. The observed frequency of residuals in group i is denoted by o_i. The number of values theoretically expected in group i if the residuals are normally distributed is denoted by d_i. The test statistic is:

$$\chi^2 = \sum_{i=1}^{m} \frac{(d_i - o_i)^2}{d_i}$$

Under the hypothesis of normality the test statistic can be compared with a χ^2 distribution with m–K–2 degrees of freedom. Further discussion of the goodness of fit test is in, for example, Klein [1974, p.372].

The Jarque-Bera [1987] test statistic is calculated as:

$$N\left[\frac{\gamma_1^2}{6}+\frac{\gamma_2^2}{24}\right]$$

In large samples, under the null hypothesis of normally distributed residuals, the test statistic can be compared with a chi-square distribution with 2 degrees of freedom.

More Information

Detailed command descriptions are in the chapters that follow. Users may also find it helpful to review the *MISCELLANEOUS COMMANDS AND INFORMATION* chapter. It gives some of the "rules and regulations" to follow when running SHAZAM

.

3. DATA INPUT AND OUTPUT

"I have traveled the length and breadth of this country and talked with the best people, and I can assure you that data processing is a fad that won't last out the year."

The editor in charge of business books for
Prentice Hall, 1957

This chapter provides information on how to assign a **FILE**, set a **SAMPLE** period, and **READ**, **PRINT**, and **WRITE** SHAZAM data.

THE FILE COMMAND

A SHAZAM session may typically require a command file and one or more data files. Other types of files include output files and procedure files. It may be necessary to specify the location of files or to assign a particular file to a SHAZAM *unit* before it can be used. The **FILE** command is used whenever some action is required on a particular file.

The format of the **FILE** command is:

FILE ***option filename***

where *option* is either one of the keywords described below or an input-output unit number (range is 11-49).

FILE CD *folder*	Change the current directory to the specified *folder* (or directory).
FILE CLOSE *filename*	Closes the specified *filename*. This is usually only needed if you wish to re-assign a file for another purpose.
FILE DELETE *filename*	Delete the specified *filename*.
FILE INPUT *filename*	Reads SHAZAM commands from *filename*.
FILE KEYBOARD *filename*	Makes a copy of all commands typed on a keyboard and writes them into *filename*.

FILE LIST *filename*	Lists the specified *filename* on the screen.
FILE OUTPUT *filename*	Puts the output in the assigned *filename*. If used, this is often the first **FILE** command. No output will appear on the screen.
FILE PATH *pathname*	Specifies a directory path to append to all filenames used in subsequent **FILE**, **READ** or **WRITE** commands or for gnuplot files.
FILE PLOTPATH *pathname*	Specifies a directory path for the gnuplot program.
FILE PRINT *filename*	Prints the specified *filename* on a printer (may not work on all computers).
FILE PROC *filename*	Loads a SHAZAM procedure from the specified *filename*.
FILE PROCPATH *pathname*	Specifies a directory path to use to search for SHAZAM procedures.
FILE PWD	Print working directory. This command displays the pathname of the current folder (or directory).
FILE SCREEN *filename*	Puts the output in the assigned filename and simultaneously displays the output on the computer screen. Alternative to **FILE OUTPUT** command.
FILE TEMP *pathname*	Specifies a directory path to use for creating and writing to temporary scratch files.

There are occasions (described in this manual) when a file must be assigned to an input-output unit number. Any unit number from 11-49 is available. Unit numbers are sometimes required as some SHAZAM commands do not allow you to write the full filename and are expecting a unit number instead. In this case you must first use the **FILE** command to assign a unit number to the file then use the unit number instead of the filename in any further SHAZAM commands. For example:

```
file 11 mydata
```

If the file is binary (as is the case for the **OUT=** option in **SYSTEM** and **NL** problems or the **BINARY** option on a **WRITE** command) a decimal point (.) should be placed next to the unit number. For example:

```
file 12. dump.dat
```

THE SAMPLE COMMAND

The **SAMPLE** command has the format:

SAMPLE *beg end*

where *beg* and *end* are numbers specifying the beginning and ending observations. For example, to input the first 10 observations of the data with the **READ** command the following **SAMPLE** command should be used:

```
sample 1 10
```

The **SAMPLE** command should not set a sample size larger than the number of observations available in the data.

An Expanded Form of the **SAMPLE** command can be used after all the data has been read to control which observations are used for estimation. Note, however that the Expanded Form is not available for use with the **READ** command described below. In the Expanded Form you can select certain groups of observations. For example:

```
sample 1 8 11 15
```

would skip observations 9 and 10. Another example is:

```
sample 1 1 5 5 12 12 15 15 20 20
```

which would only use observations 1, 5, 12, 15, and 20.

With time series data the **SAMPLE** command can specify dates. This form of the **SAMPLE** command is described in the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.

THE READ COMMAND

The **READ** command inputs SHAZAM data and assigns variable names. When data is not in a separate file the data directly follows the **READ** command. However, if the data is in a separate file the **READ** command must specify the file containing the data with either a filename or unit number. If a unit number is used there must be a previous **FILE** command which assigns the file to the unit number.

This command is for working with Fixed Format or Free Format numeric or character data or for working with Data Interchange Format (DIF) files.

Possible formats for the **READ** command are:

READ *vars* / *OPTIONS*

READ (*filename) vars* / *OPTIONS*

FILE *unit filename*
READ (*unit*) *vars* / *OPTIONS*

where *vars* is a list of variable names and *OPTIONS* is a list of desired options (see details below). In the first case, no unit number or filename is specified so the data must immediately follow the **READ** command. The second case specifies a filename for the data set. The third method specifies a unit number which was assigned on a previous **FILE** command. When the **NAMES** option is used the *vars* list is not required and the variable names must be included as the first line of the data set.

An example of loading a data set from the data file **MYDATA** is:

```
sample 1 17
read(mydata) year consume income price
```

An example of a **READ** command where the data file **MYDATA** is assigned to unit 11, and contains 4 variables is the following:

```
file 11 mydata
sample 1 17
read(11) year consume income price
```

This method may not work on binary files on some operating systems and instead the **FILE** command with the decimal point following the unit number must be used.

Some computer programs (but not SHAZAM!) will interpret a "." as a missing value. If your data file has missing data you must use an editor to change the "." to a numerical missing value. For example, you may want to set all the missing data to the value of –99999 which is the SHAZAM default missing data code. For more information, see the **SET MISSVALU=** command described in the chapter *SET AND DISPLAY*.

The following options are available on the **READ** command:

BINARY Used when the data is in Double Precision **BINARY** (unformatted). If the **FILE** command is used to assign the **BINARY** file, follow the instructions above to include a "." after the unit number on the **FILE** command. For example, the binary file **DUMP.DAT** can be assigned with:

```
FILE 11. DUMP.DAT
```

BYVAR Used when the data is to be read variable **BY VAR**iable rather than observation by observation. When this option is used, the data for each new variable must begin on a new line. If this option is not used, the data for each new observation must begin on a new line. Observations deleted by the **SKIPIF** command or the expanded form of the **SAMPLE** command are included when this option is used.

CLOSE **CLOSE**s a file that was opened for the **READ** or **WRITE** command. Most operating systems do not require that files be closed, but sometimes it is desirable to do so to free up memory or a unit number. If this option is used, the file will no longer be assigned to the unit number as specified with the **FILE** command.

DIF Reads data from a DIF file. DIF files can be created by spreadsheet programs. Refer to the section on *USING DIF FILES* in this chapter. This option is automatically in effect for filenames with the **.DIF** extension.

EOF Forces SHAZAM to read to the **E**nd **O**f the data **F**ile regardless of the **SAMPLE** command in effect. This is the default if no **SAMPLE** command has been previously specified.

FORMAT Data will be read in according to the **FORMAT** previously specified on the **FORMAT** command. The format will be stored in a SHAZAM variable called **FORMAT**. Details on the **FORMAT** command are given later in this chapter.

LIST **LIST**s all data read. This option is equivalent to a **READ** command together with a **PRINT** command. Details on **PRINT** are given below.

NAMES Specifies that the variable names are included as the first line in the data set. When this option is used the list of variable names is not required on the **READ** command. The first line of the data set cannot exceed 130 columns. A line terminated with & indicates that the next line is a continuation line.

REWIND/ NOREWIND By default the **READ** command loads the data set starting at the first record of the data file. If multiple **READ** commands are used to load data sequentially from a data file, then the **NOREWIND** option must be specified.

BEG=, END= Sets a sample range which overrides the sample size set on the latest **SAMPLE** command. The sample size set by these options will only be used for the current **READ** command. Subsequent problems will be performed according to the sample range set by the latest **SAMPLE** command.

CHARVARS= Specifies the number of character variables. This option allows the input of character variables without using the **FORMAT** option. The character variables must be the leading variables in the data set. The data is read by recognizing blanks or commas as delimiters. If the character variable contains embedded blanks then the variable must be enclosed by quotes " ".

ROWS=, COLS= Specifies the number of **ROWS** and **COL**umn**S** when a matrix is being read in. Only one matrix can be read in on a given **READ** command and if a matrix is being read in, no other variables may be read in on that **READ** command. **SKIPIF** commands are not in effect when a matrix is read. The beginning observation on the **SAMPLE** command is used as the first row number for the matrix. Use the option **BEG=1** to ensure that the matrix begins at row 1.

SKIPLINES= **SKIP**s the number of **LINES** specified before reading in any data. It is used if a data file contains a certain number of lines such as labels or comments which need to be skipped before reading in the data.

Reading from Several Data Files

If you use **READ** commands to read from several files, the command file might look like:

```
sample beg1 end1
read(data1) vars1
sample beg2 end2
read(data2) vars2
sample beg3 end3
read(data3) vars3
```

When the second **READ** command is encountered the first data file is closed. When a filename is specified on the **READ** or **WRITE** command the file is closed when another **READ** or **WRITE** command specifies a filename. If a unit number is specified then the files will stay assigned to each unit as shown in the next list of SHAZAM commands.

```
file 11 data1
file 12 data2
file 13 data3
sample beg1 end1
read(11) vars1
sample beg2 end2
read(12) vars2
sample beg3 end3
read(13) vars3
```

This method uses up three input-output units. On some computer systems there may be a limit on the number of open data files.

Using Microsoft Excel Files

To import data from other data sources such as Microsoft Excel files (.xls or .xlsx) please use the SHAZAM Environment to open the file directly into the Data Editor and then either save it as SHAZAM Data or add it to your current Workspace.

Once the file has been added to the current workspace it can be set to be automatically read using the AUTOmaticREAD facility. This facility is useful for reading all the data from a dataset but requires variable names be placed on the first line of the data set.

The SHAZAM Data Editor offers the ability to rename single or multiple variables.

THE PRINT COMMAND

The **PRINT** command is used to list variables on the screen or the SHAZAM output file. Note that this command does not direct output to the printer. The **PRINT** command has the following format:

PRINT *vars* / ***OPTIONS***

where *vars* is a list of variable names and ***OPTIONS*** is a list of desired options. The available options on the **PRINT** command are:

BYVAR	Specified variables will be printed variable **BY VAR**iable rather than observation by observation. If only one variable is printed this option is automatically in effect. It can be turned off with **NOBYVAR**. If **BYVAR** is specified, observations that have been omitted either with **SKIPIF** command or the expanded form (an example is given later in this chapter) of the **SAMPLE** command will also be printed. The **SET BYVAR** command will make **BYVAR** the default for the **PRINT** and **WRITE** commands.
FORMAT	The data will be written according to the format previously specified on the **FORMAT** command. The format will be stored in a variable called **FORMAT**. Details on the **FORMAT** command are given later in this chapter.
NONAMES	Omits printing the heading of variable **NAMES**.
WIDE/ NOWIDE	**WIDE** uses 120 columns and **NOWIDE** uses 80 columns. The default setting is described in the chapter *SET AND DISPLAY*.
BEG=, END=	Only the observations within the range specified by **BEG=** and **END=** will be printed. If these are not specified, the range from the **SAMPLE** command is used.

THE WRITE COMMAND

Selected data from a SHAZAM run can be written to a file by using the **WRITE** command. The **WRITE** command has the format:

WRITE*(filename) vars* / ***OPTIONS***

or

FILE ***unit filename***
WRITE(***unit***) ***vars*** / ***OPTIONS***

where *unit* is a unit number (assigned to a file with an operating system command or the SHAZAM **FILE** command), *filename* is the name of the data file, *vars* is a list of variable names and *options* is a list of desired options. The available units for writing data are 11-49. The available options on the **WRITE** command are similar to those for the **PRINT** and **READ** commands, that is, **BYVAR**, **CLOSE**, **DIF**, **FORMAT**, **NAMES**, **WIDE**, **BEG=**, **END=**. Additional options are:

APPEND Append the data to the existing data file. If this option is not used then if the data file exists it will be replaced with the new data.

BINARY Specified variables will be written into the specified file in Double Precision **BINARY**. If the **FILE** command is used to assign the **BINARY** file, follow the instructions for the **FILE** command in this chapter and be sure to include a decimal point (.) after the unit number.

DIF Writes data to a DIF file. Refer to the section on *USING DIF FILES* in this chapter.

REWIND/ NOREWIND When multiple **WRITE** commands are used the data file will be overwritten by each **WRITE** command. That is, the **REWIND** option is the default. The **NOREWIND** option will write data sequentially to a file that has been used in a previous **WRITE** command.

NOTE: The default on the **WRITE** command is **NONAMES**. Therefore, if a heading of variable names is desired the **NAMES** option should be used.

An example of the use of the **WRITE** command to write data to an output file is:

```
read(theil.dat) year consume income price
genr lncon=log(consume)
genr lninc=log(income)
genr lnpr=log(price)
write(mydata) year lncon lninc lnpr
```

EXAMPLES

The next examples illustrate the use of options on **READ** and **PRINT** commands. The following is a listing of the Theil textile data set typed observation by observation. The variables are, from left to right, *YEAR*, *CONSUME*, *INCOME* and *PRICE*.

```
1923  99.2  96.7 101.0
1924  99.0  98.1 100.1
1925 100.0 100.0 100.0
1926 111.6 104.9  90.6
1927 122.2 104.9  86.5
1928 117.6 109.5  89.7
1929 121.1 110.8  90.6
1930 136.0 112.3  82.8
1931 154.2 109.3  70.1
1932 153.6 105.3  65.4
1933 158.5 101.7  61.3
1934 140.6  95.4  62.5
1935 136.2  96.4  63.6
1936 168.0  97.6  52.6
1937 154.3 102.4  59.7
1938 149.0 101.6  59.5
1939 165.5 103.8  61.3
```

Suppose that the file **MYDATA** contains the data set. To input this data, all that is needed is a **SAMPLE** command and a **READ** command:

```
SAMPLE 1 17
READ(MYDATA) YEAR CONSUME INCOME PRICE
```

To print all or some of the variables the **PRINT** command is used. The sample size can be reset as many times as desired within a run. The sample size in effect will be the last one specified. This allows extensive manipulation of data. For example, in the following SHAZAM output the sample size is set twice, first to read the entire data file with the **READ** command, and then to print a subset of observations with the **PRINT** command.

```
|_SAMPLE 1 17
|_READ(MYDATA) YEAR CONSUME INCOME PRICE
|_PRINT YEAR CONSUME INCOME PRICE
     YEAR           CONSUME         INCOME           PRICE
   1923.000        99.20000        96.70000        101.0000
   1924.000        99.00000        98.10000        100.1000
   1925.000        100.0000        100.0000        100.0000
   1926.000        111.6000        104.9000        90.60000
   1927.000        122.2000        104.9000        86.50000
   1928.000        117.6000        109.5000        89.70000
   1929.000        121.1000        110.8000        90.60000
   1930.000        136.0000        112.3000        82.80000
   1931.000        154.2000        109.3000        70.10000
   1932.000        153.6000        105.3000        65.40000
   1933.000        158.5000        101.7000        61.30000
   1934.000        140.6000        95.40000        62.50000
   1935.000        136.2000        96.40000        63.60000
   1936.000        168.0000        97.60000        52.60000
   1937.000        154.3000        102.4000        59.70000
   1938.000        149.0000        101.6000        59.50000
   1939.000        165.5000        103.8000        61.30000
|_SAMPLE 12 17
|_PRINT YEAR CONSUME INCOME PRICE
     YEAR           CONSUME            INCOME          PRICE
   1934.000        140.6000        95.40000        62.50000
   1935.000        136.2000        96.40000        63.60000
   1936.000        168.0000        97.60000        52.60000
   1937.000        154.3000        102.4000        59.70000
   1938.000        149.0000        101.6000        59.50000
   1939.000        165.5000        103.8000        61.30000
|_PRINT YEAR INCOME / BEG=1 END=4
     YEAR           INCOME
   1923.000        96.70000
   1924.000        98.10000
   1925.000        100.0000
   1926.000        104.9000
```

An expanded form of the **SAMPLE** command can also be used to select portions from the larger sample. For example, to select observations 1 through 4 and 12 through 15, the following **SAMPLE** command would be used:

```
|_SAMPLE 1 4 12 15
|_PRINT YEAR CONSUME INCOME PRICE
     YEAR           CONSUME         INCOME          PRICE
   1923.000        99.20000     96.70000     101.0000
   1924.000        99.00000     98.10000     100.1000
   1925.000        100.0000     100.0000     100.0000
   1926.000        111.6000     104.9000     90.60000
   1934.000        140.6000     95.40000     62.50000
   1935.000        136.2000     96.40000     63.60000
   1936.000        168.0000     97.60000     52.60000
   1937.000        154.3000     102.4000     59.70000
```

It is possible to read in data from more than one file in a single SHAZAM run. If, for example, the Theil textile data were divided into two portions, 1923-1930 and 1931-1939, and each portion were in a different file, SHAZAM could not only read in both data files,

but could also combine the two files so the variables were complete. Since the maximum number of observations of a variable is set by the first **READ** command in the absence of a **SAMPLE** command, it is necessary to first create the variable with the **DIM** command. The **DIM** command will reserve enough space for all the observations from both data files. In this example **MYDATA1** is used for observations 1 to 8 and **MYDATA2** is used for observations 9 to 17:

```
|_DIM YEAR 17 CONSUME 17 INCOME 17 PRICE 17
|_READ(MYDATA1) YEAR CONSUME INCOME PRICE / BEG=1 END=8 LIST
   4 VARIABLES AND      8 OBSERVATIONS STARTING AT OBS     1
...SAMPLE RANGE IS NOW SET TO:         1         8
   1923.000       99.20000       96.70000       101.0000
   1924.000       99.00000       98.10000       100.1000
   1925.000       100.0000       100.0000       100.0000
   1926.000       111.6000       104.9000       90.60000
   1927.000       122.2000       104.9000       86.50000
   1928.000       117.6000       109.5000       89.70000
   1929.000       121.1000       110.8000       90.60000
   1930.000       136.0000       112.3000       82.80000
|_READ(MYDATA2) YEAR CONSUME INCOME PRICE / BEG=9 END=17 LIST
   4 VARIABLES AND      9 OBSERVATIONS STARTING AT OBS     9
   1931.000       154.2000       109.3000       70.10000
   1932.000       153.6000       105.3000       65.40000
   1933.000       158.5000       101.7000       61.30000
   1934.000       140.6000       95.40000       62.50000
   1935.000       136.2000       96.40000       63.60000
   1936.000       168.0000       97.60000       52.60000
   1937.000       154.3000       102.4000       59.70000
   1938.000       149.0000       101.6000       59.50000
   1939.000       165.5000       103.8000       61.30000
|_SAMPLE 1 17
|_PRINT YEAR CONSUME INCOME PRICE
     YEAR           CONSUME           INCOME          PRICE
   1923.000       99.20000       96.70000       101.0000
   1924.000       99.00000       98.10000       100.1000
   1925.000       100.0000       100.0000       100.0000
   1926.000       111.6000       104.9000       90.60000
   1927.000       122.2000       104.9000       86.50000
   1928.000       117.6000       109.5000       89.70000
   1929.000       121.1000       110.8000       90.60000
   1930.000       136.0000       112.3000       82.80000
   1931.000       154.2000       109.3000       70.10000
   1932.000       153.6000       105.3000       65.40000
   1933.000       158.5000       101.7000       61.30000
   1934.000       140.6000       95.40000       62.50000
   1935.000       136.2000       96.40000       63.60000
   1936.000       168.0000       97.60000       52.60000
   1937.000       154.3000       102.4000       59.70000
   1938.000       149.0000       101.6000       59.50000
   1939.000       165.5000       103.8000       61.30000
```

In the above example, the **DIM** command set the length of all four variables to 17. (For further details on the **DIM** command see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.) Then, using the **BEG=** and **END=** options, the variables *YEAR*,

CONSUME, INCOME and *PRICE* were read in from two data files, observations 1 to 8 are read in from **MYDATA1** and observations 9 to 17 are read in from **MYDATA2**.

READ *command with* BYVAR *option*

When the data is typed variable by variable the **BYVAR** option is needed on the **READ** command. The following is a listing of a file which contains the same data as that used above, but it is typed variable by variable rather than observation by observation:

```
1923  1924  1925  1926  1927  1928  1929  1930
1931  1932  1933  1934  1935  1936  1937  1938  1939
 99.2  99.0 100.0 111.6 122.2 117.6 121.1 136.0
154.2 153.6 158.5 140.6 136.2 168.0 154.3 149.0 165.5
 96.7  98.1 100.0 104.9 104.9 109.5 110.8 112.3
109.3 105.3 101.7  95.4  96.4  97.6 102.4 101.6 103.8
101.0 100.1 100.0  90.6  86.5  89.7  90.6  82.8
 70.1  65.4  61.3  62.5  63.6  52.6  59.7  59.5  61.3
```

Each variable may extend over more than one line. However, *new variables must start on a new line.* With the **BYVAR** option the **SAMPLE** command is especially important. SHAZAM needs the sample size in order to know when to stop reading observations into the first variable listed on the **READ** command and start reading them into the second, etc.

To input the above data a **SAMPLE** command and a **READ** command with the **BYVAR** option are needed:

```
sample 1 17
read(mydata) year consume income price / byvar
```

It is also possible to use the **BYVAR** option on the **PRINT** command. The variables will be printed horizontally by variable rather than in columns:

```
|_PRINT YEAR CONSUME / BYVAR
 YEAR
   1923.0   1924.0   1925.0   1926.0   1927.0   1928.0   1929.0   1930.0
   1931.0   1932.0   1933.0   1934.0   1935.0   1936.0   1937.0   1938.0
   1939.0
 CONSUME
    99.20    99.00   100.00   111.60   122.20   117.60   121.10   136.00
   154.20   153.60   158.50   140.60   136.20   168.00   154.30   149.00
   165.50
```

READ *command with* ROWS= *and* COLS= *option*

It is often useful to read data as a matrix. To read in a matrix, the **ROWS=** and **COLS=** options are needed on the **READ** command. If the original Theil textile data were all to be placed in a single matrix named *W*, the appropriate **READ** command would be:

```
read(mydata) w / rows=17 cols=4 beg=1
```

To print *W* the **PRINT** command is used:

```
|_PRINT W
 W
    17 BY      4 MATRIX
   1923.000        99.20000        96.70000        101.0000
   1924.000        99.00000        98.10000        100.1000
   1925.000        100.0000        100.0000        100.0000
   1926.000        111.6000        104.9000        90.60000
   1927.000        122.2000        104.9000        86.50000
   1928.000        117.6000        109.5000        89.70000
   1929.000        121.1000        110.8000        90.60000
   1930.000        136.0000        112.3000        82.80000
   1931.000        154.2000        109.3000        70.10000
   1932.000        153.6000        105.3000        65.40000
   1933.000        158.5000        101.7000        61.30000
   1934.000        140.6000        95.40000        62.50000
   1935.000        136.2000        96.40000        63.60000
   1936.000        168.0000        97.60000        52.60000
   1937.000        154.3000        102.4000        59.70000
   1938.000        149.0000        101.6000        59.50000
   1939.000        165.5000        103.8000        61.30000
```

It is also possible to print only one column of the matrix using the **PRINT** command with the column number of the matrix specified after a colon (:) as follows:

```
|_PRINT W:4
   101.0000  100.1000   100.0000   90.60000   86.50000
   89.70000  90.60000   82.80000   70.10000   65.40000
   61.30000  62.50000   63.60000   52.60000   59.70000
   59.50000  61.30000
```

The above **PRINT** command prints the data in the fourth column of the matrix *W*.

THE FORMAT *AND* NAMEFMT *COMMANDS*

The **FORMAT** command can precede a **READ**, **WRITE** or **PRINT** command that specifies the **FORMAT** option. For a **READ** / **FORMAT** command, the data file contains a sequence of formatted records. An individual record contains a list of variables that can contain

either character or numeric data. Each variable must be entered in a field with a fixed column width. That is, variable field widths are not allowed.

The **FORMAT** command gives the column positions of the data. The general format is:

FORMAT(*list***)**

where *list* contains edit descriptors of the form:

n**X**	advances the column position by n spaces. The maximum limit is 127X. To get more spaces use, for example, 127X,127X.
n**F**w.d	the field is w characters wide and contains a number such that d digits occur after the decimal point. The field is repeated n times. On a **READ** command fields that are all blanks are interpreted as zero values. Also, an explicit decimal point in the input field will override any specification of the decimal-point position given in the **FORMAT** command.
Aw	the field is w characters wide and contains a SHAZAM character variable. The maximum limit is A8. An example is given below.
Tn	Tab over to column n. That is, read from column n or write to column n.
/	skip to the next line.

The command line of a **FORMAT** statement must not exceed 248 characters. (Do not confuse this with the data file record length).

The **NAMEFMT** command can precede a **PRINT** / **FORMAT** command or a **WRITE** / **NAMES FORMAT** command. This command controls the layout of the header line of variable names. The general format is:

NAMEFMT(*list***)**

where *list* contains edit descriptors for the variable names.

Examples

As an example, the Theil textile data set can be loaded using a **FORMAT** command. The appropriate commands are a **SAMPLE** command, a **FORMAT** command and a **READ** command with the **FORMAT** option:

```
sample 1 17
format(f4.0,3f6.0)
read(mydata) year consume income price / format
```

The above **FORMAT** specifies to read the first variable in the first 4 columns, then read 3 variables of 6 columns each.

The next example uses a data set with the average stock prices in 1999 for a sample of six Toronto Stock Exchange companies. The first variable in the data set is a character variable containing the trading symbol and the second variable contains the average stock prices. In the SHAZAM commands below, the **FORMAT** command that precedes the **READ** command specifies an edit descriptor for each variable in the data set. The first edit descriptor `A8` specifies to read a character variable with a length of 8 characters. The second edit descriptor `F8.2` specifies to read numeric data with a field width of 8 characters and 2 digits follow the decimal point.

```
sample 1 6
format(a8,f8.2)
read symbol price / format
aec         41.66
cxy         23.27
dtc         13.48
nor         18.52
fsh         61.32
ry          69.05
format(1x,a8,f8.2)
namefmt(1x,a8,3x,a8)
print symbol price / format
```

The above command file also shows the use of a **FORMAT** and **NAMEFMT** command to control the output from a **PRINT** command that specifies the **FORMAT** option. The **FORMAT** command starts with the edit descriptor `1X`. For a **PRINT** command this may be required since the first column may be reserved as a control character. The output from the above command file is shown below.

```
|_SAMPLE 1 6
|_FORMAT(A8,F8.2)
|_READ SYMBOL PRICE / FORMAT
 READ USES FORMAT:(A8,F8.2)
|_FORMAT(1X,A8,F8.2)
|_NAMEFMT(1X,A8,3X,A8)
|_PRINT SYMBOL PRICE / FORMAT

SYMBOL      PRICE
AEC         41.66
CXY         23.27
DTC         13.48
NOR         18.52
FSH         61.32
RY          69.05
```

The next example works with a data set on the unemployment and vacancy rates of nine Canadian provinces for January 1976 as provided by Statistics Canada. Data on the 10th province, Prince Edward Island, was not available. The province name spans the first 16 characters of the data set. A SHAZAM character variable cannot exceed 8 characters. For longer names, two or more character variables can be created. In this example, the province name is assigned to the variable names *PROV1* and *PROV2*. The **FORMAT** command uses the edit descriptor `2A8` to load these character variables. The unemployment and vacancy rates are loaded in the variable names *UR* and *VR*, respectively. The **FORMAT** command specifies that each of these variables contains numeric data with a field width of 5 characters.

```
sample 1 9
format(2a8,2f5.1)
read prov1 prov2 ur vr / format
Newfoundland     14.9  4.0
Nova Scotia       9.1  5.0
New Brunswick    12.2  7.0
Quebec            9.1  6.0
Ontario           7.1  5.0
Manitoba          6.7  8.0
Saskatchewan      4.8  8.0
Alberta           5.3 11.0
British Columbia 10.0  4.0
format(1x,2a8,2f5.1)
print prov1 prov2 ur vr / format nonames
```

The SHAZAM output from this example is:

```
|_SAMPLE 1 9
|_FORMAT(2A8,2F5.1)
|_READ PROV1 PROV2 UR VR / FORMAT
 READ USES FORMAT:(2A8,2F5.1)
```

```
|_FORMAT(1X,2A8,2F5.1)
|_PRINT PROV1 PROV2 UR VR / FORMAT NONAMES
Newfoundland     14.9  4.0
Nova Scotia       9.1  5.0
New Brunswick    12.2  7.0
Quebec            9.1  6.0
Ontario           7.1  5.0
Manitoba          6.7  8.0
Saskatchewan      4.8  8.0
Alberta           5.3 11.0
British Columbia 10.0  4.0
```

USING DIF FILES

The DIF (Data Interchange Format) file format provides a standard for transferring files between programs. A DIF file typically contains information about the variable names, number of variables and number of observations. A spreadsheet file can be converted to DIF format by opening the spreadsheet and selecting the DIF file format from the Save As dialog box in the File menu. The **READ** command in SHAZAM can load DIF files.

When data is prepared in a spreadsheet program to read into SHAZAM it is best to set up the data as a rectangular matrix where each column of the matrix corresponds to a SHAZAM variable and each row of the matrix corresponds to an observation in SHAZAM. Hence, if your data contained 11 variables and 37 observations your spreadsheet program would show 11 columns and 37 rows. If the first row of the spreadsheet contains the names of the variables then the spreadsheet would have 11 columns and 38 rows. When variable names are included you should be careful that they correspond to SHAZAM variable name rules or SHAZAM may not be able to access the data properly.

Assume you have created a DIF file from another program (or from SHAZAM using the **WRITE** command with the **DIF** option). Suppose the file is called **MYDATA.DIF** and no names have been included in the first row of the spreadsheet and there are 17 rows and 3 columns. To assign the variable names *YEAR, CONSUME* and *INCOME,* the appropriate **READ** command is:

```
read(mydata.dif) year consume income / dif
```

If variable names are included in the DIF file, then you must not include them on the **READ** command. The data set is loaded and variable names are assigned with the command:

```
read(mydata.dif) / dif
```

The above two examples assume that the data file **MYDATA.DIF** is located in the same directory/folder as the SHAZAM command file. If the data file is located in a separate directory/folder called **ECONDAT** then, for the Microsoft Windows version of SHAZAM, the above **READ** commands would be:

```
read(c:\econdat\mydata.dif) year consume income / dif
 or
read(c:\econdat\mydata.dif) / dif
```

It is assumed that the hard disk on the personal computer is called "C". If the hard disk is called something other than "C" then replace it with the appropriate name.

It is extremely important to remember that you must specify exactly where the data file is located in your **READ** command. Otherwise, SHAZAM will not know where to look for the file and there will be an error.

The number of observations for each variable must be identical in the DIF file. This means that you cannot have any missing observations for any of the variables. If missing observations exist in the data file then you must enter a numeric missing value code. The SHAZAM default missing data code is –99999.

In some cases you may find that your data file has to be cleaned up a bit in the spreadsheet program before a proper DIF file can be produced. For example, you may find that your spreadsheet program has produced a large number of blank rows and columns. It may be necessary to delete these blank rows and columns first. It is required that an equal number of rows are present for each column in the spreadsheet. Some spreadsheet programs actually transpose the data matrix when writing a DIF file. If this happens it may be necessary to perform some data manipulations using SHAZAM **MATRIX** commands to transpose the data into the proper form.

SHAZAM can also create a DIF file for use in another program. For example, if your SHAZAM run currently has the variables *YEAR, PRICE* and *INCOME* and you wish to create a DIF file with these three variables you would need the following commands:

```
write(newfile.dif) year price income / dif names
```

If you did not wish to include the variable names in the DIF file then the **NAMES** option would not be used. In order to learn how to use DIF files, it is often easier to first **WRITE** a DIF file in SHAZAM and then examine it and **READ** it back into SHAZAM. If you are successful in this exercise you will understand the DIF file process.

4. DESCRIPTIVE STATISTICS

"Jupiter's moons are invisible to the naked eye and therefore can have no influence on the earth, and therefore would be useless, and therefore do not exist."

Francisco Sizzi
Professor of Astronomy, 1610

The **STAT** command computes descriptive statistics.

Mean, Variance and Other Statistics

For a variable X with N observations denoted by X_t for t = 1, . . . , N the statistics listed by the **STAT** command are calculated as:

Mean $\quad \bar{X} = \frac{1}{N}\sum_{t=1}^{N} X_t$

Variance $\quad \hat{\sigma}_X^2 = \frac{1}{N-1}\sum_{t=1}^{N}\left(X_t - \bar{X}\right)^2 = \frac{1}{N-1}\left[\sum_{t=1}^{N} X_t^2 - N\bar{X}^2\right]$

Standard deviation $\quad \hat{\sigma}_X = \sqrt{\hat{\sigma}_X^2}$

Minimum $\quad$ Min = the smallest value of X_t

Maximum $\quad$ Max = the largest value of X_t

When the **WIDE** option is specified the output also reports:

Coefficient of variation $\quad \hat{\sigma}_X / \bar{X}$

Constant digits statistic $\quad = \begin{cases} -\log_{10}\left(R / |\text{Min}|\right) & \text{if } R < |\text{Min}| \text{ and } \text{Min} \neq \text{Max} \\ 0 & \text{otherwise} \end{cases}$

where R=(Max–Min) is the sample range. The fraction R/|Min| gives the relative change from the smallest to the largest value. For discussion see Simon and Lesage [1989].

When the **PMEDIAN** option is specified the output reports the median, the mode and quartiles. Let $X_{(1)}, X_{(2)}, \ldots, X_{(N)}$ be the observations that are ordered from smallest to largest. The median is:

$$\begin{cases} X_{((N+1)/2)} & \text{if } N \text{ is odd} \\ \text{the average of } X_{(N/2)} \text{ and } X_{((N+2)/2)} & \text{if } N \text{ is even} \end{cases}$$

The mode is the most frequently occurring value in the set of observations. The quartiles are obtained as:

first quartile (lower 25%)	$X_{((N+1)/4)}$
second quartile	the median
third quartile (upper 25%)	$X_{(3(N+1)/4)}$
interquartile range	difference between the third and first quartiles (a measure of dispersion).

Interpolation is used to compute the quartiles when (N+1)/4 is not an integer. Discussion and examples are given in Newbold [1995, Chapter 2].

The **PFREQ** option reports frequencies, relative frequencies and cumulative relative frequencies. For the ordered data the frequency f_t is the number of occurrences of $X_{(t)}$. The relative frequency is f_t/N and the cumulative relative frequency is $(f_1+\ldots+f_t)/N$. Note that the frequency distribution for grouped data is obtained with a histogram display that is available with the **HISTO** option on the **GRAPH** command (see the chapter *PLOTS AND GRAPHS*).

Covariance and Correlation

Statistics involving relationships between variables are also produced with the **STAT** command. Consider K variables such that the k^{th} variable has observations X_{kt} for t = 1,. . .,N with mean $\overline{X}_k$ and standard deviation $\hat{\sigma}_{x_k}$ for k = 1, . . ., K. The cross-products (printed with the **PCP** option) are:

$$\sum_{t=1}^{N} X_{it} X_{jt} \qquad \text{for } i,j = 1, \ldots, K$$

The cross-products of deviations about the mean (printed with the **PCPDEV** option) are:

$$\sum_{t=1}^{N}(X_{it} - \overline{X}_i)(X_{jt} - \overline{X}_j) \qquad \text{for } i,j = 1, \ldots, K$$

The covariances (printed with the **PCOV** option and saved with the **COV=** option) are:

$$\text{Cov}\,(X_i, X_j) = \frac{1}{N-1}\sum_{t=1}^{N}(X_{it} - \overline{X}_i)(X_{jt} - \overline{X}_j) = \frac{1}{N-1}\left[\sum_{t=1}^{N} X_{it}X_{jt} - N\overline{X}_i\overline{X}_j\right]$$

The correlations (printed with the **PCOR** option and saved with the **COR=** option) are:

$$\text{Cor}\,(X_i, X_j) = \text{Cov}\,(X_i, X_j) / \hat{\sigma}_{x_i}\hat{\sigma}_{x_j}$$

The correlation coefficients may be seriously affected by extreme outliers. An alternative measure of correlation that is not as sensitive to extreme values is based on ranks. Let S_{it} be the rank of X_{it} where the ranking is in ascending order of values. Spearman's rank correlation coefficients are calculated as:

$$r_s(X_i, X_j) = 1 - \frac{6}{N(N^2 - 1)}\sum_{t=1}^{N}(S_{it} - S_{jt})^2$$

The **PRANKCOR** option computes the Spearman rank correlation coefficients. For further details see Gujarati [1995, p. 88 and p. 372], Newbold [1995, p. 436] or Yule and Kendall [1953, p. 455].

Testing for Equality of Mean and Variance

The **ANOVA** option constructs an analysis of variance table that gives a framework for testing for equality of population means (see, for example, Newbold [1995, pp. 598-605]). An assumption is that the random samples are independent. Consider K variables such that the k^{th} variable has observations X_{kt} for $t = 1, \ldots, n_k$. That is, the design allows for unequal sample sizes. Define:

$$N = \sum_{k=1}^{K} n_k \quad , \qquad \overline{X}_k = \frac{1}{n_k}\sum_{t=1}^{n_k} X_{kt} \qquad \text{and} \qquad \overline{X} = \frac{1}{N}\sum_{k=1}^{K} n_k\overline{X}_k$$

The analysis of variance (ANOVA) table is constructed as:

	SS	DF	MS	F
BETWEEN	$SSG = \sum_{k=1}^{K} n_k (\overline{X}_k - \overline{X})^2$	$K - 1$	$\frac{SSG}{K-1}$	$\frac{SSG \,/ K-1}{SSW \,/ N-K}$
WITHIN	$SSW = \sum_{k=1}^{K} \sum_{t=1}^{n_k} (X_{kt} - \overline{X}_k)^2$	$N - K$	$\frac{SSW}{N-K}$	
TOTAL	$SST = \sum_{k=1}^{K} \sum_{t=1}^{n_k} (X_{kt} - \overline{X})^2$	$N - 1$	$\frac{SST}{N-1}$	

The null hypothesis of equal means is rejected if the F statistic exceeds the critical value from an F distribution with (K–1, N–K) degrees of freedom.

For a sample with equal observations, a two-way analysis of variance can be constructed using the method discussed in Newbold [1995, pp. 618-9]. Denote $H = n_k$ (k = 1, . . . , K) and consider X_{kt} as the observation for group k and block t. Define the sample mean for block t (t = 1, . . . , H) by:

$$\overline{X}_{\bullet t} = \frac{1}{K} \sum_{k=1}^{K} X_{kt}$$

The calculations for the two-way analysis of variance are:

	SS	DF	MS	F
GROUPS	$SSG = H \sum_{k=1}^{K} (\overline{X}_k - \overline{X})^2$	$K - 1$	$\frac{SSG}{K-1}$	$\frac{SSG \,/ K-1}{SSE \,/ (K-1)(H-1)}$
BLOCKS	$SSB = K \sum_{t=1}^{H} (\overline{X}_{\bullet t} - \overline{X})^2$	$H - 1$	$\frac{SSB}{H-1}$	$\frac{SSB \,/ H-1}{SSE \,/ (K-1)(H-1)}$
ERROR	$SSE = SST–SSG–SSB$	$(K–1)(H–1)$	$\frac{SSE}{(K-1)(H-1)}$	
TOTAL	$SST = \sum_{k=1}^{K} \sum_{t=1}^{H} (X_{kt} - \overline{X})^2$	$N - 1$	$\frac{SST}{N-1}$	

The F statistics give tests for the null hypothesis that the K population group means are the same and the null hypothesis that the H population block means are the same.

The **BARTLETT** option computes Bartlett's statistic for testing for equality of population variance (see, for example, Judge, Griffiths, Hill, Lütkepohl and Lee [1985, p. 448]). The sample variances are:

$$\hat{\sigma}_k^2 = \frac{1}{(n_k - 1)} \sum_{t=1}^{n_k} (X_{kt} - \overline{X}_k)^2 \qquad \text{and} \qquad \hat{\sigma}^2 = \frac{1}{(N - K)} \sum_{k=1}^{K} (n_k - 1)\hat{\sigma}_k^2$$

A statistic for testing for equality of variance is: $u = \sum_{k=1}^{K} \left(\hat{\sigma}_k^2 / \hat{\sigma}^2\right)^{n_k/2}$

Bartlett's test statistic is based on a modification of $-2\ln(u)$ and is constructed as:

$$M = \frac{(N - K)\ln \hat{\sigma}^2 - \sum_{k=1}^{K} (n_k - 1)\ln \hat{\sigma}_k^2}{1 + \frac{1}{3(K-1)}\left[\sum_{k=1}^{K} \frac{1}{(n_k - 1)} - \frac{1}{(N - K)}\right]}$$

There is evidence to reject the null hypothesis of equal variances if the test statistic M exceeds the critical value from a χ^2 distribution with (K–1) degrees of freedom. This is an approximate test. The derivation of exact critical values for the case of equal sample sizes is given in Dyer and Keating [1980].

When only two variables are specified on the **STAT** command the **ANOVA** option lists some additional test statistics. Assume two independent random samples. To test for equality of population means a test statistic (see Newbold [1995, pp. 355-6]) is:

APPROXIMATE T-TEST OF EQUAL MEANS $\qquad t_A = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\hat{\sigma}_1^2 / n_1 + \hat{\sigma}_2^2 / n_2}}$

If $|t_A|$ exceeds the critical value from a standard normal distribution then there is evidence for different population means. Newbold [1995, p. 356] suggests that this test is a good approximation for sample sizes of at least 30.

With the assumption of equal population variances, a test statistic for equality of population means (see Newbold [1995, pp. 357-8]) is:

EQUAL VARIANCE T-TEST OF EQUAL MEANS

$$t_B = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\hat{\sigma}^2 (n_1 + n_2)/(n_1 n_2)}}$$

The null hypothesis of equal means is rejected if $|t_B|$ exceeds the critical value from a t-distribution with (n_1+n_2-2) degrees of freedom. Note that when $n_1=n_2$ the test statistics t_A and t_B are identical. Also, it can be shown that $t_B^2 = F$ where F is the F statistic from the one-way ANOVA table.

In the case where the random sample can be viewed as H matched pairs of observations a test statistic for equal population means (see Newbold [1995, p. 353]) is:

$$t_c = (\overline{X}_1 - \overline{X}_2) \Big/ \sqrt{\frac{1}{(H-1)H} \sum_{t=1}^{H} \left(X_{1t} - X_{2t} - (\overline{X}_1 - \overline{X}_2)\right)^2}$$

It can be shown that $t_c^2 = F$ where F is the F statistic from the two-way ANOVA table.

A statistic for testing for equality of variances from two independent normal populations (see Newbold [1995, p. 367]) is:

F-TEST OF EQUAL VARIANCES

$$\begin{cases} \hat{\sigma}_1^2 / \hat{\sigma}_2^2 & \text{if } \hat{\sigma}_1^2 > \hat{\sigma}_2^2 \quad (\text{df1} = n_1 - 1;\ \text{df2} = n_2 - 1) \\ \hat{\sigma}_2^2 / \hat{\sigma}_1^2 & \text{if } \hat{\sigma}_2^2 > \hat{\sigma}_1^2 \quad (\text{df1} = n_2 - 1;\ \text{df2} = n_1 - 1) \end{cases}$$

The null hypothesis of equal variances is rejected in favour of the alternative hypothesis of larger variance for the numerator population if the statistic exceeds the critical value from an F distribution with numerator degrees of freedom df1 and denominator degrees of freedom df2.

STAT *COMMAND OPTIONS*

In general, the format of the **STAT** command is:

STAT *vars* / *options*

where *vars* is a list of variable names and *options* is a list of desired options. The available options on the **STAT** command are:

ALL Statistics are computed for **ALL** the variables in the data. Therefore, no variable names need be specified.

ANOVA Prints an **AN**alysis **O**f **VA**riance table and an F-value that tests the null hypothesis that the means of all the variables listed on the given **STAT** command are the same. For equal sample sizes a two-way analysis of variance is constructed. When two variables are specified, additional test statistics for equal means and variances are reported. An **ANOVA** example is given later in this chapter.

BARTLETT Computes **BARTLETT**'s homogeneity of variance test statistic to test the hypothesis that the variances of all the variables listed are equal. An example of this option is given later in this chapter.

DN Uses N (number of observations) as a divisor rather than N–1 when computing variances and covariances.

MATRIX Any **MATRIX** or matrices contained in the list *vars* will be treated as a single variable if this option is used. If this option is not specified SHAZAM will treat each column of the matrix as a separate variable.

MAX Prints all the output of the **PCOR**, **PCOV**, **PCP**, **PCPDEV** and **PRANKCOR** options.

PCOR **P**rints a **COR**relation matrix of the variables listed.

PCOV **P**rints a **COV**ariance matrix of the variables listed.

PCP **P**rints a **C**ross**P**roduct matrix of the variables listed.

PCPDEV **P**rints a **C**ross**P**roduct matrix of the variables listed in **DEV**iations from the means.

PFREQ **P**rints a table of **FREQ**uencies of occurrence for each observed value in the data. Also prints the median, the mode and quartiles. This option is ***not*** recommended for large sample sizes with many different possible values since pages and pages of output would result.

PMEDIAN **P**rints the **MEDIAN**, mode and quartiles for each variable. This option could be time-consuming when the sample size is large.

PRANKCOR **P**rints a matrix of Spearman's **RANK COR**relation coefficients.

REPLICATE Used with **WEIGHT=** when the weights indicate a sample replication factor. The **REPLICATE** option is not available with **PFREQ**, **PMEDIAN**, **MEDIANS=** or **MODES=**.

SAMEOBS Restrict the sample to ensure that all observations for all variables are non-missing. This option reports descriptive statistics for the sample used in an **OLS** or other estimation command. The **SET SKIPMISS** command must be in effect.

SAMPSIZE Calculates the sample size required to obtain a 95% confidence interval with width 2e where e is a margin of error with values ranging from 0.01 to 0.10. The population size can be specified with the **NPOP=** option. The method is described in Lohr [1999, p. 40].

WIDE Requires 120 columns of output. When this option is used the coefficient of variation and the constant digits statistic will be printed in addition to the regular descriptive statistics.

BEG=, END= Specifies the **BEG**inning and **END** observations to be used in the **STAT** command.

COR= Stores the **COR**relation matrix in the variable specified.

COV= Stores the **COV**ariance matrix in the variable specified.

CP= Stores the **C**ross**P**roduct matrix in the variable specified.

CPDEV= Stores the **C**ross**P**roduct matrix in **DEV**iations from the mean in the variable specified.

MAXIM= Stores the **MAXIM**ums as a vector in the variable specified.

MEAN= Stores the **MEAN**s as a vector in the variable specified.

MEDIANS= Stores the **MEDIANS** as a vector in the variable specified.

MINIM= Stores the **MINIM**ums as a vector in the variable specified.

MODES= Stores the **MODES** as a vector in the variable specified. If there are multiple modes then the largest mode is saved. If there are no repeat values then the value saved is the maximum value.

NPOP= Specifies the population size to use with the **SAMPSIZE** option.

RANKCOR= Stores the Spearman's **RANK COR**relation matrix in the variable specified (see the **PRANKCOR** option above).

STDEV= Stores the **ST**andard **DEV**iations as a vector in the variable specified.

STEMPLOT= Specifies the number of digits in the stem (usually 1 or 2) for a stem-and-leaf display of the data. The method is described in Newbold, Carlson and Thorne [2003, pp. 19-22].

SUMS= Stores the sum of each variable as a vector in the variable specified.

VAR= Stores the **VAR**iances as a vector in the variable specified. Note that when the **DN** option is used the divisor is N instead of N–1.

WEIGHT= Specifies a variable to be used as a **WEIGHT** if weighted descriptive statistics are desired. Consider K variables such that the k^{th} variable has observations X_{kt} for t = 1, . . . , N and k = 1, . . . , K. Let W_t be the weights. The weighted statistics are:

Mean $$\bar{X}^{w}_{k} = \frac{1}{W^{*}} \sum_{t=1}^{N} W_t X_{kt} \qquad \text{where} \quad W^{*} = \sum_{t=1}^{N} W_t$$

Variance $$\hat{\sigma}^{2}_{x_k w} = \frac{c}{W^{*}} \cdot \left[\sum_{t=1}^{N} W_t X^{2}_{kt} - W^{*} \cdot (\bar{X}^{w}_{k})^{2} \right]$$

where c = N/(N–1). When the **DN** option is used then c = 1. When the **REPLICATE** option is used then $c = W^{*}/(W^{*} - 1)$. The **REPLICATE** option is not effective when **DN** is specified. The weighted covariances and correlations are computed as:

$$\text{Cov}\left(X_i, X_j\right)_w = \frac{c}{W^{*}} \cdot \left[\sum_{t=1}^{N} W_t X_{it} X_{jt} - W^{*} \cdot \bar{X}^{w}_{i} \bar{X}^{w}_{j} \right]$$

$$\text{Cor}\left(X_i, X_j\right)_w = \text{Cov}\left(X_i, X_j\right)_w / \hat{\sigma}_{x_i w}\hat{\sigma}_{x_j w}$$

It is important to note that options beginning with **P** will merely print results on the output file. Options ending in an equal sign (=) are used to store the matrix or vector in the variable specified or to specify input requirements.

EXAMPLES

The examples below use Theil's textile data to illustrate some options on the **STAT** command. The output for the **STAT** command with the **PCOR**, **PCOV** and **WIDE** options is:

```
 |_STAT CONSUME INCOME PRICE / PCOR PCOV WIDE
 NAME         N    MEAN         ST. DEV      VARIANCE     MINIMUM      MAXIMUM
 CONSUME     17    134.51         23.577        555.89        99.000       168.00
 INCOME      17    102.98         5.3010        28.100        95.400       112.30
 PRICE       17    76.312         16.866        284.47        52.600       101.00

COEF.OF.VARIATION CONSTANT-DIGITS
      .17529        .15679
      .51475E-01    .75166
      .22102        .36140E-01

   CORRELATION MATRIX OF VARIABLES -       17 OBSERVATIONS

 CONSUME    1.0000
 INCOME      .61769E-01   1.0000
 PRICE      -.94664       .17885       1.0000
              CONSUME      INCOME       PRICE

  COVARIANCE MATRIX OF VARIABLES -       17 OBSERVATIONS

 CONSUME    555.89
 INCOME     7.7201        28.100
 PRICE     -376.44        15.990       284.47
              CONSUME      INCOME       PRICE
```

The next example shows the use of the **STDEV=** and **COR=** options. The standard deviations of the variables *CONSUME, INCOME* and *PRICE* are stored in a variable called *SD*. The correlation matrix is stored in a variable called *CMATRIX*. These variables are then available for future computations.

```
 |_STAT CONSUME INCOME PRICE / STDEV=SD COR=CMATRIX
 NAME         N    MEAN         ST. DEV      VARIANCE     MINIMUM      MAXIMUM
 CONSUME     17    134.51         23.577        555.89        99.000       168.00
 INCOME      17    102.98         5.3010        28.100        95.400       112.30
 PRICE       17    76.312         16.866        284.47        52.600       101.00
```

```
|_PRINT SD CMATRIX
    SD
   23.57733       5.300972       16.86623
    CMATRIX
   1.000000
   .6176945E-01   1.000000
  -.9466377       .1788466       1.000000
```

The next example uses the fuel consumption data from Newbold [1995, p. 614]. The data set contains obervations on fuel consumption for three types of automobiles driven by drivers in six age classes. The output below shows the use of the **ANOVA** and **BARTLETT** options on the **STAT** command. Note that the **LIST** option on the **READ** command is used to obtain a listing of the fuel consumption data.

```
|_SAMPLE 1 6
|_READ A B C / LIST
   3 VARIABLES AND        6 OBSERVATIONS STARTING AT OBS       1

      A              B              C
   25.10000       23.90000       26.00000
   24.70000       23.70000       25.40000
   26.00000       24.40000       25.80000
   24.30000       23.30000       24.40000
   23.90000       23.60000       24.20000
   24.20000       24.50000       25.40000

|_STAT A B C / ANOVA BARTLETT
NAME         N    MEAN        ST. DEV      VARIANCE     MINIMUM      MAXIMUM
A            6   24.700     0.76158     0.58000       23.900       26.000
B            6   23.900     0.46904     0.22000       23.300       24.500
C            6   25.200     0.73756     0.54400       24.200       26.000

                 ANALYSIS OF VARIANCE - OVERALL MEAN=        24.600
                       SS          DF             MS             F    P-VALUE
BETWEEN            5.1600          2.         2.5800          5.7589     0.0139
WITHIN             6.7200         15.        0.44800
TOTAL              11.880         17.        0.69882

            TWO-WAY ANALYSIS OF VARIANCE - OVERALL MEAN=        24.600
                       SS          DF             MS             F    P-VALUE
GROUPS             5.1600          2.         2.5800          14.828     0.0010
BLOCKS             4.9800          5.        0.99600          5.7241     0.0095
ERROR              1.7400         10.        0.17400
TOTAL              11.880         17.        0.69882

 BARTLETTS HOMOGENEITY OF VARIANCE TEST =     1.1883
    APPROXIMATELY CHI-SQUARE WITH    2 DEGREES OF FREEDOM
```

The two-way analysis of variance table can be compared with Newbold [1995, Table 15.9, p. 621]. The above results show that Bartlett's test statistic is 1.1883. The 5% critical value from a chi-square distribution with 2 degrees of freedom is 5.99. The test statistic is less than the critical value and so the null hypothesis of equal population variances is not rejected.

The next example uses monthly seasonally adjusted data on the unemployment rate for Canada from 1980 to 1998. The SHAZAM output below reports tests for equal means and equal variances for the two sample periods January 1980 to December 1989 and January 1990 to December 1998.

```
|_SAMPLE 1 120
|_READ (URATE.DAT) DATE80 UR80
UNIT 88 IS NOW ASSIGNED TO: URATE.DAT
   2 VARIABLES AND      120 OBSERVATIONS STARTING AT OBS       1

|_SAMPLE 1 108
|_READ (URATE.DAT) DATE90 UR90 / NOREWIND
   2 VARIABLES AND      108 OBSERVATIONS STARTING AT OBS       1

|_SAMPLE 1 120
|_STAT UR80 UR90 / ANOVA
NAME         N    MEAN        ST. DEV      VARIANCE     MINIMUM      MAXIMUM
UR80         120   9.3533      1.7671      3.1225       6.9000       12.900
UR90         108   9.8000      1.1471      1.3159       7.2000       11.900

  DIFFERENCE BETWEEN TWO SAMPLES TESTS:    STATISTIC          D.F.      P-VALUE
APPROXIMATE    T-TEST OF EQUAL MEANS:       -2.2852             206    0.0233
EQUAL VARIANCE T-TEST OF EQUAL MEANS:       -2.2365             226    0.0263
F-TEST OF EQUAL VARIANCES:                   2.3729       119    107    0.0000

                ANALYSIS OF VARIANCE - OVERALL MEAN=        9.5649
                      SS          DF             MS                F    P-VALUE
BETWEEN            11.341          1.         11.341            5.0021     0.0263
WITHIN             512.38        226.         2.2672
TOTAL              523.72        227.         2.3071
```

In the next example the fuel consumption data is read in as a matrix *M*. If the **MATRIX** option is used on the **STAT** command the matrix variable will be treated as a single variable as shown in the following output.

```
 |_SAMPLE 1 6
 |_READ M / ROWS=6 COLS=3
       6 ROWS AND           3 COLUMNS, BEGINNING AT ROW       1
 |_STAT M / MATRIX
 NAME         N    MEAN        ST. DEV      VARIANCE     MINIMUM      MAXIMUM
 M             18   24.600     0.76080     0.57882       23.300       26.000
```

However, if the **MATRIX** option is not used, each column of the matrix specified will be treated as a single variable as shown in the output:

```
| _STAT M
NAME         N     MEAN         ST. DEV      VARIANCE     MINIMUM      MAXIMUM
...NOTE...TREATING COLUMNS OF M       AS VECTORS
M              6   24.700      0.76158      0.58000       23.900       26.000
M              6   23.900      0.46904      0.22000       23.300       24.500
M              6   25.200      0.73756      0.54400       24.200       26.000
```

5. PLOTS AND GRAPHS

"When the President does it, that means it is not illegal."
Richard Nixon
Former U.S. President, 1977

The **GRAPH** command provides an interface to the GNUPLOT program (Version 3.7 or later) for preparing graphs and histograms. GNUPLOT is a freely distributable command-driven plotting program developed by Thomas Williams, Colin Kelley and others. Information about gnuplot is available at the SHAZAM web site on the internet. When the **GRAPH** command is used the graph is displayed in a separate window. The **PLOT** command can be used to obtain plots that are printed with plain text characters on the SHAZAM output.

THE GRAPH COMMAND

In general, the format of the **GRAPH** command is:

GRAPH *depvars indep / options*

where *depvars* is one or more dependent variables to be plotted against a single independent variable, *indep*, and *options* is a list of desired options. Available options are:

APPEND Specifies that the gnuplot command file is to be appended to gnuplot command files created with previous commands in the SHAZAM program.

AXIS/ NOAXIS By default, an x-axis and y-axis is drawn at x=0 and y=0. Use **NOAXIS** to omit the axes.

AXISFMT Set the x-axis labels with the date format specified on the previous **AXISFMT** command. Description of the **AXISFMT** command is available later in this chapter. When this option is used the **TIMEFMT** option must also be specified.

HISTO Plots a **HISTO**gram for the variable specified on the **GRAPH** command. Only one variable can be specified. A few outliers may complicate the scaling of the histogram. Therefore, by default, the histogram is centered

around the mean with a scale of 3 standard deviations on either side of the mean. When the **RANGE** option is specified, the scale of the histogram is based on the entire range of the data. See also the **GROUPS=** option.

KEY/ NOKEY By default, a key is displayed in the upper right-hand corner. Use **NOKEY** to suppress the key.

LINE Used to draw a line connecting the data points and plot a symbol at each point.

LINEONLY Used to only draw a line connecting the data points. No symbols at the data points will be plotted.

RANGE Utilizes the entire **RANGE** of the data for plotting the histogram when the **HISTO** option is used.

TIME Plots the listed *depvars* sequentially against **TIME**. In this case, *indep* is not specified.

TIMEFMT Specifies that the *indep* variable on the **GRAPH** command is a time date variable with a format as given on the previous **TIMEFMT** command. Description of the **TIMEFMT** command is available later in this chapter.

BEG= , END= Sets a sample range for the particular **GRAPH** command. This sample size is in effect only for the **GRAPH** command with this option. If these options are not specified, the sample range for the current **SAMPLE** command is used.

COMMFILE= Gives the filename for the gnuplot command file (8 characters maximum).

DATAFILE= Gives the filename for the gnuplot data file (8 characters maximum).

DEVICE= Specifies the type of device for gnuplot output. The default is the terminal screen. The option **DEVICE=POSTSCRIPT** sends output to a PostScript file. The gnuplot command **set term** gives the complete list of valid options.

GROUPS= When the **HISTO** option is used SHAZAM normally places data into 6 groups. This option can be used to specify up to 60 groups. The values allowed for **GROUPS=** are 2, 3, 4, 5, 6, 10, 12, 15, 30 or 60.

OUTPUT= Gives the filename for the gnuplot output file (8 characters maximum) when the **DEVICE=** option is used. When **DEVICE=POSTSCRIPT** is used the default output filename is **`GNU.ps`**.

PORT= Use **PORT=NONE** to obtain gnuplot command and data files but no gnuplot output.

The **SET NOGRAPH** command suppresses the display of gnuplot graphs that are generated with the **GRAPH** command. The gnuplot command and data files with the extension **`.GNU`** will still be created.

THE AXISFMT AND TIMEFMT COMMANDS

The **TIMEFMT** command can precede a **GRAPH** command that specifies the **TIMEFMT** option. It specifies the date format of the *indep* variable. The date variable must contain numeric values. The general format is:

TIMEFMT *format*

where *format* contains descriptors from the list:

	Explanation	*Values*
%d	day of the month	1 - 31
%m	month of the year	1 - 12
%y	year	0 - 99
%Y	year	4-digit
%j	day of the year	1 - 365
%H	hour	0 - 24
%M	minute	0 - 60
%S	second	0 - 60

Note that upper case and lower case must be used correctly. The format rules generally conform to the style of gnuplot. For more details, open gnuplot and at the gnuplot command prompt type `help timefmt` or `help format`. For example, a partial listing of a data file with daily data on the Canadian / U.S. dollar exchange rate is:

```
19980406     0.70502
19980407     0.70314
19980408     0.70200
19980409     0.70175
19980413     0.69828
19980414     0.69696
```

The first column contains the date in the form YYYYMMDD. This is specified with the command:

```
timefmt %Y%m%d
```

The **AXISFMT** command can precede a **GRAPH** command that specifies the **AXISFMT** option as well as the **TIMEFMT** option. It specifies the date format of the x-axis labels. By default, the format is as specified with the **TIMEFMT** command. The general format is:

AXISFMT *format*

A separator character such as / can be used between the format descriptors. For the daily exchange rate data set, to obtain x-axis labels with the format YY/MM the following SHAZAM commands can be used.

```
sample 1 1006
read (uscan.dat) date exch
timefmt %Y%m%d
axisfmt %y/%m
graph exch date / timefmt axisfmt lineonly
```

The next example uses monthly seasonally adjusted data on the unemployment rate for Canada from 1980 to 1998. The **TIME** command is used to set a variable that contains monthly dates in the form YYYY.MM. The **TIMEFMT** and **AXISFMT** commands then set the date formats for a plot of the data with the **GRAPH** command.

```
sample 1 228
read (urate.dat) obs urate
time 1980 12 date
timefmt %Y.%m
axisfmt %y/%m
graph urate date / timefmt axisfmt lineonly
```

THE PLOT COMMAND

In general, the format of the **PLOT** command is:

PLOT *depvars indep / options*

The following options as specified for the **GRAPH** command are available: **RANGE**, **TIME**, **BEG=**, **END=** and **GROUPS=**. Additional options are:

ALTERNATE Alternates the symbols "X" and "O" in plotting columns of the histogram when the **HISTO** option is used. It is especially useful with the **GROUPS=** option.

HISTO Plots **HISTO**grams for the variables specified on the **PLOT** command. A separate histogram is done for each variable in the list. Each histogram takes one page of computer output. If **NOWIDE** is specified the histogram will be half the regular size. See also the **HISTO**, **RANGE** and **GROUPS=** options as described for the **GRAPH** command and the **ALTERNATE** option described above.

HOLD **HOLD**s the printing of the plot. The contents of this plot will be saved for the next **PLOT** command. At that time the plot will be blanked out unless the **NOBLANK** option is used.

NOBLANK Prevents the plot from being initialized with blanks, to allow the plot to be imposed on the plot previously specified with the **HOLD** option. The **HOLD** and **NOBLANK** options would be used if, for example, a plot with different symbols for each part of the sample were desired.

NOPRETTY SHAZAM attempts to make pretty intervals on the axes by checking the range of the data. This usually works, but sometimes the labels are not acceptable. The **NOPRETTY** option will tell SHAZAM not to attempt to make the axes pretty and just use the range of the data directly.

SAME/ NOSAME Plots *depvars* against the *indep* on the **SAME** plot. The two relationships are distinguishable by their differing point symbols. See the **SYMBOL=** option for further details. No more than 8 dependent variables should be plotted against the independent variable on the same plot. The default is **SAME**.

WIDE/ NOWIDE **NOWIDE** reduces the size of the plot in the printed output. All of the reduced plot can be seen on a terminal screen as it takes up less than 80 columns. The default value is explained in the chapter *SET AND DISPLAY*.

SYMBOL= Specifies the **SYMBOL**s to be used. The default symbols, in order, are * + 0 % $ # ! @.

XMIN= **XMAX=** **YMIN=** **YMAX=** Specifies the desired range for either the **X** or **Y** axis. The **NOPRETTY** option must be used with these options otherwise SHAZAM attempts to make pretty intervals on the axis by checking the range of the data. If these are not specified the computed **MIN**imum and **MAX**imum for the variables will be used.

EXAMPLES

An example of the use of the **GRAPH** command for plotting the Theil textile data is:

```
graph consume income price year / lineonly
```

The graph will appear in a separate window. This will initiate the creation of gnuplot command and data files with the extension **.GNU**. The gnuplot command file is **COMM.GNU**. When the SHAZAM run has finished the plot can be viewed again. Open windows gnuplot, click on the Open button and enter the name of the gnuplot command file.

The plot can be printed on paper or saved to a PostScript file for subsequent printing. For example, the plot can be saved to the PostScript file **MYPLOT.PS** with the SHAZAM command:

```
graph consume income price year / lineonly device=postscript output=myplot.ps
```

Customizing the Plot

SHAZAM prepares a basic gnuplot command file ready for the user to customize from within the SHAZAM Environment. For advanced customization this command file is also available for the user to customize with additional labels, arrows, titles, scaling of axes etc. to obtain report quality graphics. The gnuplot command file **COMM.GNU** may look like:

```
load "C000.GNU"
```

The file **C000.GNU** has the gnuplot commands and, for the Theil textile data example, a listing of this file is:

```
set samples  17
set key
set xlabel "YEAR    "
set ylabel
plot  "D000.GNU" using  1: 2  title "CONSUME "  w lines     ,\
      "D000.GNU" using  1: 3  title "INCOME  "  w lines     ,\
      "D000.GNU" using  1: 4  title "PRICE   "  w lines
```

This file gives a simple example of gnuplot commands for plotting time series data. The **plot** command plots the time series from the data file **D000.GNU**. If the gnuplot files are to be saved for future use then the **.GNU** files should be renamed so they will not be overwritten by future SHAZAM runs. If gnuplot files from a SHAZAM run are not needed for future work then the **.GNU** files can be deleted.

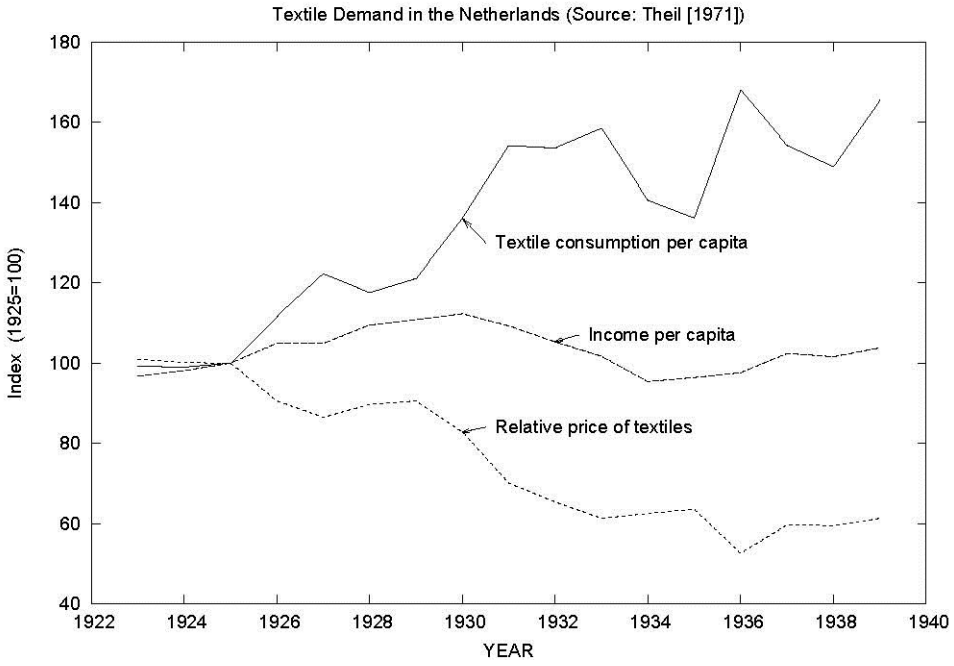
GNUPLOT is case sensitive and commands are typically lower case only. To obtain on-line documentation about gnuplot features start gnuplot and at the gnuplot command prompt type: **help**. To exit the gnuplot program type: **exit**. For more information about using gnuplot with SHAZAM see the *HOW TO RUN SHAZAM* chapters.

The gnuplot command file can be customized by using the various options available with the **set** command. This is illustrated below. The **C000.GNU** file is modified to include titles and labelling of the time series. Note that the # symbol is used for a gnuplot comment. The first two commands are required to send the output to the PostScript file **MYPLOT.PS.** The **set nokey** command is used to omit the key that gives the data description. Then, as a replacement to the key, the **set label** commands are used to place identifiers on the time series.

```
set term postscript
set output "MYPLOT.PS"
set samples          17
set title "Textile Demand in the Netherlands (Source: Theil [1971])"
set nokey
set xlabel "YEAR"
set ylabel "Index  (1925=100)"
# Put in labels for the time series
set arrow 1 from 1930.5,130 to 1930,136
set label "Textile consumption per capita" at 1930.7,130
set arrow 2 from 1932.5,107 to 1932,105.3
set label "Income per capita" at 1932.7,107
set arrow 3 from 1930.5,84 to 1930,82.8
set label "Relative price of textiles" at 1930.7,84
#
```

```
plot  "D000.GNU" using  1: 2    w lines     ,\
      "D000.GNU" using  1: 3    w lines     ,\
      "D000.GNU" using  1: 4    w lines
```

The graph is displayed below.



6. GENERATING VARIABLES

"The government are very keen on amassing statistics. They collect them, add them, raise them to the nth power, take the cube root and prepare wonderful diagrams. But you must never forget that every one of these figures comes in the first instance from the village watchman, who just puts down what he damn well pleases."

Anonymous, Quoted in Sir Josiah Stamp,
Some Economic Factors in Modern Life

This chapter describes commands for generating variables, selecting sub-sets of observations and performing numeric differentiation and integration.

THE GENR COMMAND

The **GENR** command will create new variables from old ones and do a variety of data transformations. The **SAMPLE** command defines the observation range used in **GENR** commands.

In general, the format of the **GENR** command is:

GENR *var = equation*

where *var* is the name of the variable to be generated and *equation* is any arithmetic equation which involves variables, constants and mathematical functions. The available mathematical operators are:

Priority-level	*Operator*
1	unary – (see note below)
1	unary functions (see list below)
2	** (exponentiation)
3	* , / (multiplication, division)
4	+ , – (addition, subtraction)
5	.EQ., .NE. , .GE. , .GT. , .LE. , .LT. (relational operators)
6	.NOT. (logical operator)
7	.AND. (logical operator)
8	.OR. (logical operator)

The available unary functions are:

ABS(x)	absolute value
DUM(x)	dummy variable generator
EXP(x)	e^x
INT(x)	integer truncation
LAG(x)	lag a variable one time period
LAG(x,n)	lag a variable n time periods
LGAM(x)	log gamma function, log(Γ(x))
LOG(x)	natural logs
MAX(x,y)	maximum of two variables
MIN(x,y)	minimum of two variables
MOD(x,y)	modulo arithmetic, remainder of x/y
NCDF(x)	standard normal cumulative distribution function
NOR(x)	normal random number with standard deviation x
SAMP(x)	draw a sample with replacement from the variable x starting at observation 1
SEAS(x)	seasonal dummy variable with periodicity x
SIN(x)	sine (x measured in radians)
SIN(x,–1)	arcsine (defined for x in the interval [–1,1])
SQRT(x)	square roots
SUM(x)	cumulative sum of variable x
SUM(x,n)	sum of n observations on variable x starting at observation 1
TIME(x)	time index plus x
UNI(x)	uniform random number with range (0,x)

The order of operations of mathematical expressions in SHAZAM conforms to the priority levels given above. Among operations of the same priority, expressions are executed from left to right. To avoid confusion use as many levels of parentheses as desired. Multiplication is assumed with "`)(`" and is treated as "`)*(`".

Note that the expression `X**-A` is not permitted. This should be entered as `X**(-A)`. The expression `-X**A` is evaluated as `-(X**A)`.

As a general rule, undefined values are set to a missing value code. For example, operations like **LOG** or **SQRT** of a negative number will generate undefined values. The missing value code has a default of –99999 but can be changed with the **SET MISSVALU=**

option. The temporary variable *$MISS* stores the missing value code. The **LAG** function sets zero values for initial observations.

When the **SET SKIPMISS** command is in effect the **GENR** command will assign a missing value code to results that involve a computation with a missing observation. For more details see the chapter *SET AND DISPLAY*.

A variable with a common scalar element everywhere can be created by using the command format:

GENR *newvar=var(element)*

where *element* is a number or the name of a scalar variable. For example, to generate a variable that takes on the value of the third observation of the variable *A*, the following command is appropriate:

```
genr athree=a(3)
```

The example below illustrates the use of **GENR** commands for a case where Theil's [1971, p. 102] textile data has been read in and a series of new variables needs to be created.

```
sample 1 17
read(11) year consume income price
genr price=price/100
genr pccons=(consume-lag(consume))/lag(consume)
genr t=time(0)
```

The first **GENR** command is used to change the units of the variable *PRICE*. The second **GENR** command is used to generate a variable for the percentage change of consumption from the preceding year. Note that observation 1 of *PCCONS* will not be defined properly since ***LAG****(CONSUME)* does not exist for the first observation. A warning message will be printed. The third **GENR** command is used to generate a variable *T* with observation values 1, 2, 3, . . . , 17.

DUM ***function***

The DUM(x) function will create a dummy variable equal to one when x is positive and equal to zero for observations when x is not positive. For example:

```
genr d1=dum(3)
```

```
genr d2=dum(time(0)-6)
genr d3=dum(consume-income-1)
```

The first example of the **DUM**(x) function creates a dummy variable that is always equal to one. The second creates a dummy variable that is equal to zero for the first 6 observations and equal to one otherwise. The last example uses the variables *CONSUME* and *INCOME* from Theil's textile data and creates a dummy variable equal to one when the relation inside parentheses is positive or zero and equal to zero otherwise. It is also possible to create a matrix of seasonal dummy variables. For information on this procedure, see the chapter *MATRIX MANIPULATION*.

LAG ***function***

The **LAG**(x,n) function will lag the variable x, n times. When only one lag is desired the n can be left off the function (i.e. **LAG**(x)). The first n observations are undefined when the **LAG**(x,n) function is used. SHAZAM replaces these observations with zeros. It is not necessary to change the sample size to use the **LAG**(x,n) function, but warning messages will appear whenever this function is used without proper sample commands. In fact, changing the sample size before generating new variables can cause further sample size problems. However, the sample size should be changed to start at n+1 before estimation. The **LAG**(x,n) function must be used only on predefined variables and not on functions of variables. So, for example:

```
genr x=lag(sqrt(y),3)
```

will result in an error message. To avoid this error two **GENR** commands can be used:

```
genr z=lag(y,3)
genr x=sqrt(z)
```

or one:

```
genr x=sqrt(lag(y,3))
```

It is possible to lead future variables by using a negative value for n on the **LAG**(x,n) function. For example:

```
genr xt=lag(x,-1)
```

Note that in this case the final observation of *XT* may not be defined.

The next example shows the computation of a 3-period centered moving average for the variable *P*. The result is stored in the variable *SMA3*.

```
genr sma3=(lag(p)+p+lag(p,-1))/3
```

The **GENR** command implements recursive calculations when the right hand side variables are **LAG** functions of the left hand side variable. For example, the series:

$$X_t = .8X_{t-1} + Z_{t-1}$$

can be generated with the command:

```
genr x=.8*lag(x)+lag(z)
```

Note that the command:

```
genr x=lag(x)
```

does not give the same result as the command:

```
genr y=lag(x)
```

That is, when using the **LAG** function, if a recursive calculation is not required then the left side variable must not be one of the variables in the **LAG** function.

LGAM *function*

The **LGAM**(x) function is used to compute the log of the mathematical gamma function $\Gamma(x)$ which is used in a number of probability distributions, including the gamma, beta, chi-square and F distributions.

A result from calculus is that for an integer x: $\Gamma(x) = (x-1)!$

Therefore, x! (x factorial) can be calculated with the command:

```
genr xfac=exp(lgam(x+1))
print xfac
```

This should be used with caution since as x increases the value for x! rapidly becomes extremely large.

LOG *function*

The **LOG**(x) function is used to take natural logarithms. The relationship between natural log and logarithm to the base 10 is:

$$\ln_e(x) = 2.3026 \cdot \log_{10}(x)$$

Therefore, a command that will obtain the logarithm to the base 10 of each element in the variable *A* is:

```
genr log10=log(a)/2.3026
```

MOD *function*

The **MOD**(x,y) function is used to compute the remainder of a division. For example: **MOD**(15,4)=3.

NCDF *function*

The **NCDF**(x) function returns the probability associated with the standard normal cumulative distribution function at the value x. The probability will be in the [0,1] range. The use of the **NCDF**(x) function is illustrated below. In the example *Z* is generated to take on the values 0 to 2.2 in increments of 0.2.

```
|_SAMPLE 1 12
|_GENR Z=(TIME(0)-1)/5
|_GENR P=NCDF(Z)
|_PRINT Z P
      Z              P
   0.000000      0.5000000
  0.2000000      0.5792597
  0.4000000      0.6554217
  0.6000000      0.7257469
  0.8000000      0.7881446
   1.000000      0.8413447
   1.200000      0.8849303
   1.400000      0.9192433
   1.600000      0.9452007
   1.800000      0.9640697
   2.000000      0.9772499
   2.200000      0.9860966
```

To obtain the inverse **NCDF** and save the result in the variable *Z* use the commands:

```
genr pa=1-p
distrib pa / inverse type=normal crit=z
```

Further information on the **DISTRIB** command is in the chapter *PROBABILITY DISTRIBUTIONS*.

NOR *function and* UNI *function*

The **GENR** command can be used to generate random numbers. The random number generator is usually initialized by the system clock. Users can generate all their data if they have none to be read in. The next example shows the use of the **GENR** command for generating random numbers from selected probability distributions.

```
sample 1 30
genr x1=uni(2)
genr x2=5+nor(1)
genr chi3=nor(1)**2+nor(1)**2+nor(1)**2
genr chi5=nor(1)**2+nor(1)**2+nor(1)**2+nor(1)**2+nor(1)**2
genr f35=(chi3/3)/(chi5/5)
```

The **SAMPLE** command sets the number of observations to generate. The first **GENR** command creates a new variable called *X1* with values from a uniform distribution with a range of 0 to 2. The second **GENR** command creates a new variable called *X2* with values from a normal distribution with a mean equal to 5 and a standard deviation equal to 1.

The final three **GENR** commands show how to generate values from an F-distribution. Consider $Z_1^2, Z_2^2, \ldots, Z_{n_1}^2$ as independent standard normal random variables then

$$\sum_{k=1}^{n_1} Z_k^2 \sim \chi_{n_1}^2$$

is a chi-square random variable with n_1 degrees of freedom. With $\chi_{n_1}^2$ and $\chi_{n_2}^2$ as independent chi-square random variables then a variable with the F-distribution with n_1 and n_2 degrees of freedom is constructed as:

$$F_{n_1,n_2} = \frac{n_2}{n_1}\chi_{n_1}^2 \Big/ \chi_{n_2}^2$$

The **GENR** commands generate the variables *CHI3* and *CHI5* as chi-square variables with 3 and 5 degrees of freedom respectively. The variable *F35* is then from an F-distribution with 3 and 5 degrees of freedom.

The command **SET RANFIX** (described in the chapter *SET AND DISPLAY*) typed before the **GENR** statement will prevent the random number generator from being set by the system clock. Therefore, the same set of random numbers will be generated in repeated runs. Otherwise, the random number generator will generate a different set of random numbers when requested. The normal random number algorithm is described in Brent [1974].

SAMP *function*

The **SAMP**(x) function draws a sample (with replacement) from a variable starting at observation 1. In the example below the variable *BIGX* has 100 observations. A new variable with 20 observations is generated by sampling with replacement.

```
sample 1 100
read(x.dat) bigx
sample 1 20
genr newx=samp(bigx)
```

The **SAMP** function is useful for bootstrapping experiments (see the example in the chapter *PROGRAMMING IN SHAZAM*).

SIN *function*

For any number x sine and cosine are related by the rule cos(x) = sin(x+π/2). Therefore the cosine function for a variable *X* can be generated with the command:

```
genr cosx=sin(x+$pi/2)
```

Note that *$PI* is a SHAZAM temporary variable that contains the value of π.

The tangent function can be generated using the definition: tan(x) = sin(x)/cos(x). The relation between arc sin(x) and arc cos(x) is: arc cos(x) = π/2 – arc sin(x). Therefore values for arccosine can be calculated with the command:

```
genr arccosx=($pi/2)-sin(x,-1)
```

The inverse tangent can be expressed in terms of the inverse sine function as:

$$\text{arc tan}(x) = \text{arc sin}\left(x \Big/ \sqrt{1+x^2}\right)$$

SUM *function*

The **SUM**(x) function creates a cumulative sum of the variable x. For example,

```
genr x=sum(2)
```

creates a variable that takes on the values 2, 4 ,6, . . . The **SUM** function can be used to create a capital stock series from a net investment series. For example, if the initial capital stock is 25.3 then the **GENR** command is:

```
genr kapital=25.3+sum(invest)
```

The **SUM**(x,n) function will sum up n successive observations on the variable x starting at observation 1. This can be used to convert monthly data to quarterly or yearly data, as in the following example:

```
sample 1 120
read(cpi.dat) mcpi

* Convert to annual data using 12-month averages
sample 1 10
genr ycpi=(sum(mcpi,12))/12
```

SHAZAM calculates the sum of the first 12 observations of the variable *MCPI* and then divides the sum of *MCPI* by 12 and makes this number the first observation in the new variable *YCPI.* It continues this for subsequent observations in *MCPI,* until it has created 10 observations for *YCPI.* The **SAMPLE** command which precedes the **GENR** statement tells SHAZAM the size of the new variable. This is only necessary if the new variable is to have a different length than that specified by the current **SAMPLE** command.

It is often useful to compute a variable total. The recommended way of doing this is with the **SUMS=** option on the **STAT** command (see the chapter *DESCRIPTIVE STATISTICS*). For example, to compute the sum of all elements of *X* and save the result in *TOTX* the following command can be used:

```
stat x / sums=totx
```

TIME *function*

The **TIME**(x) function creates a time index. For example, the command:

```
genr t=time(0)
```

creates a time index so that the first observation is equal to 1 and the rest are consecutively numbered. The command:

```
genr t=time(1929)
```

will create a time index so that the first observation is equal to 1930. See the **TIME** command in the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* for an alternate way to create a time index.

Relational and logical operators

The operators available for use in **GENR** commands are:

Operator	*Meaning*
.EQ.	Equal (=)
.NE.	Not equal (≠)
.GE.	Greater than or equal (≥)
.GT.	Greater than (>)
.LE.	Less than or equal (≤)
.LT.	Less than (<)
.AND.	Logical conjunction
.OR.	Logical disjunction
.NOT.	Logical negation

A value of 1 is generated if an expression is true, and a value of 0 if it is false. Parentheses should be used to ensure the correct order of processing. For example:

```
genr x5=(x1.eq.x2)
genr x6=(x1.le.x2).or.(x3.gt.0.5)
genr x7=(time(0).eq.18)
genr x8=(uni(1).le.0.6)
genr x1=(x1.ge.0)*x1
```

The first two examples create dummy variables equal to 1 if the condition is met, and 0 otherwise. The third example creates a dummy variable equal to 1 for observation 18 only. The fourth example creates a binomial random variable from the uniform random number generator. The variable will take on a value of 1 for approximately 60% of the observations and a value of 0 for the rest. Of course, the probabilities are true only for the population

and may differ for any sample. The last example will set all negative values of *X1* to 0. Note that the old *X1* is replaced by the new *X1*.

THE GEN1 COMMAND

The **GEN1** command is used to generate a scalar variable or constant. The **GEN1** command is equivalent to using both a **SAMPLE 1 1** command and a **GENR** command to generate a variable with only one observation. It is faster because it removes the need for the **SAMPLE 1 1** command. **SKIPIF** commands do not affect the **GEN1** command.

The format of the **GEN1** command is:

GEN1 *constant = equation*

For example:

```
gen1 nob=$n
```

saves the value of the number of observations in the variable *NOB*. See the chapter *ORDINARY LEAST SQUARES* for a description of the temporary variable *$N*.

An element of a vector can be saved as a scalar with the command format:

GEN1 *constant=var(element)*

The next example saves the coefficient estimates from OLS estimation in the variable *BETA*. Then element *K* of the variable *BETA* is saved in the scalar *B0*.

```
ols consume income price / coef=beta
gen1 k=$k
gen1 b0=beta(k)
```

A useful feature of the **GEN1** and **GENR** commands is to use it as a desk calculator. If the = sign is left out of the equation, SHAZAM will simply print the result. For example, the command:

```
gen1 log(2*3.14159)
```

will give the output:

```
|_GEN1 LOG(2*3.14159)
    1.8378762
```

THE IF COMMAND

The **IF** command is a conditional **GENR** command. The format of the **IF** command is:

IF(*expression*) *statement*

where *expression* is an expression in parentheses to be evaluated. If the expression is true or positive, the remainder of the **IF** command is executed. For example:

```
if(x1.ge.0) x4 = sqrt(x1)
if(2+x2/6) x2 = x2 + 12
if(x2) x3 = x2
```

The first **IF** condition states that if *X1* is greater than or equal to zero then *X4* is equal to the square root of *X1*. The second **IF** condition states that if *X2* divided by 6 plus 2 is positive then *X2* is equal to *X2* plus 12. The third **IF** condition states that if *X2* is positive then *X3* is equal to *X2*.

Note that some observations for a variable may not be defined if the **IF** condition is not true. In this case, the variable will be set to zero when the variable is initially created. For example, for the first **IF** command shown above, the variable *X4* is undefined when *X1* is less than zero and is thus equal to zero for these observations.

The **IF** command can also be used to conditionally execute any SHAZAM command on the line following the **IF** command. For example, if you only wanted to run an **AUTO** command when the Durbin-Watson statistic for the **OLS** regression is less than 1.0, you would use:

```
ols y x / rstat
if($dw .lt. 1.0)
auto y x
```

If the **IF** command is used on a vector of observations (rather than a scalar as in the example above) the command following the **IF** command will be executed if the **IF** condition is true for any observation in the sample.

A set of SHAZAM commands can be placed in a SHAZAM procedure as described in the chapter *SHAZAM PROCEDURES*. The **IF** command can then be used to conditionally execute the set of commands in the procedure. The format is:

IF(*expression*)
EXEC *proc_name*

THE IF1 COMMAND

The **IF1** command is equivalent to using both a **SAMPLE 1 1** command and a **IF** command to perform an operation only on the first observation. It is faster because it removes the need for the **SAMPLE 1 1** command. It is used primarily when checking either temporary variables or variables that have been generated with the **GEN1** command.

The format of the **IF1** command is:

IF1(*expression*) *statement*

For example:

```
gen1 sigg=$sig2
if1(sigg.gt.0) sig=sqrt(sigg)
```

THE SKIPIF COMMAND

The **SKIPIF** command is used to specify conditions under which observations are to be skipped for most commands. (The observations will still be held in memory and in data files.) The format of the **SKIPIF** command is:

SKIPIF *(expression)*

where the *expression* may use arithmetic or logical operators such as those described for **GENR** and **IF** commands. The observation will be skipped if the value of the expression is positive; otherwise, the observation is retained.

Some examples of **SKIPIF** commands are:

```
skipif (x3+x4.eq.x5)
skipif (x3.gt.x12)
skipif ((x4.eq.0).or.(x5.eq.0))
skipif (time(0).eq.6)
skipif (x1)
skipif (lag(x2+3)/12-x4)
skipif (province.eq."manitoba")
skipif (abs(x3+x4-x5).le.0.0001)
```

The first **SKIPIF** example skips the observations when the sum of *X3* and *X4 i*s equal to *X5*. The second **SKIPIF** example skips the observations when *X3* is greater than *X12*. The third **SKIPIF** example skips the observations where *X4* is equal to zero or *X5* is equal to zero. The fourth **SKIPIF** example skips the observations where the *TIME* variable is equal to 6. The fifth **SKIPIF** example will skip the observations where *X1* is positive. The sixth **SKIPIF** example will skip the observations where the result of the lagged value of *X2* plus 3 divided by 12 minus *X4* is positive. The seventh **SKIPIF** example skips the observations where the variable *PROVINCE* is equal to MANITOBA. Note that if characters are used in the command they must be enclosed in double quotes (") and only upper case comparisons are valid. The final **SKIPIF** example skips the observations where the absolute value of *X3* plus *X4* minus *X5* is less than or equal to 0.0001.

Users should be aware that the test for equality between two numbers may sometimes fail due to rounding error in the computer. The first example above is a typical case where the problem might occur. The last example gives a possible remedy.

Users should take care to express **SKIPIF** commands accurately. A common error is to skip all the observations so that no data is left, for example:

```
skipif (a.ge.0)
skipif (a.lt.0)
```

If a large number of consecutive observations are to be skipped at the beginning or end of the data, it is more efficient to omit them with the **SAMPLE** command than with **SKIPIF** commands.

The **SKIPIF** command automatically creates a special variable called *SKIP$* and initializes it to be zero for each observation. Then, for any observation to be skipped, *SKIP$* is set equal to one.

Note that the *SKIP$* variable is set at the time the **SKIPIF** command is executed. For example:

```
genr a=0
skipif(a.eq.1)
genr a=1
print a
```

would not skip any observations since all observations in *A* were zero at the time the **SKIPIF** condition was evaluated.

Sometimes the entire data set is required including those observations skipped on **SKIPIF** commands. It is possible to temporarily turn off the **SKIPIF** commands that are in effect. This requires the **SET** command:

```
set noskip
```

To put the **SKIPIF**'s back in effect the **SET** command is again used:

```
set skip
```

To permanently eliminate all **SKIPIF** commands in effect, the **DELETE** command is used:

```
delete skip$
```

If the **DELETE** command has been used to eliminate all **SKIPIF** commands it is not necessary to use the **SET SKIP** command to use the **SKIPIF** command again.

SKIPIF commands are in effect for **READ**, **WRITE** and **PRINT** commands except when a matrix is used or the **BYVAR** option is used. The **BYVAR** option is the default for **WRITE** and **PRINT** commands when only one variable is specified.

The **SKIPIF** command will print out messages that tell you which observations have been skipped. If you have many skipped observations you could get many lines of messages. These warnings can be suppressed if you use the **SET NOWARNSKIP** command.

For the automatic skipping of missing observations the **SET SKIPMISS** command can be used as described in the chapter *SET AND DISPLAY*. Then, if no longer required, a **SET NOSKIPMISS** command must be used.

THE ENDIF COMMAND

The **ENDIF** command works like the **IF** command except that when the condition is true for any observation, execution is stopped. If the **ENDIF** command is used inside a **DO**-loop, the **DO**-loop is terminated and execution will continue at the statement after **ENDO**. See the chapter *PROGRAMMING IN SHAZAM* for information on **DO**-loops.

The format of the **ENDIF** command is:

ENDIF(*expression***)**

For example:

```
endif(a.lt.0)
```

The above execution will terminate when the variable *A* is less than zero.

THE DERIV COMMAND

The **DERIV** command is used for obtaining numerical derivatives of an equation. The format of the **DERIV** command is:

DERIV *variable resultvar=equation*

where *variable* is the variable in the *equation* for which the derivative should be taken with respect to. The evaluated derivatives are placed in the variable *resultvar*. The current **SAMPLE** command will control the observations to be used. For example, if you wish to find the derivative of the equation:

```
2*(consume/income) + 3*price
```

with respect to *INCOME*, use:

```
deriv income result=2*(consume/income) + 3 * price
```

The variable *RESULT* will contain all the derivatives as in the output:

```
|_DERIV INCOME RESULT=2*(CONSUME/INCOME) + 3*PRICE
|_PRINT RESULT
    RESULT
```

```
 -0.02121723 -0.02057439 -0.02000002 -0.02028352 -0.02221009
 -0.01961595 -0.01972853 -0.02156798 -0.02581511 -0.02770540
 -0.03064908 -0.03089716 -0.02931251 -0.03527277 -0.02943039
 -0.02886880 -0.03072085
```

The derivative of the function $f(x_1, x_2, ...)$ with respect to x_1 is approximated by:

$$\frac{\partial f}{\partial x_1} = \frac{f(x_1 + 2 \cdot h, x_2, ...) - f(x_1, x_2, ...)}{2 \cdot h}$$

where h is the step-length set as $h = 1 / 10^6$.

THE INTEG *COMMAND*

The **INTEG** command can perform numeric univariate integration on any equation. The format of the **INTEG** command is:

INTEG *variable lower upper resultvar=equation*

where the integral of *equation* with respect to *variable* will be taken using the range specifed in *lower* and *upper*. The result of the integration will be placed in the variable *resultvar*. The current **SAMPLE** command will control the range of observations to be used in the integration. The variables *lower* and *upper* can either be existing variables or simple number values can be supplied.

For example, if you wish to find the cumulative probability for the standard normal distribution which has a density function of:

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-x^2/2\right)$$

use the commands:

```
sample 1 1
gen1 lower=0
gen1 upper=1
integ x lower upper answer=exp(-.5*x**2)/sqrt(2*$pi)
print answer
```

The output is:

```
|_SAMPLE 1 1
|_GEN1 LOWER=0
|_GEN1 UPPER=1
|_INTEG X LOWER UPPER ANSWER=EXP(-.5*X**2)/SQRT(2*$PI)
..NOTE..CURRENT VALUE OF $PI  =   3.1416
OBSERVATION  LOWERBOUND    UPPERBOUND     INTEGRAL
          1   .00000000     1.0000000     .34134475
|_PRINT ANSWER
    ANSWER
   .3413447
```

If the usual output from the **INTEG** command is not required, it can be suppressed with a "?" prefix, for example:

```
?integ x lower upper answer=exp(-.5*x**2)/sqrt(2*$pi)
```

EXAMPLES

Sampling Without Replacement

The **SAMP** function can be used for sampling with replacement. The SHAZAM commands in this example illustrate sampling without replacement. A variable called *X* is generated that has 300 observations of random integers between 1 and 2000 with no duplicate numbers in the data set.

```
set ranfix
sample 1 2000
genr x=time(0)
genr ran=uni(1)
sort ran x
sample 1 300
print x / nobyvar
stat x
```

The **SET RANFIX** command will prevent the random number generator from being set by the system clock so that the same set of random numbers will be generated each time this same command file is executed. The **SAMPLE** command sets the sample size. The **TIME(0)** function on the **GENR** command first generates a set of integers with a range of 1 to 2000. The next **GENR** command generates a uniform random number. The **SORT** command then sorts the variables *RAN* and *X* in ascending order according to *RAN*. The second **SAMPLE** command then selects 300 observations of the original 2000 observations. The **PRINT** command prints the variable *X* in vector format rather than **BYVAR**iable format.

Systematic Sampling

Suppose you wanted to take a sub-set of data from a larger data set that consisted of 100 observations. In this example, you want every 5th observation of variables *X* and *Y* to be in the new sub-set of data. The SHAZAM commands to achieve this result are:

```
sample 1 100
read(data) x y
skipif(mod((time(0),5)).ne.0)
stat x y
```

The **SAMPLE** command first sets the sample size to 100 observations. The **READ** command locates the data file called **DATA** and reads in the observations for variables *X* and *Y*. The **SKIPIF** command first evaluates the expression **TIME(0)** and generates a time index that begins at 1. Then the expression **MOD((TIME(0),5)** divides the time index by 5 and remainder is determined. If the remainder from the **MOD** function is not equal to 0 then the observation is skipped. If the **MOD** function is true then the observation is retained for the sub-set of data. The **STAT** command on *X* and *Y* will then print out the descriptive statistics on these two variables. The descriptive statistics should confirm that there are exactly 20 observations for each variable.

Replacing of Missing Values with the Mean

Suppose your data set has a missing observation in row 3 column 1 for the dependent variable, *Y*. There are two possible ways to deal with the missing data issue. The first method would be to omit the missing observation in the regression. For example, if your regression model is $Y = \beta_1 + \beta_2 X + \varepsilon$ and the dependent variable, *Y*, had a missing observation in row 3 then to estimate the model you would omit observation 3 for the dependent variable, *Y*, and the independent variable *X*. The number of observations in the regression must be the same for all variables. An example of the SHAZAM commands illustrating how to omit observation 3 assuming there are 7 observations in total is:

```
sample 1 7
read(data.dat) x y
sample 1 2  4 7
ols y x
```

The second **SAMPLE** command tells SHAZAM to use observations 1 to 2 omit observation 3 and use observations 4 to 7 for the OLS regression.

The second method would be to take the average and use it for the missing observation. For example, the data set consists of variables *X* and *Y* and the data:

```
X    Y
1    100
2    200
     300
4    400
5    500
6    600
7    700
```

As you will notice observation 3 is missing for variable *X*. To properly read these variables into SHAZAM you will be required to place a number such as the SHAZAM default missing data code of –99999 in row 3 column 1. SHAZAM will get confused reading in the data file if the number is not in the place of the missing value. For example, if the missing value in row 3 column 1 is left as a blank then the data would be read by SHAZAM as:

```
1       100
2       200
300     4
400     5
500     6
600     7
```

As you can see this is definitely not correct. The correct data file should look like:

```
      1       100
      2       200
 -99999       300
      4       400
      5       500
      6       600
      7       700
```

To calculate the average for the missing observation in row 3 column 1 the SHAZAM commands would be:

```
sample 1 7
read(data.dat) x y

* Skip observation 3 using the sample command
sample 1 2 4 7

* Calculate the mean of x
stat x  / mean=xbar

* Place the mean of x in row 3 column 1
sample 3 3
genr x=xbar
sample 1 7
print x y
```

The first **SAMPLE** command tells SHAZAM that there are 7 observations for variables *X* and *Y*. The **READ** command reads in the data for the variables. This includes the –99999 that is placed for the missing value. The second **SAMPLE** command tells SHAZAM to use observations 1 to 2 and 4 to 7 for the next command. The **STAT** command calculates the mean value of *X* and places it into the vector called *XBAR*. The third **SAMPLE** command then sets the sample range to observation 3. The following **GENR** command places the previous stored vector value of *XBAR* into the variable *X* at observation 3. Then the fourth **SAMPLE** command changes the sample range to the complete 7 observations and the **PRINT** command prints the results for *X* and *Y*. The SHAZAM output shows the results:

```
|_SAMPLE 1 7
|_PRINT X Y
   1.000000       100.0000
   2.000000       200.0000
   4.166667       300.0000
   4.000000       400.0000
   5.000000       500.0000
   6.000000       600.0000
   7.000000       700.0000
```

Also see the **MISSVALU=** and **SKIPMISS** options available on the **SET** command.

7. ORDINARY LEAST SQUARES

"Less is More."
Miës van der Rohe
Architect

The **OLS** command will perform **O**rdinary **L**east **S**quares regressions and produce standard regression diagnostics. This was introduced in the chapter *A CHILD'S GUIDE TO RUNNING REGRESSIONS*. In addition, the **OLS** command has an extensive list of options that provides many features for estimation and testing of the linear regression model. For example, tests on the residuals are available with the **DLAG**, **DWPVALUE**, **GF**, and **RSTAT** options. Model selection tests (including the Akaike information criterion and the Schwarz criterion) are available with the **ANOVA** option. Standard errors adjusted for heteroskedasticity or autocorrelation are computed with the **HETCOV** or **AUTCOV=** options. Linear restrictions on the coefficients can be incorporated with the **RESTRICT** option. Ridge regression is available with the **RIDGE=** option and weighted least squares can be implemented with the **WEIGHT=** option.

The SHAZAM output from the **OLS** command includes elasticities evaluated at the sample means. However, the printed elasticities may not be correct if data has been transformed, for example the data is in logarithms. In this case, the **LININV**, **LINLOG**, **LOGINV**, **LOGLIN** or **LOGLOG** options can be specified to obtain meaningful elasticities.

The results from the **OLS** command can be accepted as input by other SHAZAM commands to obtain further analysis of the results. For example, the **DIAGNOS** command is described in the chapter *DIAGNOSTIC TESTS*, the **TEST** and **CONFID** commands are described in the chapter *HYPOTHESIS TESTING AND CONFIDENCE INTERVALS* and the **FC** command is described in the chapter *FORECASTING*.

OLS *COMMAND OPTIONS*

In general, the format of the **OLS** command is:

OLS *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables and *options* is a list of desired options. If any variable in the list of independent variables is a matrix, SHAZAM will treat each column as a separate explanatory variable. To impose a lag structure on the independent variables see the chapter *DISTRIBUTED-LAG MODELS*. The available options on the **OLS** command are:

ANOVA Prints the **AN**alysis **O**f **VA**riance tables and the F-statistic for the test that all coefficients are zero. In some restricted models, the F-test from an **ANOVA** table is invalid. In these cases the F-statistic will not be printed, but may be obtained with the **TEST** command. Model Selection Tests are also printed. This option is in effect automatically when running in BATCH mode. In TALK mode, the **ANOVA** option must be specified if the table is desired. In BATCH mode these statistics can be suppressed with the **NOANOVA** option.

AUXRSQR Prints the R^2 statistics for the auxiliary regressions of each independent variable on all other independent variables. These R^2 statistics are useful in detecting multicollinearity (see, for example, the discussion in Griffiths, Hill and Judge [1993, Chapter 13.5]). This option is not implemented when the **HETCOV** option is specified.

DFBETAS Computes the **DFBETAS** statistic. Also see the **INFLUENCE** option. Further details and an example are given later in this chapter.

DLAG Computes Durbin's [1970] *h* statistic as a test for autocorrelation when there is a **LAG**ged **D**ependent variable in the regression. The lagged dependent variable ***must*** be the first listed independent variable when this option is used. The *h* statistic is useful since the Durbin-Watson statistic may not be valid in such situations. Durbin's *h* statistic cannot always be computed since the square root of a negative number may be required. In such cases, the *h* statistic will neither be computed nor printed. It is essential to remember that when lagged variables have been generated a **SAMPLE** command must be included to delete the beginning observations. The statistic is calculated as:

$$h = \hat{\rho}\sqrt{\frac{N}{1 - N\hat{\sigma}_{\alpha}^{2}}} \qquad \text{where} \qquad \hat{\rho} = \sum_{t=2}^{N} e_t e_{t-1} \Big/ \sum_{t=2}^{N} e_{t-1}^{2}$$

and e_t are the estimated residuals and $\hat{\sigma}_\alpha^2$ is the estimated variance of the coefficient on the lagged-dependent variable. The Durbin *h* statistic is asymptotically normally distributed under the null hypothesis of no autocorrelation.

DN Estimates the error variance $\hat{\sigma}^2$ by **D**ividing the residual sum of squares by **N** instead of N–K.

DUMP **DUMP**s large amounts of output mainly useful to SHAZAM consultants.

DWPVALUE Computes the **D**urbin **W**atson **P**robability-**VALUE**. Suppose the Durbin-Watson test statistic d has a calculated value of DW. For a test of the null hypothesis of no autocorrelation in the errors against the alternative hypothesis of positive autocorrelation the p-value is P(d<DW). For the alternative hypothesis of negative autocorrelation the p-value is 1–P(d < DW). This option automatically uses **METHOD=HH**. Also see the **ORDER=** option. The probability is saved in the temporary variable *$CDF*. Details on the computational method are given in the section *An Exact p-value for the Durbin-Watson Test* in the chapter *PROGRAMMING IN SHAZAM*.

GF Prints **G**oodness of **F**it tests for normality of residuals. Coefficients of skewness and excess kurtosis and the Jarque-Bera test for normality of the residuals are also computed as described in the chapter *A CHILD'S GUIDE TO RUNNING REGRESSIONS*. This option is automatically in effect when running in BATCH mode. In TALK mode, the **GF** option must be specified if this output is desired. In BATCH mode, this output can be eliminated by specifying **NOGF**.

GRAPH Prepares GNUPLOT plots of the residuals and the fitted values. For more information on this option see the chapter *PLOTS AND GRAPHS*. With the **GRAPH** option the **APPEND**, **OUTPUT=**, **DEVICE=**, **PORT=** and **COMMFILE=** options are also available as described for the **GRAPH** command.

HETCOV Uses White's [1980] **HET**eroskedastic-Consistent **COV**ariance matrix estimation to correct the estimates for an unknown form of heteroskedasticity. This option is not available with **METHOD=HH**. If this option is used, the forecast standard errors computed by the **FC** command will not be correct. The formula is:

$$V(\hat{\beta}) = N(X'X)^{-1} S_0 (X'X)^{-1} \quad \text{where} \quad S_0 = \frac{1}{N}\sum_{t=1}^{N} e_t^2 X_t X_t'$$

INFLUENCE Computes the Belsley-Kuh-Welsch [1980, Chapter 2] diagnostics for detecting influential observations. See also the **DFBETAS** and **HATDIAG=** options. The **INFLUENCE** option is not valid with the **RIDGE=**, **HETCOV**, **AUTCOV=**, **RESTRICT** or Stepwise Regression options. Further details and an example are given later in this chapter.

LININV Used when the dependent variable is **LIN**ear, but the independent variables are in **INV**erse form. This type of model is known as the reciprocal model. This option only affects the calculation of the elasticities. Consider observations on the explanatory variables as $Z_{kt} = 1/X_{kt}$. The **LININV** option reports elasticities evaluated at sample means as: $E_k = -\hat{\beta}_k \cdot \overline{Z}_k / Y$. NOTE: This option does not transform the data. The data must be transformed by the user with the appropriate **GENR** commands.

LINLOG Used when the dependent variable is **LIN**ear, but the independent variables are in **LOG** form. In this model the elasticities are estimated as $E_k = \hat{\beta}_k / \overline{Y}$. NOTE: This option does not transform the data. The data must be transformed by the user with the appropriate **GENR** commands.

LIST **LIST**s and plots the residuals and predicted values of the dependent variable and residual statistics. When **LIST** is specified **RSTAT** is automatically turned on.

LOGINV Used when the dependent variable is in **LOG** form, and the independent variables are in **INV**erse form. Consider observations on the explanatory variables as $Z_{kt} = 1/X_{kt}$. The elasticities are estimated as $E_k = -\hat{\beta}_k \cdot \overline{Z}_k$. The log-likelihood function is evaluated as in the **LOGLIN** option below. NOTE: This option does not transform the data. The data must be transformed by the user with the appropriate **GENR** commands.

LOGLIN Used when the dependent variable is in **LOG** form, but the independent variables are **LIN**ear. In this model the elasticities are estimated as $E_k = \hat{\beta}_k \overline{X}_k$. The log-likelihood function is evaluated as:

$$-\frac{N}{2}\ln\left(2\pi\tilde{\sigma}^2\right) - \frac{N}{2} - \sum_{t=1}^{N} \ln Y_t \qquad \text{where} \qquad \tilde{\sigma}^2 = \frac{1}{N}\sum_{t=1}^{N} e_t^2$$

The R^2 calculation does not take anti-logs of the dependent variable. However, if the **RSTAT** option is also used the output will report:

```
R-SQUARE BETWEEN OBSERVED AND PREDICTED =

R-SQUARE BETWEEN ANTILOGS OBSERVED AND PREDICTED =
```

NOTE: This option does not transform the data. The data must be transformed by the user with the appropriate **GENR** commands.

LOGLOG Used when the dependent variable and all the independent variables are in **LOG** form. In this model the elasticities are estimated as $E_k = \hat{\beta}_k$. The log-likelihood function and R^2 measure are evaluated as in the **LOGLIN** option above. NOTE: This option does not transform the data. The data must be transformed by the user with the appropriate **GENR** commands.

MAX Prints Analysis of Variance Tables, Variance-covariance matrix, Correlation matrix, Residuals, Residual Statistics and Goodness of Fit Test for Normality. This option is equivalent to using the **ANOVA**, **LIST**, **PCOV**, **PCOR** and **GF** options. Users should be sure the **MAX** output is necessary, otherwise unnecessary calculations are required.

NOCONSTANT There will be **NO CONSTANT** (intercept) in the estimated equation. This option is used when the intercept is to be suppressed in the regression or when the user is supplying the intercept. This option should be used with caution as some of the usual output may be invalid. In particular, the usual R^2 is not well defined and could be negative. However, when this option is used, the raw moment R^2 may be of interest. The `ANALYSIS OF VARIANCE - FROM MEAN` table will not be computed if this option is used.

NOMULSIGSQ With this option, for classical OLS estimation, the $(X'X)^{-1}$ matrix is used as the complete covariance matrix of the estimated coefficients and is **NO**t **MUL**tiplied by $\hat{\sigma}^2$.

NONORM Used with **WEIGHT=** if you do not want normalized weights. Interpretation of output is sometimes difficult when weights are not normalized. Sometimes, the weights can be viewed as a sampling replication factor. Users are expected to know exactly what their weights represent.

PCOR **P**rints the **COR**relation matrix of the estimated coefficients. This should not be confused with a correlation matrix of variables which can be

obtained with a **STAT** command. The elements of the correlation matrix are obtained as:

$$\text{Cor}\left(\hat{\beta}_i, \hat{\beta}_j\right) = \text{Cov}\left(\hat{\beta}_i, \hat{\beta}_j\right) \Big/ \sqrt{V(\hat{\beta}_i)\, V(\hat{\beta}_j)} \qquad \text{for } i,j = 1, \ldots, K$$

PCOV **P**rints the **COV**ariance matrix of the estimated coefficients (described in the chapter *A CHILD'S GUIDE TO RUNNING REGRESSIONS*). This should not be confused with the covariance matrix of variables which can be obtained with the **STAT** command.

PIL Estimation with the polynomial inverse lag technique proposed by Mitchell and Speaker [1986]. For more details see the chapter *DISTRIBUTED-LAG MODELS*.

PLUSH **P**rints the **LUSH** (**L**inear **U**nbiased with **S**calar covariance matrix using **H**ouseholder transformation) residuals. The Householder transformation method first finds the N x N orthogonal matrix P such that:

$$P'X = \begin{bmatrix} P_1'X \\ P_2'X \end{bmatrix} = \begin{bmatrix} R \\ 0 \end{bmatrix} \qquad \text{and} \qquad X'X = R'R$$

where P_1 is NxK, P_2 is Nx(N–K) and R is a KxK upper triangular matrix. Note that R′R is a Cholesky factorization of X′X. The OLS coefficient vector is formed by solving $R\hat{\beta} = P_1'Y$. The $N - K$ vector of LUSH residuals is computed as $P_2'Y$. For further details see Golub and Styan [1973].

REPLICATE Used with **WEIGHT=** when the weights indicate a sample replication factor. When this option is specified the **NONORM** option is often desirable. The effective sample size will then be adjusted upward to be equal to the sum of the weights. The **UT** option is automatically in effect.

RESTRICT Forces linear **RESTRICT**ions into the regression. It tells SHAZAM that **RESTRICT** commands follow. An example of this option is shown later in this chapter. Restrictions must be linear. This option may not be used with **DWPVALUE** or **METHOD=HH**.

RSTAT Prints **R**esidual Summary **STAT**istics. The output includes the Durbin-Watson statistic and related residual test statistics. It also includes the

Runs Test described in the chapter *A CHILD'S GUIDE TO RUNNING REGRESSIONS*. When the **LIST** option is specified **RSTAT** is automatically turned on.

UT Used with **WEIGHT=** for residuals and predicted values that are **U**n**T**ransformed. The estimated coefficients are used with unweighted data to obtain predicted values. The regression estimates are not affected by this option. This option is not available with **METHOD=HH**.

WIDE Uses a wider page width for the display of results. For printing, the page set-up should be set to landscape orientation.

AUTCOV= Specifies the lag length L to use in the Newey and West [1987] variance estimator for models with autocorrelated disturbances. This option is the autocorrelation equivalent to the **HETCOV** option. An explanation of the method is in Greene [2003, p. 267]. Use **AUTCOV=1** until you really understand the method. The variance-covariance matrix of the estimated coefficients is estimated as:

$$V(\hat{\beta}) = N(X'X)^{-1} S_* (X'X)^{-1} \qquad \text{where}$$

$$S_* = S_0 + \frac{1}{N}\sum_{j=1}^{L}\sum_{t=j+1}^{N} w_j e_t e_{t-j}\left(X_t X'_{t-j} + X_{t-j} X'_t\right) \qquad \text{and} \qquad w_j = 1 - \frac{j}{L+1}$$

and S_0 is the White estimator described for the **HETCOV** option.

BEG=, END= Specifies the **BEG**inning and **END**ing observations to be used in estimation. This option overrides the **SAMPLE** command and defaults to the sample range in effect.

COEF= Saves the **COEF**ficients in the variable specified. If there is an intercept it will be stored as the last coefficient.

COV= Saves the **COV**ariance matrix of coefficients in the variable specified.

FE=, FX= Specifies the entering and exiting criteria in terms of **F**-values rather than probability levels when running Stepwise regressions. SHAZAM will use the user-supplied values of **FE=** and **FX=** only when no values are supplied for **PE=** or **PX=**. If either **FE=** or **FX=** is not specified, it will be

defaulted to the other value. If **FX>FE** then SHAZAM will set **FX** to **FE**. If **FE=0** then all of the step variables will be allowed to enter. If the value specified for **FX=** is a large number then all of the step variables may be removed. See the section on *STEPWISE REGRESSION* in this chapter for further details.

HATDIAG= Saves the diagonal elements of the Hat matrix $X(X'X)^{-1}X'$ in the variable specified**.**

IDVAR= Specifies a character variable that contains observation labels to be displayed when the **LIST** or **MAX** options are used. The **WIDE** option should also be specified. For information on reading and printing character variables, see the chapter *DATA INPUT AND OUTPUT*. An example of the **IDVAR=** option in use can be found later in this chapter.

INCOEF= Specifies a vector of **COEF**ficients to **IN**put if you know what the coefficients are and do not want to estimate the equation. An example of this option is shown in the chapter *PROGRAMMING IN SHAZAM* for computing the Power of a Test. Also see the **INSIG2=** option described below.

INCOVAR= Specifies a **COVAR**iance matrix to be used with the **INCOEF=** option. The covariance matrix must be a symmetric matrix stored in lower-triangular form such as that produced by the **COV=** option or the **SYM** function on the **MATRIX** command. When this option is used the **NOMULSIGSQ** option is automatically in effect.

INDW= Specifies a value for the Durbin-Watson test statistic. This option may be used with the **DWPVALUE** option to get a p-value for the Durbin-Watson statistic.

INSIG2= Specifies a value of σ^2 to use. Also see the **INCOEF=** option described above.

METHOD= Specifies the computational **METHOD** to use on the **OLS** command. The default is **GS** (Gram Schmidt) as described in Farebrother [1974] and the alternatives are **HH** (Householder transformations), and **NORMAL** (Choleski solution of Normal equations). All methods should yield nearly identical results with most data, however **GS** is probably the most accurate in most situations.

NPOP= Specifies the population size N^P. When this option is used it is considered that the sample size N is not a small fraction of the population size. The error variance $\hat{\sigma}^2$ is multiplied by the finite population correction factor $(N^P - N)/N^P$. Some discussion is available in Newbold *et al.* [2003, p. 708].

ORDER= Specifies the order of autocorrelation to test if the **DWPVALUE** option is used. The default is **ORDER=1**.

PCINFO= This option is only used on **OLS** commands in conjunction with the **PC** command. For more information on this option see the chapter *PRINCIPAL COMPONENTS AND FACTOR ANALYSIS*.

PCOMP= This option is only used on **OLS** commands in conjunction with the **PC** command. For more information on this option see the chapter *PRINCIPAL COMPONENTS AND FACTOR ANALYSIS*.

PE=, PX= These options are similar to the **FE=** and **FX=** options described above, and are used to specify the **P**robability levels for **E**ntering (**PE=**) and e**X**iting (**PX=**) variables that may be stepped into the equation when a Stepwise regression is being run. If a variable is more significant than the **PE** level, it will be included. If at any step a variable becomes less significant than **PX** it will be deleted from the equation. The default values are **PE=.05** and **PX=.05**. If either value is not specified it will be defaulted to the other value. If **PX< PE** then **PX** will be set to equal **PE**. If **PE=1** then all of the step variables will be allowed to enter. If **PX=0** then all of the step variables will be removed. See the section on *STEPWISE REGRESSION* in this chapter for further details.

PREDICT= Saves the **PREDICT**ed values of the dependent variable in the variable specified.

RESID= Saves the values of the **RESID**uals from the regression in the variable specified.

RIDGE= Specifies a value of k to use to convert the **OLS** regression to a **RIDGE** regression. This option only permits ordinary **RIDGE** regression where the diagonal elements of the $X'X$ matrix are augmented by k. In order to do a **RIDGE** regression, a value for k must be specified. It may be necessary to run several regressions using different values of k in order to examine the stability of the coefficients. The value of k should be

between zero and one. A value of k equal to zero will be an **OLS** regression. The user should be familiar with **RIDGE** regression before using this option. Watson and White [1976] provide good examples of ridge regression. This option automatically uses **METHOD=NORMAL**. See the *PROGRAMMING IN SHAZAM* chapter for a ridge regression example. The ridge coefficients and covariance matrix are estimated as:

$$\hat{\beta}_R = [X'X + kI]^{-1} X'Y \quad \text{and} \quad V(\hat{\beta}_R) = \hat{\sigma}^2 [X'X + kI]^{-1} (X'X) [X'X + kI]^{-1}$$

The **INSIG2=** option can be used to specify a value of σ^2 to use in the estimate of the covariance matrix for the ridge regression estimates (see the discussion in Greene [2003, p. 58; 1997, p. 424]).

STDERR= Saves the values of the **ST**an**D**ard **ERR**ors of the coefficients in the variable specified.

TRATIO= Saves the values of the **T-RATIO**s in the variable specified.

WEIGHT= Specifies a variable to use as the weight for a **WEIGHT**ed Least Squares regression. **OLS** with the **WEIGHT=** option is similar to a **GLS** regression with a diagonal Omega matrix. Users should also examine the **NONORM**, **UT** and **REPLICATE** options described above which can be used with the **WEIGHT=** option. More details and an example are given later in this chapter.

OLS *TEMPORARY VARIABLES*

There are many temporary variables available on the **OLS** command. These variables contain useful statistics from the most recent regression command in the SHAZAM run. (For a list of the temporary variables available on each regression command see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.)

The temporary variables available after the **OLS** command are:

$ADR2 R-Square Adjusted

$DF Degrees of Freedom

$ERR Error Code

$K Number of Coefficients

$LLF Log of the Likelihood Function

$N Number of observations used in the estimation

$R2 R-Square

$RAW Raw-Moment R-Square

$SIG2 Variance of the Estimate - `SIGMA**2`

$SSE Sum of Squared Errors - SSE

From the `ANALYSIS OF VARIANCE - FROM MEAN` table in the **OLS** output:

$ANF ANOVA F Statistic

$SSR Regression - SS

$SST Total - SS

From the `ANALYSIS OF VARIANCE - FROM ZERO` table in the **OLS** output:

$ZANF ANOVA F Statistic

$ZSSR Regression - SS

$ZSST Total - SS

If the **ANOVA** option is used the model selection test statistics available in temporary variables are:

$FPE Akaike (1969) final prediction error

$LAIC Akaike (1973) information criterion - log AIC

$LSC Schwarz (1978) criterion - log SC

$GCV Craven-Wahba (1979) generalized cross validation

$HQ Hannan and Quinn (1979) criterion

$RICE Rice (1984) criterion

$SHIB Shibata (1981) criterion

$SC Schwarz (1978) criterion - SC

$AIC Akaike (1974) information criterion - AIC

If the **DLAG** option is used an available temporary variable is:

$DURH Durbin's h statistic

If the **GF** option is used an available temporary variable is:

$JB Jarque-Bera normality test statistic

If the **RSTAT**, **LIST** or **MAX** option is used the statistics available in temporary variables are:

$DW Durbin-Watson statistic

$R2AN R-Square between antilogs observed and predicted

$R2OP R-Square between observed and predicted

$RHO Residual autocorrelation coefficient

If the **DWPVALUE** option is used then an available temporary variable is:

$CDF p-value for the Durbin-Watson statistic.

EXAMPLES

Calculating a Chow Test

This example shows how temporary variables can be useful. Consider computing the Chow test statistic to test the hypothesis that the coefficients are the same in two regimes

(this test is described in the chapter *DIAGNOSTIC TESTS*). The SHAZAM commands that compute the Chow test statistic are:

```
?ols consume income price
gen1 csse=$sse
gen1 df1=$k
gen1 df2=$n-2*$k
sample 1 9
?ols consume income price
gen1 sse1=$sse
sample 10 17
?ols consume income price
gen1 sse2=$sse
gen1 f=((csse-(sse1+sse2))/df1)/((sse1+sse2)/df2)
sample 1 1
print f
distrib f / type=f df1=df1 df2=df2
```

In the above example three **OLS** regressions are run. The first uses all the observations in the sample. This is the combined regression. The second and third use the first 9 and last 8 observations in the sample, respectively. The purpose of the Chow test is to test the hypothesis that the coefficients are the same in each of the separate samples. The ? symbol that appears before the **OLS** commands serves to suppress the output, since only the *$SSE* variables are of interest in this problem. (For details on the ? output suppressor and temporary variables, see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.) The contents of the *$SSE* temporary variable are saved in a permanent variable before another regression is run because only the most recent values of temporary variables are saved. When saving a scalar variable, like *$SSE*, the **GEN1** rather than **GENR** command should be used. The variable *F* contains the Chow test statistic computed as:

$$F = ((CSSE-(SSE1+SSE2))/K)/((SSE1+SSE2)/(N1+N2-2K))$$

where SSE1 is the sum of squared errors for the OLS regression using the first 9 observations, SSE2 is the sum of squared errors for the OLS regression using the last 8 observations, CSSE is the sum of squared errors for the combined OLS regression, K is the number of parameters, N1 is the number of observations in the first separate OLS regression, and N2 is the number of observations in the last separate OLS regression.

The SHAZAM output from the above commands is:

```
 |_?OLS CONSUME INCOME PRICE
 |_GEN1 CSSE=$SSE
```

```
..NOTE..CURRENT VALUE OF $SSE =    433.31
|_GEN1 DF1=$K
..NOTE..CURRENT VALUE OF $K   =    3.0000
|_GEN1 DF2=$N-2*$K
..NOTE..CURRENT VALUE OF $N   =    17.000
..NOTE..CURRENT VALUE OF $K   =    3.0000
|_SAMPLE 1 9
|_?OLS CONSUME INCOME PRICE
|_GEN1 SSE1=$SSE
..NOTE..CURRENT VALUE OF $SSE =    61.671
|_SAMPLE 10 17
|_?OLS CONSUME INCOME PRICE
|_GEN1 SSE2=$SSE
..NOTE..CURRENT VALUE OF $SSE =    189.35
|_GEN1 F=((CSSE-(SSE1+SSE2))/DF1)/((SSE1+SSE2)/DF2)
|_SAMPLE 1 1
|_PRINT F
   2.662815
|_DISTRIB F / TYPE=F DF1=DF1 DF2=DF2
F DISTRIBUTION- DF1=    3.0000     DF2=    11.000
MEAN=    1.2222     VARIANCE=    1.7072     MODE=    .28205

                DATA         PDF         CDF         1-CDF
  F
 ROW     1      2.6628     0.78980E-01   .90020     0.99796E-01
```

The final step is to calculate the p-value as the area under the probability density function of the $F_{(K,N1+N2-2K)}$ distribution to the right of the calculated F-test statistic of 2.662815. The calculation is done with the **DISTRIB** command (described in the chapter *PROBABILITY DISTRIBUTIONS)*. On the SHAZAM output this area is listed in the column marked `1-CDF`. The reported p-value of .099796 suggests that the null hypothesis can be rejected at the 10% level but not at the 5% level.

Note that the **DIAGNOS** command (see the chapter *DIAGNOSTIC TESTS*) automatically computes Chow test statistics. The above result can be obtained with the commands:

```
ols consume income price
diagnos / chowone=9
```

The **CHOWONE=** option on the **DIAGNOS** command specifies the number of observations in the first group.

Labelling Residual Output with the IDVAR= option

The commands below show the use of the **IDVAR=** option on the **OLS** command. The variables are the unemployment and vacancy rates for some provinces of Canada for January 1976 (the data set was illustrated in the chapter *DATA INPUT AND OUTPUT*).

```
sample 1 9
```

```
format(2a8,2f5.1)
read prov1 prov2 ur vr / format
Newfoundland     14.9  4.0
Nova Scotia       9.1  5.0
New Brunswick    12.2  7.0
Quebec            9.1  6.0
Ontario           7.1  5.0
Manitoba          6.7  8.0
Saskatchewan      4.8  8.0
Alberta           5.3 11.0
British Columbia 10.0  4.0
ols ur vr / idvar=prov1 list wide
```

A selected portion of the **OLS** estimation output is:

OBSERVATION NO.	OBSERVED VALUE	PREDICTED VALUE	CALCULATED RESIDUAL
1 Newfound	14.900	11.087	3.8132
2 Nova Sco	9.1000	10.151	-1.0513
3 New Brun	12.200	8.2803	3.9197
4 Quebec	9.1000	9.2158	-0.11579
5 Ontario	7.1000	10.151	-3.0513
6 Manitoba	6.7000	7.3447	-0.64474
7 Saskatch	4.8000	7.3447	-2.5447
8 Alberta	5.3000	4.5382	0.76184
9 British	10.000	11.087	-1.0868

RESTRICTED LEAST SQUARES

The general command format for restricted least squares is:

OLS *depvar indeps* / **RESTRICT** *options*

RESTRICT *equation1*

RESTRICT *equation2*

...

END

The **RESTRICT** option informs SHAZAM that **RESTRICT** commands are to follow. More than one is allowed provided each is typed on a separate line. The *equation* is a linear function of the variables (that represent coefficients) involved in the estimation. NOTE: The restrictions ***must*** be a linear function of the coefficients. The **END** command marks the end of the list of **RESTRICT** commands and is required. Each restriction will add one degree of freedom.

Consider estimating the model $Y = X\beta + \varepsilon$ subject to the restrictions $R\beta = r$ where R is a known matrix and r is a known vector. The restricted least squares estimator is:

$$\hat{\beta}_r = \hat{\beta} + (X'X)^{-1}R'\left[R(X'X)^{-1}R'\right]^{-1}\left(r - R\hat{\beta}\right) \qquad \text{where} \qquad \hat{\beta} = (X'X)^{-1}X'Y$$

The covariance matrix of the restricted estimator is estimated as:

$$V\left(\hat{\beta}_r\right) = \hat{\sigma}^2(X'X)^{-1} - \hat{\sigma}^2(X'X)^{-1}R'\left[R(X'X)^{-1}R'\right]^{-1}R(X'X)^{-1}$$

The R^2 measure is calculated as: $$1 - \frac{e'e}{Y'Y - N\bar{Y}^2}$$

where e are the residuals from the restricted estimation. Note that this formula is not bounded at zero.

An example of the use of the **RESTRICT** option on the **OLS** command and **RESTRICT** commands is shown with Theil's textile data:

```
ols consume income price / restrict
restrict income+price=0
end
```

The SHAZAM output is:

```
|_OLS CONSUME INCOME PRICE / RESTRICT
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17
|_RESTRICT INCOME+PRICE=0
|_END
F TEST ON RESTRICTIONS=   1.4715     WITH    1 AND   14 DF
```

```
  R-SQUARE =     .9462     R-SQUARE ADJUSTED =     .9426
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   31.924
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.6501
 SUM OF SQUARED ERRORS-SSE=   478.86
 MEAN OF DEPENDENT VARIABLE =   134.51
 LOG OF THE LIKELIHOOD FUNCTION = -52.4966

VARIABLE   ESTIMATED  STANDARD   T-RATIO       PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      15 DF  P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.3691      .8433E-01   16.24     .000  .973      .3078     1.0482
PRICE     -1.3691      .8433E-01  -16.24     .000 -.973     -.9794     -.7768
CONSTANT   97.991      2.634       37.21     .000  .995      .0000      .7285
```

An example of estimating equations with restrictions in a system of equations is shown in the chapter *TWO-STAGE LEAST SQUARES AND SYSTEMS OF EQUATIONS*.

WEIGHTED REGRESSION

Weighted regression has a number of applications and can be implemented in SHAZAM with the **WEIGHT=** option on the **OLS** command. One application is as a correction for heteroskedasticity. A second application is for replicated data that may be produced from a sample survey. First consider the regression model with heteroskedastic disturbances stated as:

$$Y_t = X_t'\beta + \varepsilon_t \qquad \text{with} \qquad E(\varepsilon_t^2) = \sigma^2/W_t \qquad \text{for } t = 1, \ldots, N$$

and Y_t, X_t and W_t are observed and the unknown parameters are β and σ^2. The values for W_t are specified in the **WEIGHT=** variable. SHAZAM normalizes the weights to sum to the number of observations. Each observation of the dependent and explanatory variables is multiplied by the square root of the normalized weight variable and the weighted least squares estimate $\hat{\beta}_w$ is then obtained by applying OLS to the transformed model:

$$\sqrt{W_t/\overline{W}}\,Y_t = \sqrt{W_t/\overline{W}}\,X_t'\beta + v_t \qquad \text{where} \qquad \overline{W} = \frac{1}{N}\sum_{t=1}^{N} W_t$$

The weighted residuals (that can be saved with the **RESID=** option) are calculated as:

$$\hat{v}_t = \sqrt{W_t/\overline{W}}\,(Y_t - X_t'\hat{\beta}_w)$$

The residual variance (reported as `SIGMA**2` on the SHAZAM output) is estimated as:

$$\frac{1}{N-K}\sum_{t=1}^{N}(\hat{v}_t)^2$$

If the **DN** option is used the divisor is N instead of N–K. The log-likelihood function is evaluated as:

$$-\frac{N}{2}\ln\left(2\pi\tilde{\sigma}^2\right)-\frac{N}{2}+\frac{1}{2}\sum_{t=1}^{N}\log(\left|W_t/\overline{W}\right|) \qquad \text{where} \qquad \tilde{\sigma}^2=\frac{1}{N}\sum_{t=1}^{N}(\hat{v}_t)^2$$

An example, with the Theil textile data set, of applying weighted least squares to the heteroskedastic error model follows. The assumption is that the error variance is directly related to the *PRICE* variable. The SHAZAM output is:

```
 |_GENR PW=1/PRICE
 |_OLS CONSUME INCOME PRICE / WEIGHT=PW
  OLS ESTIMATION
       17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
 ...NOTE..SAMPLE RANGE SET TO:     1,   17
 SUM OF LOG(SQRT(ABS(WEIGHT)))  =  -.19396

  R-SQUARE =     .9426     R-SQUARE ADJUSTED =     .9344
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   33.810
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.8146
 SUM OF SQUARED ERRORS-SSE=   473.34
 MEAN OF DEPENDENT VARIABLE =   138.98
 LOG OF THE LIKELIHOOD FUNCTION = -52.5920

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.1543      .2922       3.950      .001  .726      .2626      .8529
PRICE     -1.4060      .9272E-01  -15.16      .000 -.971    -1.0080     -.7374
CONSTANT   122.92      28.96       4.244      .001  .750      .0000      .8845
```

Now consider an application to replicated data where each observation Y_t, X_t is replicated N_t times. The values for N_t are specified in the **WEIGHT=** variable. The weighted least squares estimate $\hat{\beta}_w$ is obtained by applying OLS to the transformed model:

$$\sqrt{N_t}Y_t=\sqrt{N_t}X_t'\beta+v_t$$

When the **REPLICATE** option is used the residuals are calculated as the untransformed residuals:

$$e_t=Y_t-X_t'\hat{\beta}_w$$

When the **NONORM** option is used the residual variance (reported as SIGMA**2 on the SHAZAM output) is estimated as:

$$\frac{1}{\sum_{t=1}^{N} N_t - K} \sum_{t=1}^{N} N_t e_t^2$$

DETECTING INFLUENTIAL OBSERVATIONS

SHAZAM output from the **OLS** command with the **INFLUENCE** option is:

```
 |_OLS CONSUME INCOME PRICE / INFLUENCE
  OLS ESTIMATION
       17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
 ...NOTE..SAMPLE RANGE SET TO:    1,   17

  R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
 SUM OF SQUARED ERRORS-SSE=   433.31
 MEAN OF DEPENDENT VARIABLE =   134.51
 LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME      1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE      -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT    130.71      27.09       4.824      .000  .790      .0000      .9718

         RESIDUAL    RSTUDENT    HT    COVRAT     DFFITS   DFFIT
     1    5.5076       1.2294    .3279   1.3360    .8588   2.6871
     2    2.5765        .5287    .2720   1.6096    .3232   .96281
     3    1.4210        .2804    .2249   1.5824    .1510   .41223
     4   -5.1814       -.9842    .1065   1.1268   -.3399  -.61780
     5   -.25169       -.0456    .0858   1.3655   -.0140 -0.23617E-01
     6   -5.3100      -1.0548    .1746   1.1827   -.4851  -1.1230
     7   -1.9455       -.3829    .2167   1.5418   -.2014  -.53833
     8    .57462        .1152    .2522   1.6652    .0669   .19384
     9    4.3958        .8588    .1694   1.2743    .3879   .89668
    10    1.5426        .2833    .1047   1.3696    .0969   .18045
    11    4.5946        .8670    .1088   1.1839    .3029   .56078
    12   -4.9571      -1.0007    .2072   1.2609   -.5116  -1.2953
    13   -8.8975      -1.9206    .1734    .7142   -.8798  -1.8670
    14    6.4156       1.3427    .2200   1.0846    .7131   1.8099
    15   -2.5614       -.4770    .1197   1.3469   -.1759  -.34841
    16   -7.2886      -1.4519    .1214    .9056   -.5396  -1.0067
    17    9.3650       1.9629    .1147    .6475    .7064   1.2131
```

The HT values are defined as the diagonal values of the Hat matrix (described in Belsley, Kuh, and Welsch [1980, Equations 2.2 and 2.15]) as:

$$h = \text{DIAG}\left(X(X'X)^{-1}X'\right)$$

where DIAG() takes the diagonal of the matrix (the **HATDIAG=** option can be used to save the diagonal). The DFFIT values measure the effect of the coefficient change and measure the change in fit due to a deletion of one observation. Denote X(t) as the observation matrix with row t deleted and b(t) as the estimate obtained with this matrix. Then:

$$\text{DFFIT}_t = \hat{Y}_t - \hat{Y}_t(t) = X_t'\left[\hat{\beta} - b(t)\right] = \frac{h_t e_t}{1 - h_t} \qquad \text{where} \quad \hat{Y}(t) = X(t)b(t) \quad \text{and}$$

$$\text{DFFITS}_t = \left[\frac{h_t}{1 - h_t}\right]^{1/2} \frac{e_t}{\hat{\sigma}(t)\sqrt{1 - h_t}} \qquad \text{where} \quad \hat{\sigma}^2(t) = \frac{1}{N - K - 1}\sum_{k \neq t}\left[Y_k - X_k' b(t)\right]^2$$

For further reference on these statistics see Belsley, Kuh, and Welsch [1980, Equations 2.10 and 2.11].

The RSTUDENT values are the Studentized Residuals as described in Belsley, Kuh, and Welsch [1980, Equation 2.26]:

$$e_t^* = \frac{e_t}{\hat{\sigma}(t)\sqrt{1 - h_t}}$$

The COVRAT statistic is defined as: $$\text{COVRAT}_t = \frac{\hat{\sigma}(t)^{2K}}{\hat{\sigma}^{2K}}\left\{\frac{\det\left[X'(t)X(t)\right]^{-1}}{\det\left(X'X\right)^{-1}}\right\}$$

and compares the covariance matrices $\hat{\sigma}^2(X'X)^{-1}$ and $\hat{\sigma}^2(t)[X'(t)X(t)]^{-1}$ as described in Belsley et al. [1980, Equation 2.36].

If the **DFBETAS** option is specified the SHAZAM output is:

```
|_OLS CONSUME INCOME PRICE / DFBETAS
  OLS ESTIMATION
       17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
 ...NOTE..SAMPLE RANGE SET TO:    1,   17

  R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
 SUM OF SQUARED ERRORS-SSE=   433.31
 MEAN OF DEPENDENT VARIABLE =   134.51
 LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
```

```
INCOME      1.0617       .2667        3.981       .001   .729       .2387       .8129
PRICE      -1.3830       .8381E-01   -16.50       .000  -.975      -.9893      -.7846
CONSTANT    130.71       27.09         4.824       .000   .790       .0000       .9718

 DFBETAS:       INCOME    PRICE     CONSTANT
              -.5514     .6385     .4262
              -.1847     .2480     .1362
              -.0659     .1218     .0418
              -.0556    -.2070     .0927
              -.0031    -.0065     .0041
              -.3208    -.1693     .3511
              -.1455    -.0641     .1574
               .0572     .0024    -.0569
               .3011    -.1392    -.2610
               .0421    -.0552    -.0260
              -.0193    -.1976     .0773
               .3667     .1608    -.4232
               .5942     .2854    -.6951
              -.2951    -.4729     .4291
              -.0086     .1247    -.0269
               .0325     .3740    -.1399
               .1661    -.4864    -.0284
```

The DFBETAS statistic yields a scaled measure of the change on the estimated regression coefficients when row t is deleted (see Belsley et al. [1980, Equation 2.7]).

$$\text{DFBETAS}_{tj} = (\hat{\beta}_j - b_j(t)) \Big/ \left(\hat{\sigma}(t) \sqrt{(X'X)^{-1}_{jj}} \right) \qquad \text{for } j = 1, \ldots, K$$

The DFBETAS statistic yields a scaled measure of the change on the estimated regression coefficients when row t is deleted (see Belsley et al. [1980, Equation 2.7]). Another statistic not listed on the SHAZAM output is:

$$\text{DFBETA}_t = \hat{\beta} - b(t) = (X'X)^{-1} X_t e_t / (1 - h_t)$$

This statistic measures the effect of deleting row t on the estimated regression coefficients (see Belsley et al. [1980, Equation 2.1]).

STEPWISE REGRESSION

Stepwise regression is not widely used in econometrics because most economists use economic theory to determine which variables belong in the model. However it is available for those who think a computer algorithm is smart enough to replace economic theory. For stepwise regression, a slight modification of the **OLS** command is required. The format of the **OLS** command is:

OLS *depvar indeps (stepvars) / options*

where *depvar* is the name of the dependent variable, *indeps* (if present) are the names of the independent variables that are always forced into the equation, *stepvars* are the names of the the variables that may be stepped into the equation, and *options* are the desired options. (See the **FE=**, **FX=**, **PE=**, **PX=** options described above.) The **DWPVALUE**, **HETCOV**, **RESTRICT** and **RIDGE=** options may not be used when stepwise regressions are being run. Stepwise regression automatically uses **METHOD=NORMAL**.

```
|_OLS CONSUME (INCOME PRICE YEAR)
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

**** STEPWISE REGRESSION ****
***PARAMETERS FOR STEPWISE REGRESSION:   PE= .050000 AND  PX= .050000.

-------------------------------------------------------------------------
|**** STEPPING SEQUENCE ****|             |     |      |             |
|      |          |          |             |     |      |             |
| STEP |VARIABLE  |          |             |D.F. | D.F. |             |
|NUMBER| LABEL    | STATUS   |     F-VALUE |NUM. | DEN. |F-PROBABILITY|
|-----------------------------------------------------------------------|
|    1 |PRICE     |STEPPED IN|    129.4014 |   1 |    15|     0.000000|
|    2 |INCOME    |STEPPED IN|     15.8508 |   1 |    14|      .001365|
|*SUMMARY FOR POTENTIAL VARIABLES NOT ENTERED INTO THE REG. EQUATION*|
|      |YEAR      |IF ENTERED|       .3869 |   1 |    13|      .544702|
|**** END OF STEPPING SEQUENCE ****       |     |      |             |
-------------------------------------------------------------------------

 R-SQUARE =    .9513     R-SQUARE ADJUSTED =    .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME      1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE      -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT    130.71      27.09       4.824      .000  .790      .0000      .9718
```

8. HYPOTHESIS TESTING AND CONFIDENCE INTERVALS

"God Himself could not sink this ship."

Titanic deckhand
April 10, 1912

The **TEST** command in SHAZAM can be used for linear or nonlinear hypothesis testing on regression coefficients after model estimation. The **CONFID** command can be used to compute confidence intervals and a confidence ellipse for two coefficients. Several **TEST** or **CONFID** commands can follow an estimation command. A discussion of hypothesis testing and interval estimation can be found in a good econometrics textbook.

HYPOTHESIS TESTING

The **TEST** command follows an estimation command and the general format is:

estimation command
TEST *equation*

where *equation* is an equation made up of combinations of the variables involved in the estimation and represents the hypothesis to be tested. The equation must **not** contain logical relations such as .EQ., .NE., .LT., etc. (Note that to estimate models with inequality restrictions see the features in the chapter *INEQUALITY RESTRICTIONS*.) The equation may only include a variable in the estimated equation or one that has been previously generated with a **GEN1** or **GENR** command. Each single hypothesis must be placed on a separate **TEST** command.

If a *joint test* is required that involves several hypotheses, these should be grouped together with a blank **TEST** command to introduce them and an **END** command to mark the end of the group. The general command format for a joint hypothesis test is:

TEST
TEST *equation* 1
TEST *equation* 2
. . .
END

The **TEST** command calculates t, F, and chi-square test statistics. The appropriate test statistic to use depends on the form of the hypothesis. For example, when the hypothesis specified on the **TEST** command involves non-linear functions of the coefficients then the Wald chi-square statistic is generally used.

An example of hypothesis testing after OLS estimation is given in the next list of SHAZAM commands.

```
stat consume / mean=cbar
stat income  / mean=ibar
ols consume income price
test income=1
test income*(ibar/cbar)
test
  test income=1
  test price=-1
end
test income*price=-1
```

The Theil textile demand data set is analyzed. The first **TEST** command tests the hypothesis that the coefficient on *INCOME* is equal to one. The second **TEST** command computes a standard error for the income elasticity. Note that the equation does not have an = sign. This **TEST** command calculates a value for the function of the coefficients on the variables and then computes a standard error. The third **TEST** command initiates a joint hypothesis test. The final **TEST** command is an example of a non-linear hypothesis and the test statistics computed give a test for the hypothesis that the coefficient on *INCOME* multiplied by the coefficient on *PRICE* is equal to –1. SHAZAM output from the above **TEST** commands is included in the discussion that follows in this chapter.

If the **TEST** command follows non-linear estimation with the **NL** command then *equation* is an equation made up of combinations of the coefficients involved in the estimation. The next list of SHAZAM commands shows how to test hypotheses following nonlinear estimation with the **NL** command (see the chapter *NONLINEAR REGRESSION*). The commands estimate a CES production function with the variables *LOGQ, L* and *K*. The coefficients to estimate are B1, B2, B3, and B4. The **TEST** command specifies a test for $H_0 : B3 = -1$.

```
nl 1 / ncoef=4
eq logq=b1 + b4 * log ( b2*l**b3 + (1-b2)*k**b3)
coef b1 .5 b2 .5 b3 -3 b4 -1
end
test b3=-1
```

For instructions on doing hypothesis testing on systems of equations, see the chapter *TWO STAGE LEAST SQUARES AND SYSTEMS OF EQUATIONS*.

Temporary variables available following a **TEST** command are:

$CHI	Wald chi-square statistic
$DF1, $DF2	Numerator and denominator degrees of freedom for the F distribution
$ERR	Error code
$F	F statistic

When a single hypothesis test is specified the temporary variables available are:

$DF	Degrees of freedom for the t distribution
$STES	Standard error of test value
$T	t statistic
$VAL	test value

When a joint test with exactly two **TEST** commands is used the temporary variables *$VAL1, $VAL2, $CT11, $CT22, $CT12* store values used to construct the test statistic (discussed later in this chapter). For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.

Testing a Single Linear Combination of Coefficients

Consider testing $H_0 : R_1\beta = r$ where R_1 is a known (1 x K) row vector and r is a known scalar. The simplest case is a hypothesis involving a single coefficient of the form $H_0 : \beta_k = r$ (that is, R_1 has a 1 in element k and 0's for all other elements). The test statistic is:

$$t = \frac{R_1\hat{\beta} - r}{\sqrt{R_1 V(\hat{\beta}) R_1'}}$$

where $V(\hat{\beta})$ is the (K x K) estimated variance-covariance matrix of $\hat{\beta}$.

On the SHAZAM output the numerator is reported as the `TEST VALUE` and the denominator is reported as the `STD. ERROR OF TEST VALUE`. Under the assumption that $H_0 : R_1\beta = r$ is true and with normally distributed errors the test statistic can be compared with a t-distribution with (N–K) degrees of freedom.

SHAZAM output from a **TEST** command is given below. The null hypothesis is that the coefficient on the variable *INCOME* is equal to 1.

```
| _TEST INCOME=1
TEST VALUE =    .61709E-01 STD. ERROR OF TEST VALUE    .26667
T STATISTIC =    .23140356     WITH    14 D.F.     P-VALUE=  .82035
F STATISTIC =    .53547606E-01 WITH     1 AND    14 D.F.  P-VALUE=  .82035
WALD CHI-SQUARE STATISTIC =    .53547606E-01 WITH     1 D.F.  P-VALUE=  .81700
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 1.00000
```

The SHAZAM output reports a p-value for a two-tailed test. In this example:

$$\text{p-value} = \Pr[t_{(14)} < -0.2314 \text{ and } t_{(14)} > 0.2314] = 2(1 - \Pr[t_{(14)} < 0.2314]) = 0.82035.$$

The calculated p-value exceeds any standard significance level (such as 0.10 or 0.05) and so there is no evidence to reject the null hypothesis that the coefficient on *INCOME* is equal to one.

In the case of a single linear hypothesis the test statistic values satisfy:

$$t^2 = \text{F statistic} = \text{Wald chi-square statistic.}$$

Now consider calculating a standard error for an elasticity. With a linear regression equation an elasticity of the dependent variable with respect to the k^{th} explanatory variable can be estimated as:

$$E_k = \hat{\beta}_k \, \overline{X}_k / \overline{Y}$$

where $\hat{\beta}_k$ is the estimated coefficient and $\overline{X}_k$ is the sample mean of variable k, and $\overline{Y}$ is the mean of the dependent variable. The SHAZAM OLS estimation output reports this value in the column labelled `ELASTICITY AT MEANS`. However, it may also be of interest to compute a standard error for this statistic. The standard error can be estimated as:

$$\sqrt{V(\hat{\beta}_k)\left(\overline{X}_k / \overline{Y}\right)^2}$$

An example of the use of the **TEST** command to do this calculation is given below. With the Theil textile demand equation the standard error of the elasticity of consumption with respect to income is reported as the `TEST VALUE`. The `T STATISTIC` gives a test for the null hypothesis that the income elasticity is equal to zero.

```
| _TEST INCOME*(IBAR/CBAR)
TEST VALUE =    .81288     STD. ERROR OF TEST VALUE    .20417
T STATISTIC =    3.9813016     WITH    14 D.F.     P-VALUE=  .00137
F STATISTIC =    15.850762     WITH     1 AND    14 D.F.  P-VALUE=  .00137
WALD CHI-SQUARE STATISTIC =    15.850762     WITH     1 D.F.  P-VALUE=  .00007
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY =  .06309
```

Testing More Than One Linear Combination of Coefficients

A general linear hypothesis can be expressed as $H_0 : R\beta = r$ where R is a known (q x K) matrix and r is a (q x 1) column vector. The test statistic is:

$$F = \frac{1}{q}(R\hat{\beta} - r)'[RV(\hat{\beta})R']^{-1}(R\hat{\beta} - r)$$

When the null hypothesis is true the F-statistic has an F-distribution with (q, N–K) degrees of freedom. For q=2, the temporary variables *$VAL1, $VAL2* store the elements of $R\hat{\beta} - r$ and *$CT11, $CT22, $CT12* store the elements of the matrix $RV(\hat{\beta})R'$.

With the Theil textile demand equation consider testing the joint hypothesis that the coefficients on *INCOME* (β_1) and *PRICE* (β_2) are 1 and –1 respectively. The intercept parameter is β_3. In matrix notation this can be written as:

$$H_0 : \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The next SHAZAM output shows the results from the **TEST** command.

```
|_TEST
|_  TEST INCOME=1
|_  TEST PRICE=-1
|_END
F STATISTIC =   10.617226     WITH    2 AND   14 D.F.  P-VALUE=  .00156
WALD CHI-SQUARE STATISTIC =   21.234451     WITH    2 D.F.  P-VALUE=  .00002
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY =  .09419
```

The F test statistic is 10.617226. By consulting statistical tables it is found that the 5% critical value for F(2,14) is 3.74. The test statistic exceeds the critical value and so the conclusion is that the hypothesis is not supported by the data. The reported p-value of 0.00156 suggests that the null hypothesis is rejected even at a 1% significance level.

Note that in general the Wald chi-square statistic is equivalent to the F statistic multiplied by the number of hypotheses q and is distributed χ^2 with q degrees of freedom.

Testing Non-Linear Functions of Coefficients

Non-linear hypotheses can be stated in the general form $H_0 : h(\beta) = 0$ where $h(\beta)$ is a set of q non-linear functions of the model parameters. The (q x K) matrix of partial derivatives is:

$$G = \frac{\partial h(\beta)}{\partial \beta}$$

The j^{th} row of G is a vector of partial derivatives of the j^{th} function with respect to β. The Wald test statistic is:

$$\text{Wald} = h(\hat{\beta})' \left[\hat{G} V(\hat{\beta}) \hat{G}' \right]^{-1} h(\hat{\beta})$$

where $\hat{G}$ is the matrix G evaluated at $\beta = \hat{\beta}$. When the null hypothesis is true the Wald statistic has an asymptotic chi-square distribution with q degrees of freedom. When q=1 then $t = \sqrt{\text{Wald}}$ has an asymptotic standard normal distribution under the null hypothesis.

Users should be aware that the value of the test statistic for a non-linear hypothesis is sensitive to the form of the equation (see Gregory and Veall [1985] and Lafontaine and White [1986]). Since the small sample distributions of non-linear test statistics are not known, the statistics reported as t, F, and Wald chi-square should be interpreted with caution.

SHAZAM uses analytical derivatives for the computation of the Wald test statistic. The nonlinear hypothesis specified on the **TEST** command can contain the functions LOG( ) and EXP( ). Other functions are not allowed. If a hypothesis with a square root function of coefficients is required then write the function in the form (*expression*)**.5 and not SQRT(*expression*).

The next SHAZAM output shows the results of a test of the non-linear hypothesis that the product of two coefficients is equal to –1.

```
| _TEST INCOME*PRICE=-1
 TEST VALUE =  -.46833     STD. ERROR OF TEST VALUE   .39456
 T STATISTIC =  -1.1869717     WITH   14 D.F.    P-VALUE=  .25499
 F STATISTIC =   1.4089019     WITH    1 AND   14 D.F.  P-VALUE=  .25499
 WALD CHI-SQUARE STATISTIC =   1.4089019     WITH    1 D.F.  P-VALUE=  .23524
 UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY =  .70977
```

The Chebychev Inequality

A summary of the distributions of test statistics is given in the table:

	Linear Model $Y = X\beta + \varepsilon$	**Non-linear Model** $Y = f(X,\beta) + \varepsilon$
Linear Hypothesis H_0: $R\beta=r$	$t_{(N-K)}$ for q=1, $F_{(q,N-K)}$ for q>1	asymptotic N(0,1) for q=1, asymptotic $\chi^2_{(q)}$ for q>1
Non-linear Hypothesis $H_0 : h(\beta) = 0$	asymptotic N(0,1) for q=1, asymptotic $\chi^2_{(q)}$ for q>1	asymptotic N(0,1) for q=1, asymptotic $\chi^2_{(q)}$ for q>1

In cases other than testing linear hypotheses of coefficients from the linear regression model the small sample distributions of the test statistics are unknown. We need to depend on the asymptotic distributions which are valid only in large samples. When applied to small samples the only probability known with certainty is the upper bound obtained from the Chebychev inequality.

For a random variable Z with zero mean and unit variance the univariate Chebychev inequality may be written:

$$\Pr[|Z| > D] \leq 1/D^2$$

where D is a positive constant. This gives an upper bound on the probability of $1/D^2$. For a t test statistic, in a circumstance in which we are uncertain if a t distribution is appropriate, an upper bound for the p-value is: $1/t^2$ (the maximum is 1.0).

For a sequence $\{Z_n\}$ for n = 1, . . . , q the multivariate Chebychev inequality states:

$$\Pr[g(Z_n) > D] \leq \frac{q}{D}$$

where g() is any non-negative continuous function. For a discussion, see Dhrymes [1978, pp. 382-384]. This result allows the computation of an upper bound for the p-value of a chi-square test statistic obtained from a joint hypothesis test.

The SHAZAM output below shows the results of a joint hypothesis test involving a non-linear function of the coefficients.

```
|_TEST
|_  TEST PRICE=-1
|_  TEST INCOME*PRICE=-1
|_END
F STATISTIC =   10.658128     WITH    2 AND   14 D.F.  P-VALUE=  .00154
WALD CHI-SQUARE STATISTIC =   21.316256     WITH    2 D.F.  P-VALUE=  .00002
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY =  .09383
```

The calculated chi-square statistic is 21.316256 and the reported p-value is 0.00002. This suggests rejection of the null hypothesis. But, with a small sample, we may be reluctant to rely on the asymptotic chi-square distribution. The Chebychev inequality yields an upper bound on the p-value as 2 / 21.316256 = 0.09383. Therefore, the null hypothesis is not rejected at a 5% level, but there is evidence to reject at a 10% level.

CONFIDENCE INTERVALS

The **CONFID** command computes confidence intervals. In general, the format of the **CONFID** command is:

estimation command
CONFID *var1 var2 ... / options*

where *var1* and *var2* are variable names and *options* is a list of available options. When the **CONFID** command follows a **STAT** command confidence intervals are estimated for the population means of the variables. When the **CONFID** command follows regression estimation confidence intervals are estimated for the regression coefficients of the variables. Following a **NL** command the **CONFID** command must specify the names of the coefficients involved in the estimation.

Options available on the **CONFID** command are:

NORMAL Specifies that the normal distribution rather than the t distribution should be used in computing confidence regions. Often, we only know that the coefficients are asymptotically normally distributed and the use of the t or F distribution would be inappropriate.

DF= Specifies the degrees of freedom to use for obtaining 5% and 10% critical values from the t-distribution. The default setting is the degrees of freedom from the previous estimation command.

TCRIT= Specifies the t-distribution critical value for calculating confidence intervals. This value can be obtained from tables of the t-distribution or

from output associated with use of the **DISTRIB** command. If this option is not specified then SHAZAM computes the critical values for 90% and 95% confidence intervals. An example of the use of the **TCRIT=** option is given later in this chapter.

When only 2 coefficients are listed, the **CONFID** command will also compute a plot of the confidence ellipse. The available **PLOT** options (see the chapter *PLOTS AND GRAPHS*) are:

HOLD, **NOBLANK**, **NOWIDE**, **SYMBOL**, **WIDE**, **XMAX=**, **XMIN=**, **YMAX=** and **YMIN=.**

Other options available on the **CONFID** command are:

GRAPH Prepares a gnuplot plot of the joint confidence region when 2 coefficients are specified. For more information on this option see the chapter *PLOTS AND GRAPHS*. With the **GRAPH** option the **APPEND**, **AXIS**, **NOAXIS**, **COMMFILE=**, **DEVICE=**, **OUTPUT=** and **PORT=** options are also available as described for the **GRAPH** command.

NOFPLOT Omits the joint confidence region plot when two coefficients are specified. This option would be used if you only wanted the individual confidence intervals for the coefficients.

NOMID Omits the display of a symbol at the center of the confidence ellipse.

NOTPLOT Omits the computation of the confidence intervals for individual coefficients. This might be used if only two coefficients were specified and you only want the joint confidence region plot (**FPLOT**). Normally, the individual confidence intervals are shown on the joint plot with a plus symbol (+) to show the outline of the confidence rectangle. The **NOTPLOT** option will suppress the drawing of the (+) symbol.

DF= Specifies the degrees of freedom to use for obtaining 5% and 10% critical values from the t-distribution. With 2 coefficients this will be the denominator degrees of freedom for obtaining a 5% critical value from the F-distribution. This option is also described above.

FCRIT= Specifies the critical value from an F-distribution to be used for the joint confidence region plot. This option is similar to the **TCRIT=** option described below and is usually used to complement it. If the **FCRIT=** option is not specified a 5% critical value will be used.

POINTS= SHAZAM usually constructs the confidence ellipse by evaluating the ellipse at approximately 200-205 points. The number used can be changed with this option if more or fewer points are desired to obtain a better looking plot.

In some cases, users would like to compute the confidence region for coefficients that have been estimated in an earlier run and do not wish to re-estimate a model. This is possible for any set of two coefficients where you tell SHAZAM the estimated values of the coefficients, their estimated variances, and the covariance between the coefficients. In this case, the **CONFID** command does not need to follow an estimation command, but the following options ***must*** be included:

COEF1= Specifies the coefficient estimate for the first coefficient you wish to plot. This coefficient will appear on the Y-axis of the plot.

COEF2= Specifies the coefficient estimate for the second coefficient you wish to plot. This coefficient will appear on the X-axis of the plot.

COVAR12= Specifies the estimated covariance between the two coefficients.

DF= Specifies the number of degrees of freedom. This is typically (N–K). This option is also described above.

VAR1= Specifies the estimated variance of the first coefficient.

VAR2= Specifies the estimated variance of the second coefficient.

Interval Estimation for a Population Mean

A $100\cdot(1-\alpha)\%$ confidence interval for a population mean is: $\overline{X} \pm t_c \, \hat{\sigma}_X / \sqrt{N}$

where t_c is the $\alpha/2$ critical value from a t-distribution with (N–1) degrees of freedom. SHAZAM sets values of t_c to give 95% and 90% confidence interval estimates.

The example below uses a sample of 6 observations on fuel consumption for cars from Example 8.4 in Newbold [1995, pp. 286-7]. The results show 95% and 90% interval estimates for the population mean fuel consumption.

```
|_SAMPLE 1 6
|_READ FUEL / BYVAR LIST
   1 VARIABLES AND        6 OBSERVATIONS STARTING AT OBS       1
    FUEL
   18.60000       18.40000       19.20000       20.80000       19.40000
   20.50000

|_STAT FUEL
NAME         N     MEAN         ST. DEV      VARIANCE     MINIMUM      MAXIMUM
FUEL           6   19.483      0.98065     0.96167       18.400       20.800

|_CONFID FUEL
USING 95% AND 90% CONFIDENCE INTERVALS
CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH   5 D.F.
     - T CRITICAL VALUES =   2.571 AND   2.015
NAME   LOWER 2.5%   LOWER 5%   COEFFICIENT   UPPER 5%   UPPER 2.5%   STD. ERROR
FUEL       18.45       18.68       19.483       20.29       20.51       0.400
```

Interval Estimation for a Single Regression Coefficient

A $100\cdot(1-\alpha)\%$ confidence interval for a coefficient β_k is: $\hat{\beta}_k \pm t_c\sqrt{V(\hat{\beta}_k)}$

where t_c is the $\alpha/2$ critical value from a t-distribution with (N–K) degrees of freedom. SHAZAM sets values of t_c to give 95% and 90% confidence interval estimates. However, other values for t_c can be specified with the **TCRIT=** option on the **CONFID** command.

The next output shows the use of the **CONFID** command to compute interval estimates for the coefficients of the Theil textile demand equation.

```
|_OLS CONSUME INCOME PRICE
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17
 R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME      1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE      -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT    130.71      27.09       4.824      .000  .790      .0000      .9718

|_CONFID INCOME PRICE CONSTANT
USING 95% AND 90% CONFIDENCE INTERVALS
CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH  14 D.F.
     - T CRITICAL VALUES =   2.145 AND   1.761
NAME   LOWER 2.5%   LOWER 5%    COEFFICENT   UPPER 5%   UPPER 2.5%  STD. ERROR
```

INCOME	.4897	.5921	1.0617	1.531	1.634	.267
PRICE	-1.563	-1.531	-1.3830	-1.235	-1.203	.084
CONSTANT	72.59	82.99	130.71	178.4	188.8	27.094

The 95% interval estimate for the coefficient on *INCOME* is:

$$1.0617 \pm 2.145 \cdot 0.267 = [0.4897 , 1.634]$$

Interval Estimation for the Error Variance

An interval estimate for the error variance can be obtained by specifying the name *$SIG2* on the **CONFID** command. For example,

```
ols consume income price
confid $sig2
```

A $100 \cdot (1-\alpha)\%$ interval estimate for σ^2 is calculated as:

$$\left[(N-K)\hat{\sigma}^2 \Big/ \chi^2_{N-K,\alpha/2} \; , \; (N-K)\hat{\sigma}^2 \Big/ \chi^2_{N-K,1-\alpha/2} \right]$$

where $\chi^2_{N-K,\alpha/2}$ is the critical value from a chi-square distribution with N–K degrees of freedom.

Estimation of a Joint Confidence Region for Two Regression Coefficients

Suppose β^* contains 2 coefficients from the β parameter vector and $V(\hat{\beta}^*)$ is a (2 x 2) sub-matrix of the full covariance matrix. A $100 \cdot (1-\alpha)\%$ confidence ellipse for β^* is the set of values that satisfies:

$$\frac{1}{2}(\hat{\beta}^* - \beta^*)'[V(\hat{\beta}^*)]^{-1}(\hat{\beta}^* - \beta^*) \le F_c$$

where F_c is the α critical value from a F-distribution with (2, N–K) degrees of freedom. The rectangle that is formed by the two individual confidence intervals each with probability 1–α gives a confidence region with probability ≥ 1–2α (see Theil [1971, p. 132]). SHAZAM sets F_c to give a 95% confidence ellipse. Other values for F_c can be set with the **FCRIT=** option.

The next example shows the use of the **CONFID** command to obtain a joint confidence region for the coefficients on the *INCOME* and *PRICE* variables from the Theil textile

demand equation. Note that critical values are specified with the **TCRIT=** and **FCRIT=** options to give 99% interval estimates and a 99% confidence ellipse.

```
|_CONFID INCOME PRICE / TCRIT=2.977 FCRIT=6.51

CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH  14 D.F.
     - T CRITICAL VALUE =   2.977
NAME         LOWER      COEFFICENT     UPPER       STD. ERROR
 INCOME      .26782       1.0617       1.8556       .26667
 PRICE     -1.6325       -1.3830      -1.1335       .83814E-01

CONFIDENCE REGION PLOT FOR INCOME   AND PRICE
 USING F DISTRIBUTION WITH 2 AND   14 D.F.     F-VALUE =   6.510

      205 OBSERVATIONS
                M=MULTIPLE POINT
   2.4000        |
   2.2737        |
   2.1474        |
   2.0211        |                  M
   1.8947        |             MM**     ****M
   1.7684        |          +MM              *MM*   +
   1.6421        |        MM                    MM*
   1.5158        |       M*                       MM
   1.3895        |      MM                         M
   1.2632        |      M                           M
   1.1368        |      M                            M
   1.0105        |      M               *            M
   .88421        |       M                           *M
   .75789        |       MM                          MM
   .63158        |        MM                         M
   .50526        |         MM                       MM
   .37895        |           MM                     MM
   .25263        |         +   MMM                MM+
   .12632        |                M*M*        ***M*
   .66613E-15    |                     * M   *
                 ___________________________________________
            -1.800    -1.600    -1.400    -1.200    -1.000
                              PRICE
```

The individual confidence intervals are shown on the above plot with a + symbol to show the outline of the confidence rectangle. The point estimate is displayed at the center of the confidence ellipse with the * symbol.

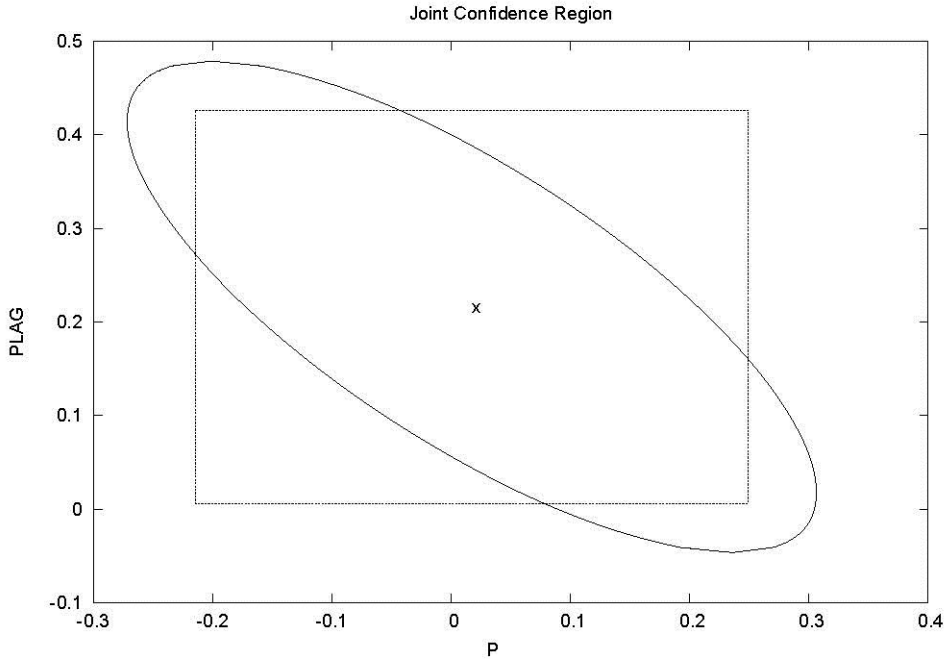
A GNUPLOT plot of the confidence ellipse can be obtained by specifying the **GRAPH** option on the **CONFID** command. This is illustrated by showing the use of the **CONFID** command after the **2SLS** command. The estimation is for the first equation of the Klein model (see the chapter *TWO STAGE LEAST SQUARES AND SYSTEMS OF EQUATIONS)*. Since the 2SLS estimated coefficients are assumed to be asymptotically normally distributed and the small sample properties are unknown, the **NORMAL** option is specified. The **NOAXIS** option specifies to omit the vertical and horizontal lines drawn at 0. The SHAZAM commands are as follows:

```
sample 1 21
2sls c plag p wgwp (wg t g time plag klag xlag) / dn
confid plag p / normal graph noaxis
```

The SHAZAM output from the **CONFID** command is:

```
|_CONFID PLAG P / NORMAL GRAPH NOAXIS
USING 95% AND 90% CONFIDENCE INTERVALS

CONFIDENCE INTERVALS BASED ON NORMAL DISTRIBUTION WITH
CRITICAL VALUES=   1.960 AND   1.645
NAME   LOWER 2.5%   LOWER 5%    COEFFICENT   UPPER 5%   UPPER 2.5%  STD. ERROR
PLAG       .5989E-02   .3978E-01   .21623       .3927       .4265       .107
P         -.2141      -.1769       .17302E-01   .2115       .2487       .118
CONFIDENCE REGION PLOT FOR PLAG     AND P
USING CHI-SQUARE DISTRIBUTION WITH CRITICAL VALUE=   5.990
```

For the plot of the joint confidence region the point estimate in the center of the confidence ellipse is marked with the label "x".



9. INEQUALITY RESTRICTIONS

"However, inequality constrained linear regression is not an option of any of the more popular econometrics software packages. Many practitioners may lack the gall (if not the resources) to determine the solution using ordinary least squares regression packages."

John Geweke, 1986

The **BAYES** command provides a procedure for estimation with inequality restrictions. The methodology is described in Geweke [1986]; Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 20]; and Chalfant, Gray and White [1991]. The idea is to compute Bayes estimates as the mean of the truncated multivariate t posterior. Suppose that unrestricted estimation gives parameter estimates $\hat{\beta}$ with a variance-covariance estimate $V(\hat{\beta})$. The method uses a Monte Carlo numerical integration procedure that is implemented by generating replications from a multivariate t distribution. At replication i draw the vector w_i from a N(0, $V(\hat{\beta})$) distribution and draw z_i from a χ^2 distribution with v degrees freedom (for the linear regression model, v = N–K). Then compute the replication:

$$\beta_i^A = \hat{\beta} + w_i / \sqrt{z_i^2 / v}$$

The antithetic replication is computed as:

$$\beta_i^B = \hat{\beta} - w_i / \sqrt{z_i^2 / v}$$

The antithetic replication simply changes the sign to essentially give an additional replication at low cost and ensuring a symmetric distribution (see Geweke [1988] for further details). The inequality constrained estimates and standard errors are reported as the mean and standard deviation of the values that satisfy the inequality restrictions.

Suppose that R is the number of replications and s is the number that satisfy the restrictions. On the SHAZAM output the `PROPORTION` is computed as $\hat{p} = s/R$ and this gives the probability that the restrictions are true. The `NUMERICAL STANDARD ERROR OF PROPORTION` (a standard error for numerical accuracy) is computed as $\sqrt{\hat{p}(1-\hat{p})/R}$. The column with the label `NUMERICAL SE` is the standard deviation of the mean computed as the standard deviation divided by $\sqrt{s}$.

The method is computer intensive and initial experimentation with relatively few replications is recommended. Different machines will yield different results due to the

machine differences in random number generation. The **SET RANFIX** command can be used to ensure the same set of random numbers in repeated runs.

BAYES *COMMAND OPTIONS*

In general, the format for estimation with inequality restrictions is:

estimation command
BAYES / *options* **NSAMP=**
RESTRICT *inequality restriction*
. . .

END

where *estimation command* is an estimation command such as **OLS** and *options* is a list of options. **NSAMP=** is the number of replications. The **NSAMP=** option is required. The **BAYES** command is followed by **RESTRICT** commands that specify the inequality restrictions, and an **END** command. The inequality restrictions must use one of the operators .LT., .LE., .GT. or .GE.. The inequality restrictions can be linear or non-linear.

The available options on the **BAYES** command are:

NOANTITHET Omits the **ANTITHETIC** replications.

NORMAL Assumes the coefficients are normally distributed rather than t distributed.

PSIGMA Prints the `ORIGINAL COVARIANCE MATRIX ESTIMATE` and the `NUMERICAL COVARIANCE MATRIX`. The latter is computed as:

$$\frac{1}{R}\sum_{i=1}^{R} w_i w_i'$$

DF= Specifies the degrees of freedom for the t distribution. The default setting is N–K.

NSAMP= Specifies the number of replications. The **NSAMP=** option is required. This should be a small number (for example 100), while the user is experimenting. Readers of the Geweke article will realize that **NSAMP=** is often very high.

OUTUNIT= Specifies an output unit number to write all the replicated coefficients that satisfy the inequality restrictions. The output unit should be assigned with the SHAZAM **FILE** command. If **NSAMP=** is a large number the resulting file could be quite large.

EXAMPLES

Linear Regression with Inequality Restrictions

This example of the **BAYES** command uses the rental data set of Pindyck and Rubinfeld [1998, Table 2.1, p. 54] that is also analyzed in the Geweke [1986] article.

```
|_SAMPLE 1 32
|_READ RENT NO RM S DUM
   5 VARIABLES AND       32 OBSERVATIONS STARTING AT OBS       1
|_GENR Y=RENT/NO
|_GENR R=RM/NO
|_GENR SR=S*R
|_GENR OSR=(1-S)*R
|_GENR SD=S*DUM
|_GENR OSD=(1-S)*DUM
|_* Estimation by OLS - results from column 1 of Geweke's Table ii.
|_OLS Y SR OSR SD OSD
 OLS ESTIMATION
      32 OBSERVATIONS     DEPENDENT VARIABLE = Y
...NOTE..SAMPLE RANGE SET TO:      1,     32

 R-SQUARE =   0.4525     R-SQUARE ADJUSTED =   0.3713
VARIANCE OF THE ESTIMATE-SIGMA**2 =   1395.5
STANDARD ERROR OF THE ESTIMATE-SIGMA =   37.356
SUM OF SQUARED ERRORS-SSE=   37678.
MEAN OF DEPENDENT VARIABLE =   138.17
LOG OF THE LIKELIHOOD FUNCTION = -158.544

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
SR          103.55      38.47       2.692     0.012 0.460     1.0083     0.2264
OSR         122.04      37.36       3.267     0.003 0.532     1.1597     0.5290
SD          3.3151      1.961       1.690     0.102 0.309     0.3463     0.0555
OSD        -1.1535     0.5714      -2.019     0.054-0.362    -0.3480    -0.0900
CONSTANT    38.562      32.22       1.197     0.242 0.224     0.0000     0.2791

|_* Now use Geweke's method to impose inequality restrictions.
|_* Use 1000 antithetic pairs (2000 replications).
|_* This gives results similar to column 3 of Geweke's table ii.
|_* Since random numbers are used, different results will be obtained
|_* in different runs and on different computers.
|_SET CPUTIME
|_BAYES / NSAMP=1000
BAYESIAN (GEWEKE) INEQUALITY CONSTRAINED ESTIMATION
|_RESTRICT SR.GT.0
|_RESTRICT OSR.GT.0
|_RESTRICT SD.LT.0
|_RESTRICT OSD.LT.0
```

```
|_END

NUMBER OF INEQUALITY RESTRICTIONS =   4
NUMBER OF COEFFICIENTS=  5
NUMBER OF REPLICATIONS=      1000
ANTITHETIC REPLICATIONS ALSO INCLUDED
DEGREES OF FREEDOM FOR T DISTRIBUTION =       27
ORIGINAL COEFFICIENT ESTIMATES
   103.55        122.04        3.3151       -1.1535        38.562
      2000 REPLICATIONS        95 SATISFIED
PROPORTION= 0.04750 NUMERICAL STANDARD ERROR OF PROPORTION= 0.00476
                    ASYMPTOTIC STANDARD ERROR OF PROPORTION= 0.00476

VARIABLE  AVERAGE       STDEV       VARIANCE     NUMERICAL SE
SR         141.97       35.315       1247.1       3.6232
OSR        129.52       38.475       1480.4       3.9475
SD        -1.1777       1.0566       1.1163      0.10840
OSD       -1.3733      0.66244      0.43883      0.67965E-01
CONSTANT   35.130       33.957       1153.0       3.4839
|_DISPLAY CPUTIME
CPUTIME     0.60000E-01

|_* Now attempt to impose the restrictions directly using the trick
|_* of squaring a coefficient to force it positive. This requires
|_* nonlinear estimation.
|_* This gives results similar to column 2 of Geweke's table ii.
|_SET CPUTIME
|_NL 1 / NC=5 PITER=50
...NOTE..SAMPLE RANGE SET TO:     1,    32
|_EQ Y = A + (B**2)*SR + (C**2)*OSR - (D**2)*SD - (E**2)*OSD
|_END
   5 VARIABLES IN  1 EQUATIONS WITH   5 COEFFICIENTS
        32 OBSERVATIONS

COEFFICIENT STARTING VALUES
A          1.0000      B          1.0000      C          1.0000
D          1.0000      E          1.0000
      100 MAXIMUM ITERATIONS, CONVERGENCE =   0.100000E-04

INITIAL STATISTICS :
TIME =      0.000 SEC.   ITER. NO.     0   FUNCT. EVALUATIONS     1
LOG-LIKELIHOOD FUNCTION=   -207.0281
COEFFICIENTS
   1.000000       1.000000       1.000000       1.000000       1.000000
GRADIENT
  0.1960723      0.1349835      0.2307822      -1.165898      -4.093769

FINAL STATISTICS :
TIME =      0.020 SEC.   ITER. NO.    33   FUNCT. EVALUATIONS    45
LOG-LIKELIHOOD FUNCTION=   -160.1530
COEFFICIENTS
   37.63371       11.40220       11.09239      0.8808439E-08  -1.073957
GRADIENT
 -0.1582881E-07 -0.8567296E-07 -0.1860299E-06 -0.2527607E-06  0.1327808E-07

MAXIMUM LIKELIHOOD ESTIMATE OF SIGMA-SQUARED =   1302.0
SUM OF SQUARED ERRORS =   41665.
GTRANSPOSE*INVERSE(H)*G  STATISTIC  -  =  0.53927E-13

      COEFFICIENT    ST. ERROR   T-RATIO
A         37.634      30.255      1.2439
```

```
B          11.402       1.4500       7.8637
C          11.092       1.5915       6.9697
D        0.88084E-08  0.72146     0.12209E-07
E         -1.0740      0.25274     -4.2492
|_END
|_DISPLAY CPUTIME
CPUTIME     0.20000E-01
|_* The estimated coefficients are found by squaring each
|_* of the parameters.
|_* TEST commands can be used to obtain the coefficients and standard errors
|_* in the output labeled "TEST VALUE" and "STD. ERROR OF TEST VALUE".
|_* Be careful when using these standard errors.  For details
|_* see Lafontaine and White, "Obtaining Any Wald Statistic
|_* You Want", Economics Letters, 1986.
|_TEST B**2
TEST VALUE =   130.01     STD. ERROR OF TEST VALUE   33.066
ASYMPTOTIC NORMAL STATISTIC =   3.9318297      P-VALUE= 0.00008
WALD CHI-SQUARE STATISTIC =   15.459285     WITH    1 D.F.  P-VALUE= 0.00008
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 0.06469
|_TEST C**2
TEST VALUE =   123.04     STD. ERROR OF TEST VALUE   35.308
ASYMPTOTIC NORMAL STATISTIC =   3.4848358      P-VALUE= 0.00049
WALD CHI-SQUARE STATISTIC =   12.144080     WITH    1 D.F.  P-VALUE= 0.00049
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 0.08234
|_TEST -(D**2)
TEST VALUE = -0.77589E-16 STD. ERROR OF TEST VALUE  0.12710E-07
ASYMPTOTIC NORMAL STATISTIC = -0.61046306E-08  P-VALUE= 1.00000
WALD CHI-SQUARE STATISTIC =  0.37266515E-16 WITH    1 D.F.  P-VALUE= 1.00000
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 1.00000
|_TEST -(E**2)
TEST VALUE =  -1.1534     STD. ERROR OF TEST VALUE  0.54287
ASYMPTOTIC NORMAL STATISTIC =  -2.1245979      P-VALUE= 0.03362
WALD CHI-SQUARE STATISTIC =   4.5139163     WITH    1 D.F.  P-VALUE= 0.03362
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 0.22154
```

The resulting coefficient estimates satisfy the inequality constraints specified. However, in this example, the probability that the restrictions are true is only .0475. Note, that the **BAYES** command requested only 1000 antithetic pairs (2000 replications). The examples in the Geweke paper used from 10,000 to 250,000 antithetic pairs. An increase in the number of replications will increase the computer time required to complete the estimation.

Seemingly Unrelated Regression with Inequality Restrictions

This example considers the estimation of firm investment relations using the data set in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Table 11.1, p. 453] (from an article by Boot and Dewitt and discussed by Theil). Data is available for General Electric and Westinghouse on investment (*IGE*, *IWH*), market value (*VGE*, *VWH*) and capital stock (*KGE*, *KWH*). Consider estimation of the system:

$$IGE = \beta_{11} + \beta_{12} VGE + \beta_{13} KGE + \varepsilon_1$$
$$IWH = \beta_{21} + \beta_{22} VWH + \beta_{23} KWH + \varepsilon_2$$

subject to the inequality restrictions $\beta_{12} > 0$, $\beta_{13} > 0$, $\beta_{22} > 0$ and $\beta_{23} > 0$.

The commands that follow show how to set-up the estimation in SHAZAM. The **SET RANFIX** command is used to ensure the same random numbers in repeated runs. The **SYSTEM** command is used to obtain seemingly unrelated regression estimates. Note that when this is combined with the **BAYES** command the variable names should be different across equations. If the same variable is used across equations then the **GENR** command can be used to create a duplicate variable with a different name. With the **SYSTEM** command, SHAZAM saves the coefficients and variance-covariance matrix in a form that does not include the intercept terms. Therefore, if inequality restrictions on the intercept terms are required then a constant term must be generated and included as a variable and the **NOCONSTANT** option must then be specified on the **SYSTEM** command.

The **BAYES** command in this example specifies the **NORMAL** option. This may be appropriate when the estimation procedure is not OLS. If the **NORMAL** option is not used then the **DF=** option can be used to set the degrees of freedom for the t distribution. The **NSAMP=** option requests 5000 replications. SHAZAM will then generate 5000 antithetic pairs to effectively give 10,000 replications.

```
set ranfix
sample 1 20
read (table11.1) ige vge kge iwh vwh kwh
system 2 / dn
  ols ige  vge kge
  ols iwh  vwh kwh
  bayes / normal nsamp=5000
  restrict vge.gt.0
  restrict kge.gt.0
  restrict vwh.gt.0
  restrict kwh.gt.0
end
stop
```

10. ARIMA MODELS

"The end of the decline of the Stock Market will ... probably not be long, only a few more days at most."

Irving Fisher, Economist
November 14, 1929

The **ARIMA** command provides features for the Box-Jenkins approach (see Box and Jenkins [1976]) to the analysis of AutoRegressive Integrated Moving Average models of univariate time series. SHAZAM uses a modified version of programs written by Charles Nelson and described in Nelson [1973]. Other references are Harvey [1981]; Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 7] and Griffiths, Hill and Judge [1993, Chapter 20]. The **ARIMA** command has three stages. The identification stage reports the sample autocorrelation function and the sample partial autocorrelation function that can be inspected to determine a specification for an ARIMA model. The estimation stage estimates the parameters of an ARIMA model and gives diagnostic tests for checking the model adequacy. The Box-Jenkins method is to repeat the identification and estimation stage until a suitable model is found. The forecasting stage provides point forecasts and confidence intervals.

In general, the format of the **ARIMA** command is:

ARIMA *var / options*

where *var* is a time series variable and *options* is a list of desired options. The options specified determine the stage as identification, estimation or forecasting. Options that start with **GRAPH** prepare graphs with the GNUPLOT software as described in the *PLOTS AND GRAPHS* chapter.

IDENTIFICATION

For a time series z_t, $t = 1, \ldots, N$ with sample mean $\bar{z}$ the sample autocovariances are:

$$c_j = \frac{1}{N} \sum_{t=j+1}^{N} \left[(z_t - \bar{z})(z_{t-j} - \bar{z}) \right] \qquad j = 0, 1, 2, \ldots$$

The sample autocorrelations are: $r_j = c_j / c_0 \qquad j = 1, 2, \ldots$

A plot of the r_j gives the sample correlogram. If the order of the moving average process is q then the distribution of r_j for $j > q$ is approximately normal with zero mean and variance (Bartlett's formula):

$$\frac{1}{N}\left(1 + 2\sum_{j=1}^{q} r_j^2\right)$$

The partial autocorrelations give information about the order of an autoregressive process. The j^{th} partial autocorrelation coefficient is the estimated coefficient of z_{t-p} from a p^{th} order autoregressive model with p = j. SHAZAM computes these by Durbin's recursive method (further discussion of this method is given at the end of this section). If the autoregressive order is p the higher order partial autocorrelations are approximately normally distributed with zero mean and standard deviation $1/\sqrt{N}$.

A test for a white noise process is given by the Ljung-Box-Pierce (see Box and Pierce [1970] and Ljung and Box [1978]) portmanteau test statistic. This test is constructed from the first J squared autocorrelations as:

$$Q = N(N+2)\sum_{j=1}^{J} \frac{1}{N-j} r_j^2$$

The choice of J is arbitrary (see, for example, the discussion in Harvey [1981, p. 148]). For a white noise process Q has an asymptotic χ_J^2 distribution.

For a stationary time series the correlogram function will decline at higher lags. If the correlogram shows slow decay this gives evidence for nonstationarity. Differencing is a technique to transform a nonstationary time series to a stationary process. The backward operator B is defined by:

$$B^d z_t = z_{t-d}$$

The first difference of z_t is given by $(1–B)z_t$ and the second difference is:

$$(1-B)^2 z_t = (z_t - z_{t-1}) - (z_{t-1} - z_{t-2})$$

When s is the number of periods in the seasonal cycle the first seasonal difference is:

$$(1-B^s) z_t = z_t - z_{t-s}$$

The general differencing transformation to consider is: $(1 - B)^d (1 - B^s)^D z_t$

where d is the order of differencing and D is the order of seasonal differencing.

ARIMA ***Identification Stage Command Options***

The following options are available in the identification stage of the **ARIMA** command:

ALL	Computes the time series properties for **ALL** orders of differencing up to the values specified with **NDIFF**= and **NSDIFF**=.
IAC	Computes inverse autocorrelations as described in Cleveland [1972]. The number of lags is as specified with the **NLAG**= option (the maximum is 20).
GRAPHAC	Plots the sample autocorrelation function using the gnuplot program. The number of lags is specified with the **NLAG**= option. An approximate 95% confidence interval for the autocorrelations is calculated based on ± 2 standard errors.
GRAPHDATA	Plots the data using the gnuplot program. If the **LOG**, **NDIFF**= or **NSDIFF**= options are used then the plot is of the transformed data.
GRAPHPAC	Plots the sample partial autocorrelation function using the gnuplot program. The number of lags is specified with the **NLAGP**= option. An approximate 95% confidence interval for the partial autocorrelations is calculated based on ± 2 standard errors.
LOG	Takes logs of the data.
PLOTAC	Plots the sample autocorrelation function with text characters on the SHAZAM output. Also see the **GRAPHAC** option.
PLOTDATA	Plots the data with text characters on the SHAZAM output. Also see the **GRAPHDATA** option.
PLOTPAC	Plots the sample partial autocorrelation function with text characters on the SHAZAM output. Also see the **GRAPHPAC** option.
WIDE **NOWIDE**	Sets the output width. **NOWIDE** uses 80 columns and **WIDE** uses 120 columns.

ACF= Saves the sample AutoCorrelation Function in the variable specified.

BEG=
END= Sets the sample period. This option overrides the **SAMPLE** command. The values may be given as an observation number or as a date.

NDIFF= Specifies the order of differencing to transform the data.

NLAG= Specifies the number of lags to consider in the calculation of the autocorrelations. The default is 24.

NLAGP= Specifies the number of lags to consider in the calculation of the partial autocorrelations. The default is 12. (The value for **NLAGP=** must not exceed that for **NLAG=**).

NSDIFF= Specifies the order of seasonal differencing. If this is specified then **NSPAN=** must be set.

NSPAN= Specifies the number of periods in the seasonal cycle. For example, set **NSPAN**=4 for quarterly data and **NSPAN**=12 for monthly data.

PACF= Saves the sample Partial AutoCorrelation Function in the variable specified.

TESTSTAT= Saves the Ljung-Box-Pierce statistics in the variable specified.

After the **ARIMA** identification command the temporary variables available are *$ERR* (error code) and *$N* (number of observations used in the identification phase). For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

Example

The **ARIMA** command can be illustrated with the Theil textile data set (although a larger sample size is desirable). The output below gives the results from the ARIMA identification stage for the variable *CONSUME*.

```
|_ARIMA CONSUME / NLAG=8 NLAGP=8 PLOTAC PLOTPAC
     IDENTIFICATION SECTION - VARIABLE=CONSUME
NUMBER OF AUTOCORRELATIONS =   8
NUMBER OF PARTIAL AUTOCORRELATIONS =   8
```

```
              0     0 0
SERIES  (1-B) (1-B  )  CONSUME

  NET NUMBER OF OBSERVATIONS =   17
MEAN=    134.51       VARIANCE=    555.89     STANDARD DEV.=    23.577

  LAGS                      AUTOCORRELATIONS                      STD ERR
  1  -8     .72  .53  .42  .21  .16  .04 -.18 -.29                   .24

 MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS  (CHI-SQUARE)
    LAG    Q    DF  P-VALUE       LAG    Q    DF  P-VALUE
     1   10.59   1  .001           5   22.43   5  .000
     2   16.64   2  .000           6   22.48   6  .001
     3   20.64   3  .000           7   23.53   7  .001
     4   21.74   4  .000           8   26.51   8  .001

  LAGS                   PARTIAL AUTOCORRELATIONS                 STD ERR
  1  -8     .72  .01  .06 -.24  .17 -.22 -.26 -.12                   .24

                                                 0     0 0
AUTOCORRELATION FUNCTION OF THE SERIES    (1-B) (1-B  )  CONSUME

 1  .72 .               +               RRRRRRRRRRRRRRRRRRRRRRRRR
 2  .53 .       +                       RRRRRRRRRRRRRRRRRR     +
 3  .42 .    +                          RRRRRRRRRRRRRR            +
 4  .21 .   +                           RRRRRRR                    +
 5  .16 .  +                            RRRRRR                      +
 6  .04 .  +                            RR                          +
 7 -.18 .  +                        RRRRRRR                         +
 8 -.29 .  +                    RRRRRRRRRR                          +

                                                       0     0 0
PARTIAL AUTOCORRELATION FUNCTION OF THE SERIES  (1-B) (1-B  )  CONSUME

 1  .72 .               +               RRRRRRRRRRRRRRRRRRRRRRRRR
 2  .01 .               +               R               +
 3  .06 .               +               RRR             +
 4 -.24 .               +       RRRRRRRRR               +
 5  .17 .               +               RRRRRRR         +
 6 -.22 .               +       RRRRRRRRR               +
 7 -.26 .               +      RRRRRRRRRR               +
 8 -.12 .               +           RRRRR               +                .
```

The correlogram shows that the sample autocorrelation function declines at higher lags to indicate a stationary process. Some judgement is required to determine a model specification that may describe this data. For example, to consider the suitability of an ARMA(1,1) process, the above results can be compared with theoretical patterns of autocorrelations and partial correlations of ARMA(1,1) processes that are given in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, pp.698-699]. (More formal tests of stationarity are available in the chapter *COINTEGRATION AND UNIT ROOT TESTS*).

A Note on Calculating the Partial Autocorrelation Function

The partial autocorrelation function is approximated by a computationally fast recursive method of calculation described in Box and Jenkins [1976, Appendix A3.2, pp.82-84]. Box-Jenkins comment that "these estimates of the partial autocorrelations differ somewhat from the maximum likelihood values obtained by fitting autoregressive processes of successively higher order. They are very sensitive to rounding errors particularly when the process approaches nonstationarity." The potential for rounding error can be illustrated with monthly data on the 3-month treasury bill rate used in Pindyck and Rubinfeld [1991, Chapter 15]. The evidence is that the series is non-stationary but first differencing yields a stationary time series. The following SHAZAM commands first save the partial autocorrelation function generated by the **ARIMA** command in the variable *PACREC*. Then a set of SHAZAM commands computes the partial autocorrelation function by an exact method and saves the result in the variable *PACOLS*. This is then repeated for the first differences. The commands show SHAZAM features described in the chapter *PROGRAMMING IN SHAZAM*.

```
sample 1 462
arima tbill / nlagp=6 pacf=pacrec
dim pacols 6 beta 6  pacdols 6

do #=1,6
  * Use ? to request no listing of ols results
  ?ols tbill tbill(1.#) / coef=beta
  gen1 pacols:#=beta:#
endo

* Now analyze the first differences
arima tbill / nlagp=6 pacf=pacdrec ndiff=1
sample 2 462
genr tbilld=tbill-lag(tbill)
do #=1,6
  ?ols tbilld tbilld(1.#) / coef=beta
  gen1 pacdols:#=beta:#
endo

* Compare the results
sample 1 6
print pacols pacrec pacdols pacdrec
```

The output below shows the numerical differences between the two computational methods. The variables *PACDOLS* and *PACDREC* are similar to show that the recursive

approximation of the partial autocorrelation function is accurate for the first differences of the *TBILL* time series.

```
|_PRINT PACOLS PACREC PACDOLS PACDREC
      PACOLS          PACREC          PACDOLS         PACDREC
    .9853103        .9849570        .3211617        .3210678
   -.3292023       -.2853001       -.2370160       -.2369215
    .2268389        .1769929       -.1309987E-02   -.1195264E-02
   -.6588303E-02    .1690129E-01   -.2410939E-01   -.2402593E-01
    .1629670E-01    .5984347E-02    .2512562E-01    .2511988E-01
   -.3295990E-01   -.2934607E-01   -.2968467       -.2964889
```

ESTIMATION

A simple example of a time series model is the ARMA(1,1) process given by:

$$z_t - \phi_1 z_{t-1} = u_t - \theta_1 u_{t-1} + \delta$$

where u_t is a random error and the unknown parameters are ϕ_1, θ_1 and δ. To generalize this define polynomials in the backward operator as:

$$\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p \qquad \text{and} \qquad \theta(B) = 1 - \theta_1 B - \ldots - \theta_q B^q$$

where p is the order of the autoregressive process and q is the order of the moving average process. The ARIMA(p, d, q) model has the form:

$$\phi(B)(1 - B)^d z_t = \theta(B) u_t + \delta$$

Note that some authors define lag polynomials with opposite signs from that used here. Therefore, the computer output must be interpreted carefully.

A feature of economic time series is serial correlation at the seasonal lag. Seasonality can be modelled by defining the polynomials:

$$\Gamma(B^s) = 1 - \Gamma_1 B^s - \ldots - \Gamma_P B^{sP} \qquad \text{and} \qquad \Delta(B^s) = 1 - \Delta_1 B^s - \ldots - \Delta_Q B^{sQ}$$

where P is the order of the seasonal AR process and Q is the order of the seasonal MA process. The multiplicative seasonal ARIMA model has the general form:

$$\Gamma(B^s)\phi(B)(1 - B)^d (1 - B^s)^D z_t = \Delta(B^s)\theta(B) u_t + \delta$$

Let $\beta' = [\phi' \; \theta' \; \Gamma' \; \Delta' \; \delta]$ be a K x 1 vector of the model parameters where K = p+q+P+Q+1. Parameter estimates are obtained as the values that minimize the sum of squares function:

$$S(\beta) = \sum_{t=1}^{N} u_t(\beta)^2$$

A pure autoregressive model can be estimated by OLS (see the chapter *DISTRIBUTED LAG MODELS*). The inclusion of seasonal and moving average components means that nonlinear estimation methods are required. The ARIMA estimation uses a version of Marquardt's algorithm [1963]. Each iteration uses a gradient estimate that is calculated by numerical approximation. Let $\hat{\beta}^{(i)}$ be the parameter estimates at iteration i. The approximation used is:

$$\frac{\partial u_t}{\partial \beta_k}\left(\hat{\beta}^{(i)}\right) \approx \frac{1}{h_k}\left[u_t\left(\hat{\beta}^{(i)} + h_k e_k\right) - u_t\left(\hat{\beta}^{(i)}\right)\right] \qquad \text{for } k = 1, \ldots, K$$

where e_k is a K x 1 zero vector with 1 in the k^{th} element and $h_k = h\hat{\beta}_k^{(i)}$ where the scalar h is referred to as the step length. In SHAZAM h can be specified with the **STEPSIZE=** option and the default value is h = 0.01.

At each iteration the residuals are computed recursively. Presample values are required to start the recursion. The ARIMA estimation computes the presample values by a back-forecasting method. The treatment of presample values means that the parameter estimates from the **ARIMA** command will be different from estimates obtained with the **OLS** or **AUTO** SHAZAM commands (where applicable).

The ARIMA nonlinear estimation converges when one of two convergence criteria is met. The first criteria is based on the relative change in each parameter and requires:

$$\left|\hat{\beta}_k^{(i)} - \hat{\beta}_k^{(i-1)}\right| \Big/ \left|\hat{\beta}_k^{(i)}\right| < 0.0001 \qquad \text{for all } k$$

The second convergence criteria is based on the relative change in the sum of squares and requires:

$$\left|S\left(\hat{\beta}^{(i)}\right) - S\left(\hat{\beta}^{(i-1)}\right)\right| \Big/ S\left(\hat{\beta}^{(i-1)}\right) < 0.000001$$

As in any non-linear optimization, convergence to a global minimum is not guaranteed and the choice of good starting values is important. If the parameter estimates move into

unacceptable regions then SHAZAM may terminate with overflow errors. If this occurs then the estimation can be restarted with different starting values or a different model specification. Users should carefully check their output for evidence of model convergence.

The variance of the estimated residuals (reported as `SIGMA**2` on the SHAZAM output) is calculated as:

$$\hat{\sigma}_u^2 = \frac{1}{N-K}\sum_{t=1}^{N} u_t(\hat{\beta})^2$$

If the **DN** option on the **ARIMA** command is used then the divisor is N instead of (N–K). The variance-covariance matrix of the parameter estimates is estimated as:

$$\hat{\sigma}_u^2 \, (X'_{\hat{\beta}} X_{\hat{\beta}})^{-1}$$

where the individual elements of the X matrix are: $x_{kt} = -\frac{\partial u_t}{\partial \beta_k}(\hat{\beta})$

The calculation of the X matrix uses numerical derivatives as described above.

Following model estimation the SHAZAM output reports a number of statistics that are useful for evaluating the model adequacy. The Ljung-Box-Pierce statistic (see the identification stage) is constructed from the squared autocorrelations of the estimated residuals. This gives a test for white noise errors. With J lagged residual autocorrelations the test statistic can be compared with a χ^2_{J-K+1} distribution. If there is no constant term in the model (when the **NOCONSTANT** option is used) then the test statistic has an approximate χ^2_{J-K} distribution under the null hypothesis of white noise errors.

For model selection purposes the Akaike information criterion (AIC) and the Schwarz criterion (SC) are computed as:

$$AIC(K) = \log\left(\hat{\sigma}_u^2\right) + 2 \cdot K / N$$
$$SC(K) = \log\left(\hat{\sigma}_u^2\right) + K \log(N)/N$$

For the differenced series $y_t = (1-B)^d(1-B^s)^D z_t$ the cross-covariances between y_t and the estimated residuals $\hat{u}_{t-j}$ are:

$$c_{(y\hat{u})j} = \frac{1}{N}\sum_{t=j+1}^{N}(y_t - \bar{y})(\hat{u}_{t-j} - \bar{u}) \qquad j = 0, \pm 1, \pm 2, \ldots$$

where $\bar{y} = \frac{1}{N}\sum_{t=1}^{N} y_t$ and $\bar{u} = \frac{1}{N}\sum_{t=1}^{N} \hat{u}_t$

The cross-correlations are computed as:

$$r_{(y\hat{u})j} = c_{(y\hat{u})j} \Big/ \sqrt{c_{(yy)0}\, c_{(\hat{u}\hat{u})0}} \qquad j = 0, \pm 1, \pm 2, \ldots$$

The theoretical u_t are correlated with current and future values of y_t but are not correlated with past values. The estimated cross-correlations reported on the SHAZAM output can be inspected to see if they exhibit such a pattern.

ARIMA ***Estimation Stage Command Options***

In general, the format of the **ARIMA** command for the estimation stage is:

ARIMA *var* / **NAR=** **NMA=** *options*

where *var* is a time series variable and *options* is a list of desired options. The options as described for the identification stage are:

LOG, **WIDE**/**NOWIDE**, **BEG=**, **END=**, **NDIFF=**, and **NSDIFF=**.

Options described for the **OLS** command that may be used are:

ANOVA, **PCOR**, **PCOV**, **COV=**, **STDERR=**, and **TRATIO=**.

Other options are:

DN Uses a divisor of N instead of (N–K) when estimating SIGMA**2.

GRAPHRES Prepares a gnuplot plot of the residuals.

NOCONSTANT Excludes the constant term.

PITER Prints the parameter estimates and the value of the sum of squares function at every iteration. By default only the results at the final iteration are listed.

PLOTRES Plots the residuals with text characters on the SHAZAM output. Also see the **GRAPHRES** option.

RESTRICT Uses zero starting values as zero restrictions. Also see the **START** and **START=** options. For example, with a quarterly time series *IINV*, an AR(5) process with zero coefficients for the 2nd and 3rd autoregressive coefficients may be estimated with the commands:

```
arima iinv / nar=5 start restrict beg=1950.1 end=1985.4
.5  0  0  .1  .1  5
```

START Indicates that parameter starting values are entered on the line immediately following the **ARIMA** command. This option will not work when the **ARIMA** command is in a **DO**-loop. Also see the **START=** option below.

ACF= Saves the AutoCorrelation Function of the estimated residuals in the variable specified.

COEF= Saves the estimated Coefficients in the variable specified. These values can be used as input to the **ARIMA** forecasting stage. This option should not be confused with the **COEF=** option used in the forecasting stage. In the estimation stage **COEF=** saves the coefficients while, in the forecasting stage, the **COEF=** option uses the coefficients as input parameters.

ITER= Sets the maximum number of iterations. The default is 50. If this iteration limit is exceeded it may be sensible to try different starting values or a different model specification.

NAR= Specifies the order of the AR (autoregressive) process.

NMA= Specifies the order of the MA (moving average) process.

NSAR= Specifies the order of the Seasonal AR process. If this is specified then **NSPAN=** must be set.

NSMA= Specifies the order of the Seasonal MA process. If this is specified then **NSPAN=** must be set.

NSPAN= Specifies the number of periods in the seasonal cycle. For example, set **NSPAN**=4 for quarterly data and **NSPAN**=12 for monthly data.

PREDICT= Saves the Predicted values in the variable specified. These are the implied one-step-ahead forecasts.

RESID= Saves the estimated Residuals in the variable specified.

START= Specifies a vector of starting values. The input sequence for the starting values must correspond to **NAR=**, **NMA=**, **NSAR=**, and **NSMA=** with the constant term given as the last value. The calculation of numeric derivatives means that zero starting values can cause problems in starting the iterations. Therefore, SHAZAM sets a zero value to 0.01 to get the iterations started. Also see the **RESTRICT** option above.

STEPSIZE= Specifies the stepsize to use in calculating the numeric derivatives. The default value is 0.01. The estimated covariance matrix of the parameter estimates may be sensitive to this value.

TESTSTAT= Saves the Ljung-Box-Pierce statistics in the variable specified.

Following model estimation the available temporary variables as described for the **OLS** command are:

$ERR, $K, $LAIC, $LSC, $N, $R2, $SIG2, $SSE, $SSR, and *$SST*.

The choice of good starting values will facilitate model convergence. If the **START** or **START=** option are not specified then SHAZAM will set default starting values. If **NAR=** or **NMA=** do not exceed 1 then default starting values of 0.5 are used for the coefficients. Otherwise, default starting values of 0.1 are used for the coefficients. The default starting value for the constant term is 1.0.

Example

The ARIMA estimation stage can be demonstrated with the *CONSUME* variable that was inspected in the identification stage. The example below shows the estimation of an

ARMA(1,1) model. When the **START** option is used the starting values for the coefficients are entered on the line after the **ARIMA** command as follows:

```
arima consume / nar=1 nma=1 start
0.5  -0.2   100
```

The first value is the AR parameter, the second value is the MA parameter and the final value is the starting value for the constant term. Alternatively, starting values for the model estimation can be entered in a variable, say *ALPHA*, as shown in the next listing of SHAZAM commands.

```
dim alpha 3
gen1 alpha:1= 0.5
gen1 alpha:2= -.2
gen1 alpha:3= 100
arima consume / nar=1 nma=1 start=alpha
```

The output below gives results from the estimation of an ARMA(1,1) model.

```
|_ARIMA CONSUME / NAR=1 NMA=1 START=ALPHA
ESTIMATION PROCEDURE
STARTING VALUES OF PARAMETERS ARE:
  .50000      -.20000       100.00

MEAN OF SERIES =   134.5
VARIANCE OF SERIES =   555.9
STANDARD DEVIATION OF SERIES =   23.58

INITIAL SUM OF SQUARES =      17958.972

ITERATION STOPS - RELATIVE CHANGE IN EACH PARAMETER LESS THAN .1E-03

NET NUMBER OF OBS IS   17
DIFFERENCING: 0 CONSECUTIVE, 0 SEASONAL WITH SPAN  0
CONVERGENCE AFTER 30 ITERATIONS
INITIAL SUM OF SQS=   17958.972         FINAL SUM OF SQS=   2527.1076

 R-SQUARE =   .7159     R-SQUARE ADJUSTED =   .6753
VARIANCE OF THE ESTIMATE-SIGMA**2 =   166.02
STANDARD ERROR OF THE ESTIMATE-SIGMA =   12.885
AKAIKE INFORMATION CRITERIA -AIC(K) =   5.4650
SCHWARZ CRITERIA- SC(K) =   5.6121

             PARAMETER ESTIMATES        STD ERROR     T-STAT
              AR( 1)    .59682          .2130          2.802
              MA( 1)   -.94992          .9190E-01     -10.34
            CONSTANT    55.041          28.77          1.913

                            RESIDUALS
   LAGS                    AUTOCORRELATIONS                        STD ERR
1 -12  -.37  .04  .23 -.12  .12 -.04  .00 -.16  .16 -.06 -.06 -.06   .24
13-16  -.05  .00 -.08 -.03                                          .30
```

```
 MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS  (CHI-SQUARE)
    LAG     Q     DF  P-VALUE        LAG     Q     DF  P-VALUE
     3    3.96    1  .047           10    6.93    8  .545
     4    4.34    2  .114           11    7.12    9  .624
     5    4.73    3  .193           12    7.37   10  .690
     6    4.77    4  .311           13    7.60   11  .748
     7    4.77    5  .444           14    7.61   12  .815
     8    5.69    6  .458           15    8.72   13  .793
     9    6.75    7  .456           16    9.09   14  .825

CROSS-CORRELATIONS BETWEEN RESIDUALS AND (DIFFERENCED) SERIES
CROSS-CORRELATION AT ZERO LAG =  .41
  LAGS                    CROSS CORRELATIONS Y(T),E(T-K)
  1-12     .49  .06  .24  .12 -.06  .13 -.04 -.09 -.12 -.08 -.14 -.12
 13-16    -.11 -.13 -.08 -.07
 LEADS                    CROSS CORRELATIONS Y(T),E(T+K)
  1-12     .04  .26  .10  .08  .06 -.07 -.11 -.04  .01 -.15 -.03 -.21
 13-16    -.11 -.04 -.15 -.05
```

The estimation converged in 30 iterations. The estimated equation (where z_t denotes the variable *CONSUME*) is:

$$z_t - .59682\ z_{t-1} = \hat{u}_t + .94992\ \hat{u}_{t-1} + 55.041$$

Note that with different starting values the estimation may converge to different parameter estimates. Therefore, re-estimation with different starting values may be useful. The above output shows that the Ljung-Box-Pierce test statistics are less than the critical values from a chi-square distribution at any reasonable significance level and so the hypothesis of white noise errors is not rejected.

The *CONSUME* variable is annual data and therefore modelling of seasonal effects need not be considered. In economic analysis there is interest in modelling seasonally unadjusted quarterly and monthly data. A multiplicative seasonal autoregressive process that may be considered is:

$$(1 - \Gamma_1 B^4)(1 - \phi_1 B)z_t = u_t + \delta$$

That is, $$z_t - \Gamma_1 z_{t-4} - \phi_1 z_{t-1} + \Gamma_1 \phi_1 z_{t-5} = u_t + \delta$$

Suppose that quarterly seasonally unadjusted data is available in the variable *ZQ*. The SHAZAM command for estimation of the above model is:

```
ARIMA ZQ / NSPAN=4 NSA=1 NAR=1
```

FORECASTING

An ARIMA model can be used to forecast future values of a time series. The goal is to obtain estimates $\hat{z}_{T+l}$ as the forecast at origin date T ($p \le T \le N$) for lead time l ($l \ge 1$)) of z_{T+l}. The forecasting exercise is facilitated by recognizing that the data generation process for z_{T+l} can be expressed in different forms. The first form to consider is the difference-equation form. For example, the ARMA(1,1) process can be expressed as:

$$z_{T+l} = \phi_1 z_{T+l-1} + u_{T+l} - \theta_1 u_{T+l-1} + \delta$$

The second form to analyze is the random shock form. Any ARIMA process can be expressed as a linear function of the current and past random shocks as:

$$z_{T+l} = \mu + u_{T+l} + \Psi_1 u_{T+l-1} + \Psi_2 u_{T+l-2} + \ldots$$

where the constant μ and the weights $\Psi_1, \Psi_2, \ldots$ are determined as functions of the model parameters. The forecast can be written in the form:

$$\hat{z}_{T+l} = \mu + \Psi_l u_T + \Psi_{l+1} u_{T-1} + \ldots$$

The l-step ahead forecast error is:

$$e_{T+l} = [z_{T+l} - \hat{z}_{T+l}] = u_{T+l} + \Psi_1 u_{T+l-1} + \ldots + \Psi_{l-1} u_{T+1}$$

The error has zero expectation and variance: $V[e_{T+l}] = \sigma_u^2 \left(1 + \Psi_1^2 + \ldots + \Psi_{l-1}^2\right)$

In SHAZAM the forecasting procedure is made operational as follows. Input requirements from the estimation stage are the model parameter estimates $\hat{\beta}$ and the estimate of the error variance $\hat{\sigma}_u^2$ (the SHAZAM output labels this as `SIGMA**2`). If a moving average component is present then estimated residuals enter the forecast. Therefore, a sequence of one-step-ahead prediction errors is computed to the end of the sample period (including back-forecasting of pre-sample residuals). Point forecasts are calculated recursively from the difference-equation form of the process.

For example, for the ARMA(1,1) model the $l = 1$ period ahead forecast is:

$$\hat{z}_{T+1} = \hat{\phi}_1 z_T - \hat{\theta}_1 \hat{u}_T + \hat{\delta}$$

The $l = 2$ periods ahead forecast is: $\hat{z}_{T+2} = \hat{\phi}_1 \hat{z}_{T+1} + \hat{\delta}$

The $l = 3$ periods ahead forecast is: $\hat{z}_{T+3} = \hat{\phi}_1 \hat{z}_{T+2} + \hat{\delta}$

The Ψ_i weights are calculated (these are listed on the SHAZAM output in the column marked `PSI WT`) and are used to estimate the forecast variance as $\hat{V}[e_{T+l}]$. A 95% confidence interval is computed as:

$$\hat{z}_{T+l} \pm 1.96\, \hat{V}[e_{T+l}]^{1/2}$$

Users should be aware that the 95% confidence interval is approximate. If the sample period set for the forecasting stage extends beyond the sample period used in the estimation stage then Nelson [1973, p. 224] notes that "the computations of the variance of residuals is restricted to the sample period so that any postsample deterioration in the fit of the model will not be reflected in standard errors and confidence intervals."

ARIMA *Forecasting Stage Command Options*

In general, the format of the **ARIMA** command for the forecasting stage is:

ARIMA *var* / **NAR=** **NMA=** **COEF=** **SIGMA=** **FBEG=** **FEND=** *options*

where *var* is the variable to forecast and *options* is a list of desired options. The options for differencing the data as used in the identification stage are:

NDIFF=, **NSDIFF=**, and **NSPAN=**.

The options for model specification as used in the estimation stage are:

NOCONSTANT, **BEG=**, **END=**, **NAR=**, **NMA=**, **NSAR=**, and **NSMA=**.

Other options are:

DN Uses a divisor of N instead of (N–K) when estimating `SIGMA**2`. Also see the **SIGMA=** option below.

GRAPHFORC Prepares gnuplot plots of the forecast.

LOG Takes logs of data. The data z_t is transformed to $y_t=\log(z_t)$. If this option is specified then it must also have been used in the estimation stage. SHAZAM first generates forecasts in log form, $\hat{y}_{T+l}$, and then

generates antilog forecasts, $\hat{z}_{T+l}$. Some statistical theory is needed to understand how the forecasts in antilog form are obtained. Consider a random variable Y distributed as $N(\mu,\sigma^2)$ and define the random variable Z=exp(Y). Then the random variable Z has mean $\exp(\mu+\sigma^2/2)$ (see, for example, Mood, Graybill and Boes [1974]). Therefore, the conditional expectation forecast of $z_{T+l} = \exp(y_{T+l})$ is:

$$\hat{z}_{T+l} = \exp\left(\hat{y}_{T+l} + \hat{\sigma}_u^2/2\right)$$

For the derivation of the standard errors of the anti-log forecasts, see Nelson [1973, p. 163]. Note that the confidence interval for $\hat{z}_{T+l}$ is not symmetric.

PLOTFORC Plots the forecast with 95% confidence intervals with text characters on the SHAZAM output. Also see the **GRAPHFORC** option.

COEF= Gives the model coefficients. If this is not specified then values must be entered on the line following the **ARIMA** command. The input sequence for the coefficients must correspond to **NAR=**, **NMA=**, **NSAR=**, and **NSMA=** with the constant term given as the last value.

FBEG=
FEND= Specifies the origin date (**FBEG=**) and the last observation (**FEND=**) of the forecast. The origin date may not be greater than the final observation of the **SAMPLE** command or the value set with **END=**. Furthermore, the origin date must exceed the first observation of the **SAMPLE** command (or the value set with **BEG=**) by an amount at least equal to the maximum AR lag. These values may be given as an observation number or as a date. This option is required.

FCSE= Saves the forecast standard errors in the variable specified.

PREDICT= Saves the predictions in the variable specified.

PSI= Saves the Ψ_i weights in the variable specified.

RESID= Saves the forecast residuals in the variable specified.

SIGMA= Gives the value of **SIGMA** to use in calculating the forecast standard errors. By default the data for the current sample period will be used to estimate **SIGMA**. A recommended approach is to follow estimation with the command:

```
gen1 s=sqrt($sig2)
```

Then use the option **SIGMA=S** for the ARIMA forecasting. When this option is used the **DN** option is not applicable.

Example

A listing of SHAZAM commands for estimation and forecasting of an ARMA(1,1) model for the *CONSUME* variable is:

```
arima consume / nar=1 nma=1 coef=beta start=alpha
gen1 s=sqrt($sig2)
arima consume / nar=1 nma=1 coef=beta fbeg=14 fend=19 sigma=s
graphforc
```

In the above commands the parameter estimates from the model estimation are saved in the variable *BETA* and the value of `SIGMA**2` is available in the temporary variable *$SIG2*. These become input for the forecasting. The origin date for the forecast is set at observation 14 and the final forecast period is set at observation 19. This gives a 3 period ex post (within sample) forecast and a 2 period ex ante (out of sample) forecast.

SHAZAM output from the **ARIMA** forecast is:

```
|_ARIMA CONSUME / NAR=1 NMA=1 COEF=BETA FBEG=14 FEND=19 SIGMA=S GRAPHFORC

    ARIMA FORECAST
          PARAMETER VALUES ARE:
               AR( 1)=    .59682
               MA( 1)=   -.94992
               CONSTANT =    55.041

FROM ORIGIN DATE  14, FORECASTS ARE CALCULATED UP TO    5 STEPS AHEAD

FUTURE DATE    LOWER       FORECAST       UPPER       ACTUAL       ERROR
     15       141.881       167.136      192.390      154.300     -12.8356
     16       108.277       154.791      201.306      149.000     -5.79115
     17       95.3940       147.424      199.453      165.500      18.0763
     18       89.1687       143.027      196.885
     19       85.9081       140.402      194.897

     STEPS AHEAD     STD ERROR      PSI WT
           1          12.88         1.0000
           2          23.73         1.5467
           3          26.55          .9231
           4          27.48          .5509
           5          27.80          .3288

VARIANCE OF ONE-STEP-AHEAD ERRORS-SIGMA**2 =    166.0
STD.DEV. OF ONE-STEP-AHEAD ERRORS-SIGMA    =    12.88
```

11. AUTOCORRELATION MODELS

"It can be predicted with all security that in fifty years light will cost one fiftieth of its present price, and in all the big cities there will be no such thing as night."

J.B.S. Haldane
British Scientist, 1927

The **AUTO** command provides features for estimation of models with autocorrelated errors. Consider the linear model:

$$Y_t = X_t'\beta + \varepsilon_t \qquad \text{for} \quad t = 1, \ldots, N$$

where Y_t is an observation on the dependent variable, X_t is a vector of observations on K explanatory variables, and β is a vector of unknown parameters. Disturbances that follow an autoregressive process of order p (an AR(p) process) have the form:

$$\varepsilon_t = \sum_{j=1}^{p} \rho_j \varepsilon_{t-j} + v_t$$

Disturbances that follow a moving average process of order q (an MA(q) process) have the form:

$$\varepsilon_t = v_t + \sum_{j=1}^{q} \theta_j v_{t-j}$$

where the ρ_j and θ_j are unknown parameters and v_t is a random error. The order of the autoregressive process or the moving average process is specified with the **ORDER=** option on the **AUTO** command.

ESTIMATION WITH AR(1) ERRORS

The model with first-order autoregressive errors has the form:

$$Y_t = X_t'\beta + \varepsilon_t \qquad \text{and} \qquad \varepsilon_t = \rho\,\varepsilon_{t-1} + v_t \qquad \text{with} \qquad |\rho| < 1$$

where ρ is the autocorrelation parameter and v_t is a random disturbance. Using the assumptions $E[v_t] = 0$, $E[v_t^2] = \sigma_v^2$, $E[v_t v_s] = 0$ for $t \neq s$, the covariance matrix for ε can be expressed as:

$$E(\varepsilon\varepsilon') = \sigma_v^2 \Omega = \frac{\sigma_v^2}{1-\rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & . & \rho^{N-1} \\ \rho & 1 & \rho & . & \rho^{N-2} \\ \rho^2 & \rho & 1 & . & . \\ . & . & . & . & . \\ \rho^{N-1} & \rho^{N-2} & . & . & 1 \end{bmatrix}$$

SHAZAM provides a number of methods for estimating the parameters of the model with AR(1) errors.

Cochrane-Orcutt Iterative Estimation

The SHAZAM default estimation method for the model with AR(1) errors is Cochrane-Orcutt iterative estimation. The method proposed by Cochrane-Orcutt [1949] is extended to include the first observation with the Prais-Winsten [1954] transformation and the estimation is iterated until a convergence criteria is satisfied. The steps in the estimation procedure are as follows.

STEP 1: Estimate the regression equation by OLS and obtain residuals e_t.

STEP 2: Run the regression: $e_t = \rho e_{t-1} + v_t^*$

This gives the least squares estimate of ρ as: $\hat{\rho} = \dfrac{\Sigma_{t=2}^{N} e_t e_{t-1}}{\Sigma_{t=2}^{N} e_{t-1}^2}$

STEP 3: Use the estimate $\hat{\rho}$ to obtain transformed observations Y^* and X^* as:

$$Y_1^* = \sqrt{1-\hat{\rho}^2}\, Y_1, \quad X_1^* = \sqrt{1-\hat{\rho}^2}\, X_1 \qquad \text{and}$$

$$Y_t^* = Y_t - \hat{\rho} Y_{t-1}, \quad X_t^* = X_t - \hat{\rho} X_{t-1} \qquad \text{for } t = 2, 3, \ldots, N$$

An estimate of β is obtained from an OLS regression of Y^* on X^*. A new set of residuals is then calculated and steps 2 and 3 are repeated until successive estimates of ρ differ by less

than .001 or the value set with the **CONV=** option on the **AUTO** command. The maximum number of iterations is 19 and this can be changed with the **ITER=** option. If the **DROP** option is used, the estimates are obtained by dropping the first observation.

The final estimate of ρ, say $\tilde{\rho}$, can be used to construct an estimate $\tilde{\Omega}$ of the matrix Ω. Then the final parameter estimates are given by:

$$\tilde{\beta} = \left(X'\tilde{\Omega}^{-1}X\right)^{-1}X'\tilde{\Omega}^{-1}Y$$

The transformed residuals are:

$$\left.\begin{aligned}\tilde{v}_1 &= \sqrt{1-\tilde{\rho}^2}\,\tilde{e}_1 \\ \tilde{v}_t &= \tilde{e}_t - \tilde{\rho}\,\tilde{e}_{t-1} \qquad t = 2, \ldots, N\end{aligned}\right\} \qquad \text{where} \quad \tilde{e} = Y - X\tilde{\beta}$$

The estimate of the covariance matrix of $\tilde{\beta}$ is: $V(\tilde{\beta}) = \tilde{\sigma}_v^2\left(X'\tilde{\Omega}^{-1}X\right)^{-1}$

where $\tilde{\sigma}_v^2 = \dfrac{1}{N-K}(\tilde{v}'\tilde{v}) = \dfrac{1}{N-K}(Y - X\tilde{\beta})'\tilde{\Omega}^{-1}(Y - X\tilde{\beta})$

The R^2 measure is: $1 - \dfrac{\tilde{v}'\tilde{v}}{Y'Y - N\bar{Y}^2}$

The predicted values that are saved with the **PREDICT=** option are computed as:

$$\hat{Y}_1 = X_1'\tilde{\beta} \qquad \text{and}$$

$$\hat{Y}_t = X_t'\tilde{\beta} + \tilde{\rho}\,(\hat{Y}_{t-1} - X_{t-1}'\tilde{\beta}) \qquad \text{for} \quad t = 2, \ldots, N$$

The residuals that are saved with the **RESID=** option are obtained as $Y - \hat{Y}$.

After estimation it may be useful to test for an AR(1) error term by testing the null hypothesis H_0: $\rho = 0$ against the alternative H_1: $\rho \neq 0$. Harvey [1990, pp. 200-201] discusses a Wald test that can be constructed as:

$$t = \frac{\tilde{\rho}}{\sqrt{V(\tilde{\rho})}} \qquad \text{where} \qquad V(\tilde{\rho}) = (1 - \tilde{\rho}^2)/N$$

This test statistic has an asymptotic standard normal distribution under the null hypothesis and is reported as the ASYMPTOTIC T-RATIO on the SHAZAM output.

Grid Search Estimation

The **GS** option on the **AUTO** command implements a Hildreth-Lu [1960] least squares grid search procedure. SHAZAM considers values of ρ ranging from –0.9 to 0.9 in increments of 0.1. For each of these values a regression of Y^* on X^* is estimated and the error sum of squares is computed. The value of ρ for which the sum of squared errors is smallest is chosen. Then, SHAZAM refines the value of ρ by searching in the neighbourhood of the chosen ρ in increments of 0.01. The refined value of ρ associated with the minimum sum of squared errors is selected as the optimal ρ. This method is illustrated later in this chapter.

Maximum Likelihood Estimation by the Beach-MacKinnon Method

Under the assumption of normality the log-likelihood function for the model with autocorrelated errors can be expressed as:

$$L(\beta,\Omega,\sigma_v^2) = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln\sigma_v^2 - \frac{1}{2}\ln|\Omega| - \frac{1}{2\sigma_v^2}(Y - X\beta)'\Omega^{-1}(Y - X\beta)$$

With the AR(1) error model $\left|\Omega^{-1}\right| = 1 - \rho^2$. The Beach and MacKinnon [1978] estimation procedure is to first maximize the log-likelihood function with respect to β holding ρ fixed (initially ρ=0) and then maximize L with respect to ρ considering β fixed. The solutions to these two maximization problems are developed in Beach and MacKinnon. The estimation is iterated until two successive estimates of ρ differ by less than .001 or the value set with the **CONV=** option.

Maximum Likelihood Estimation by Grid Search

The **GS** and **ML** options on the **AUTO** command give a variation on the previous grid search method. Dhrymes [1971, p. 70] shows that maximizing the likelihood function is equivalent to minimizing:

$$\frac{\sigma_v^2(\rho)}{(1-\rho^2)^{1/N}}$$

The minimum is located using the same grid search method described previously. Note that the previous method was concerned with minimizing $\sigma_v^2(\rho)$. Thus the two methods

are based on different objective functions. Asymptotically the two grid search methods are equivalent since:

$$\lim_{N \to \infty} (1 - \rho^2)^{1/N} = 1 \qquad \text{when} \qquad |\rho| < 1$$

Nonlinear Least Squares

The **PAGAN** option on the **AUTO** command gives estimation by a nonlinear least squares method that is described in more detail later in this chapter.

TESTS FOR AUTOCORRELATION AFTER CORRECTING FOR AR(1) ERRORS

After correcting for first-order autocorrelation it is a good idea to check whether any autocorrelation remains in the v_t residuals. If so then it is possible that higher order autocorrelation exists. A simple procedure is to use a modification of the Durbin [1970] *h* test which is appropriate since a model corrected for autocorrelation could actually be rewritten as a lagged dependent variable model. Denote ϕ as the first order autoregressive parameter of the series v_t. An estimate is obtained as:

$$\hat{\phi} = \frac{\Sigma_{t=2}^{N} \tilde{v}_t \tilde{v}_{t-1}}{\Sigma_{t=2}^{N} \tilde{v}_{t-1}^2}$$

Consider testing the null hypothesis H_0: $\phi = 0$ against the alternative H_1: $\phi \neq 0$. When the **RSTAT**, **LIST**, or **MAX** options is used with the AR(1) error model SHAZAM computes the test statistic:

$$h' = \hat{\phi} \sqrt{\frac{N}{1 - N\, V(\tilde{\rho})}}$$

This statistic is printed on the SHAZAM output as:

```
DURBIN H STATISTIC (ASYMPTOTIC NORMAL)
  MODIFIED FOR AUTO ORDER=1
```

If the original model has a lagged dependent variable then another modified Durbin *h* statistic can be derived. This model is:

$$Y_t = \gamma Y_{t-1} + X_t'\beta + \varepsilon_t \qquad \text{with} \qquad \varepsilon_t = \rho\varepsilon_{t-1} + v_t$$

Consider testing the null hypothesis H_0: $\phi = 0$ in: $v_t = \phi v_{t-1} + w_t$

The modified Durbin *h* test statistic is: $h' = \hat{\phi}\sqrt{\dfrac{N}{1 - N\,[V(\tilde{\rho}) + 2\,\mathrm{Cov}\,(\tilde{\rho},\tilde{\gamma}) + V(\tilde{\gamma})]}}$

In this case, if the **DLAG** option is used, the *h* statistic is labeled on the output as:

```
DURBIN H STATISTIC (ASYMPTOTIC NORMAL)
 MODIFIED FOR AUTO ORDER=1 WITH LAGGED DEPVAR
```

In both cases the *h* statistics can be compared with a standard normal distribution.

ESTIMATION WITH AR(2) ERRORS

The model with AR(2) errors has the form:

$$Y_t = X_t'\beta + \varepsilon_t \qquad \text{with} \qquad \varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + v_t$$

where $E[v_t] = 0$, $E[v_t v_s] = 0$ for , $t \neq s$ and $E[v_t^2] = \sigma_v^2$. This process will be stationary if:

$$\rho_1 + \rho_2 < 1, \quad \rho_2 - \rho_1 < 1, \quad \text{and} \quad \rho_1, \rho_2 \in (-1, 1).$$

The region in ρ_1 and ρ_2 space defined by these conditions is called the stability triangle. A discussion of the stability triangle is in Box and Jenkins [1976, pp. 58-65]. The covariance matrix of ε is $E[\varepsilon\varepsilon'] = \sigma_v^2\Omega$. The form of Ω for the model with AR(2) errors is discussed in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Section 8.2.2]. As in the AR(1) error case SHAZAM offers a number of estimation methods.

Cochrane-Orcutt Iterative Estimation

The SHAZAM default estimation method for the model with AR(2) errors is a Cochrane-Orcutt iterative estimation procedure that is implemented as follows.

STEP 1: Estimate the regression equation by OLS and obtain residuals e_t.

STEP 2: Get estimates $\hat{\rho}_1$ and $\hat{\rho}_2$ of ρ_1 and ρ_2 from the regression:

$$e_t = \rho_1 e_{t-1} + \rho_2 e_{t-2} + v_t^* \qquad \text{for} \qquad t = 3, 4, \ldots, N$$

STEP 3: Obtain transformed observations Y^* and X^* as:

$$Y_1^* = \sqrt{\hat{a}}\,Y_1 \;, \quad X_1^* = \sqrt{\hat{a}}\,X_1 \qquad \text{where} \qquad \hat{a} = (1+\hat{\rho}_2)[(1-\hat{\rho}_2)^2 - \hat{\rho}_1^2]/(1-\hat{\rho}_2)$$

$$Y_2^* = \sqrt{1-\hat{\rho}_2^2}\left(Y_2 - \frac{\hat{\rho}_1}{1-\hat{\rho}_2}Y_1\right), \quad X_2^* = \sqrt{1-\hat{\rho}_2^2}\left(X_2 - \frac{\hat{\rho}_1}{1-\hat{\rho}_2}X_1\right) \qquad \text{and}$$

$$Y_t^* = Y_t - \hat{\rho}_1 Y_{t-1} - \hat{\rho}_2 Y_{t-2} \;, \quad X_t^* = X_t - \hat{\rho}_1 X_{t-1} - \hat{\rho}_2 X_{t-2} \qquad \text{for} \quad t = 3, 4, \ldots, N$$

An estimate of β is obtained from an OLS regression of Y^* on X^*. A new set of residuals is then calculated and steps 2 and 3 are repeated. The iterations stop when successive estimates of both ρ_1 and ρ_2 differ by less than .001 or the value set with the **CONV=** option.

Grid Search Estimation

When the **GS** and **ORDER=2** options are specified on the **AUTO** command the estimation method for ρ_1 and ρ_2 is by a grid search. The search is made within the stability triangle using initial spacing of .25 for the two values of ρ. The method is not guaranteed to yield a global maximum, particularly in small samples, but it usually works quite well. The **GS** option requires over 100 iterations for the search and so can be computationally expensive.

Maximum Likelihood Estimation by Grid Search

When the **ML** and **ORDER=2** options are specified on the **AUTO** command the model estimation considers maximizing the likelihood function under the assumption of normal disturbances. Schmidt [1971] discusses that the maximum likelihood estimators are those which minimize:

$$\sigma_v^2(\rho_1,\rho_2)\Big/\left|\Omega^{-1}\right|^{1/N} \qquad \text{where} \qquad \left|\Omega^{-1}\right| = (1+\rho_2)^2\left[(1-\rho_2)^2 - \rho_1^2\right]$$

Estimation is by a grid search within the stability triangle as discussed above. An application can be found in Savin [1978].

Nonlinear Least Squares

The **PAGAN** and **ORDER=2** options on the **AUTO** command gives estimation by a nonlinear least squares method that is described in the next section.

ESTIMATION WITH HIGHER ORDER AR OR MA ERRORS

For the general estimation of models with autoregressive or moving average errors Pagan [1974] proposes the use of an iterative Gauss-Newton algorithm for the minimization of S = v′v. Using matrix notation the model with autoregressive errors can be expressed in the form:

$$v = M(\rho)(Y - X\beta) \qquad \text{where} \qquad \rho' = (\rho_1, \rho_2, \ldots, \rho_p)$$

and the model with moving average errors can be expressed in the form:

$$v = M(\theta)^{-1}(Y - X\beta) \qquad \text{where} \qquad \theta' = (\theta_1, \theta_2, \ldots, \theta_q)$$

M and M^{-1} are N x N lower triangular matrices. Specifically:

$$M(\rho) = \begin{bmatrix} 1 & 0 & . & . & . & 0 \\ \rho_1 & 1 & 0 & . & . & 0 \\ . & \rho_1 & 1 & . & & . \\ \rho_p & & . & . & . & . \\ & . & & . & . & 0 \\ 0 & & \rho_p & . & \rho_1 & 1 \end{bmatrix}$$

Note that the implementation sets the pre-sample values of ε equal to zero.

Matrix differentiation gives expressions for the computation of first derivatives and these are then used in the updating steps of the Gauss-Newton estimation algorithm.

The convergence criteria is based on the relative change in the error sum of squares. Let $S^{(i)}$ be the error sum of squares at iteration i. The estimation converges when:

$$\left| S^{(i)} - S^{(i-1)} \right| \Big/ S^{(i-1)} \leq \frac{\delta}{1000}$$

where δ can be specified with the **CONV=** option and the default is δ = 0.001.

AUTO *COMMAND OPTIONS*

In general, the format of the **AUTO** command is:

AUTO *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of desired options. Options as defined for **OLS** that are available are:

ANOVA, **DUMP**, **GF**, **LININV**, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **MAX**, **NOCONSTANT**, **NOWIDE**, **PCOR**, **PCOV**, **RESTRICT**, **RSTAT**, **WIDE**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=** and **TRATIO=**.

Additional options available with the **AUTO** command are:

DLAG Used with first-order models to tell SHAZAM that the first independent variable is a **LAG**ged **D**ependent variable. With this option the estimated variances will be calculated using Dhrymes [1971, Theorem 7.1]. An example using this method can be found in Savin [1976].

DN Computes the estimated variance of the regression line by **D**ividing the residual sum of squares by **N** instead of N–K. If the user believes that the model only has good large-sample properties and wants the maximum likelihood estimates of the variances, this option should be used.

DROP **DROP**s the first observation in estimation. (In a second-order model, the first two observations will be dropped.) While this yields less efficient estimates, there are cases where this option might be chosen. If this option is not used, the beginning observations are saved with the usual transformation. See any good econometrics text, and Poirier [1978] for discussions of this. If the **DROP** option is used, the **ML** option may not be used.

GS Uses a **G**rid **S**earch to estimate ρ. For models with AR(1) errors 39 iterations will be required by a grid search. For models with AR(2) errors over 100 iterations will be required. The grid search will yield an accuracy of 0.01.

MISS Adjusts the maximum likelihood estimates for any **MISS**ing observations that were deleted with **SKIPIF** commands. The method used is described in Savin and White [1978] and in Richardson and White [1979].

ML Does **M**aximum **L**ikelihood estimation for models with first or second order autoregressive errors. A reference is Dhrymes [1971, Theorem 4.4]. For the model with AR(1) errors the estimation method is a modified Cochrane-Orcutt procedure as developed by Beach and MacKinnon [1978]. With the **GS** option a grid search is used. A model with AR(2) errors is specified with the **ORDER=2** option and the estimation uses a grid search. Note that, with the **ML** option, the **DN** option will automatically be in effect and the **DROP** option may not be used.

NOPITER Suppresses intermediate output from iterations.

PAGAN May be used with **ORDER=1** or **ORDER=2** to estimate the model using Pagan's [1974] procedure.

CONV= Used to set a **CONV**ergence criterion. The default is .001. This option is ineffective for a grid search.

GAP= Provides an alternative way to indicate a single **GAP** in the data. It is used when the missing observations were not accounted for in the data. You should specify the observation number immediately before the gap. For example, **GAP=29** means that a gap exists in the data after observation 29. It will be assumed that only one observation is missing, unless the number of missing observations is specified with the **NMISS=** option.

ITER= May be used when doing an **ITER**ative Cochrane-Orcutt procedure to control the number of iterations. If a value of **ITER=** is specified, this will be the maximum number of iterations allowed. **ITER=** should be a number between 2 and 99. The default is 19.

NMISS= Used with the **GAP=** option to indicate the **N**umber of **MISS**ing observations from the data. For example, **GAP=29 NMISS=3** means that 3 observations were missing from the data after observation 29. This option is not to be confused with the **MISS** option described above.

NUMARMA= Specifies the **NUM**ber of **AR** or **MA** coefficients to be estimated if some of the coefficients are to be restricted to be zero. For example, when **ORDER=4** is specified, but it is desired that the second autoregressive coefficient should be zero and not estimated then use **NUMARMA=3** and in the line following the **AUTO** command include the 3 coefficents to be estimated. For example:

```
auto consume income / order=4 numarma=3
1 3 4
```

If the **NUMARMA=** option is used with **ORDER=2** then the **PAGAN** option must also be specified.

ORDER=n Used to estimate models with second or higher-**ORDER** autoregressive errors. If **ORDER=2** is specified, an iterative Cochrane-Orcutt procedure is done unless the **GS** or **ML** options are also specified. The missing data options are not valid for **ORDER=1** estimation.

If an **ORDER** larger than 2 is specified the model will be estimated using the least squares procedure described in Pagan [1974]. The **DLAG**, **DROP**, **GAP**, **GS**, **MISS**, **ML**, **NMISS**, **RESTRICT**, **RHO** and **SRHO** options are not permitted when **ORDER** is larger than 2. Also, the **FC** command will not work after an **AUTO** command that uses this option.

ORDER=–n If a negative **ORDER** is specified the model will be estimated using a moving-average error model instead of an autoregressive error model. A least squares procedure is used as described in Pagan [1974]. For example, to estimate a model with first-order moving average errors use the option **ORDER=–1**. The **MISS**, **GAP**, **RHO**, **SRHO**, **DLAG**, **DROP**, **GS**, **ML** and **RESTRICT** options are not permitted. The **FC** command will not work after an **AUTO** command that uses this option.

RHO= Allows the specification of any value of ρ desired for the regression. With the **RHO=** option neither a Cochrane-Orcutt nor a maximum likelihood estimation is done since SHAZAM already has the value of ρ. This is useful when ρ is already known as it eliminates expensive iterations.

SRHO= Used with the **ORDER=2** and **RHO=** options when the **S**econd-order ρ is to be specified for a model with AR(2) errors.

The available temporary variables on the **AUTO** command are: *$ADR2, $DF, $ERR, $K, $LLF, $N, $R2, $RAW, $SIG2, $SSE, $SSR, $SST, $ZDF, $ZSSR* and *$ZSST*.

With the **RSTAT** option the available temporary variables are: *$DURH, $DW, $R2OP* and *$RHO*.

For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

EXAMPLES

The output below uses the Theil textile data set and shows the estimation results for a model with AR(1) errors. The estimation method is by an iterative Cochrane-Orcutt procedure with the modification that retains the first observation.

```
|_AUTO CONSUME INCOME PRICE

DEPENDENT VARIABLE =  CONSUME
..NOTE..R-SQUARE,ANOVA,RESIDUALS DONE ON ORIGINAL VARS

LEAST SQUARES ESTIMATION             17 OBSERVATIONS
BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.00100

    ITERATION          RHO               LOG L.F.            SSE
        1             0.00000        -51.6470             433.31
        2            -0.18239        -51.3987             420.00
        3            -0.19480        -51.3972             419.80
        4            -0.19535        -51.3972             419.80

 LOG L.F. =   -51.3972       AT RHO =    -0.19535

                    ASYMPTOTIC  ASYMPTOTIC  ASYMPTOTIC
          ESTIMATE    VARIANCE    ST.ERROR     T-RATIO
RHO       -0.19535     0.05658     0.23786    -0.82129

 R-SQUARE =   0.9528     R-SQUARE ADJUSTED =   0.9461
VARIANCE OF THE ESTIMATE-SIGMA**2 =   29.986
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.4759
SUM OF SQUARED ERRORS-SSE=   419.80
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.3972

VARIABLE    ESTIMATED   STANDARD   T-RATIO       PARTIAL STANDARDIZED ELASTICITY
  NAME     COEFFICIENT    ERROR      14 DF P-VALUE  CORR.  COEFFICIENT  AT MEANS
INCOME     1.0650      .2282       4.667      .000  .780      .2394      .8154
PRICE     -1.3751      .7105E-01  -19.35      .000 -.982     -.9837     -.7802
CONSTANT   129.62      23.05       5.624      .000  .833      .0000      .9637
```

When the **ML** option is chosen the method of estimation is a modified Cochrane-Orcutt procedure developed by Beach and MacKinnon [1978] and based on maximum likelihood estimation. Note that the **DN** option is automatically in effect when the **ML** option is chosen. The **DROP** option may not be used with the **ML** option. The output of the **ML** option with the **RSTAT** option is given below.

```
|_AUTO CONSUME INCOME PRICE / ML RSTAT
 DEPENDENT VARIABLE =  CONSUME
 ..NOTE..R-SQUARE,ANOVA,RESIDUALS DONE ON ORIGINAL VARS
 DN OPTION IN EFFECT - DIVISOR IS N

 MAXIMUM LIKELIHOOD ESTIMATION             17 OBSERVATIONS
 BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.00100

     ITERATION          RHO               LOG L.F.            SSE
         1             0.00000        -51.6470             433.31
         2            -0.18491        -51.3982             419.95
         3            -0.19741        -51.3971             419.77
         4            -0.19797        -51.3971             419.77

  LOG L.F. =    -51.3971       AT RHO =     -0.19797

                     ASYMPTOTIC  ASYMPTOTIC  ASYMPTOTIC
           ESTIMATE    VARIANCE    ST.ERROR     T-RATIO
 RHO       -0.19797     0.05652     0.23774    -0.83275

  R-SQUARE =    0.9528     R-SQUARE ADJUSTED =    0.9461
 VARIANCE OF THE ESTIMATE-SIGMA**2 =    24.692
 STANDARD ERROR OF THE ESTIMATE-SIGMA =    4.9691
 SUM OF SQUARED ERRORS-SSE=    419.77
 MEAN OF DEPENDENT VARIABLE =    134.51
 LOG OF THE LIKELIHOOD FUNCTION = -51.3971

                              ASYMPTOTIC
 VARIABLE    ESTIMATED   STANDARD   T-RATIO       PARTIAL STANDARDIZED ELASTICITY
   NAME     COEFFICIENT    ERROR   ***** DF P-VALUE  CORR.  COEFFICIENT  AT MEANS
 INCOME      1.0650      .2067       5.152      .000  .809      .2395      .8154
 PRICE      -1.3750      .6436E-01  -21.37      .000 -.985     -.9836     -.7801
 CONSTANT    129.61      20.88       6.209      .000  .856      .0000      .9636

 DURBIN-WATSON = 1.8559    VON NEUMANN RATIO = 1.9719    RHO = -0.05282
 RESIDUAL SUM =    1.0847      RESIDUAL VARIANCE =    24.761
 SUM OF ABSOLUTE ERRORS=    73.221
 R-SQUARE BETWEEN OBSERVED AND PREDICTED = 0.9527
 RUNS TEST:     7 RUNS,     9 POSITIVE,     8 NEGATIVE, NORMAL STATISTIC = -1.2423
 DURBIN H STATISTIC (ASYMPTOTIC NORMAL) =  -1.1000
  MODIFIED FOR AUTO ORDER=1
```

12. BOX-COX REGRESSIONS

"No model exists for him who seeks what he has never seen."
Paul Eluard
Artist

The **BOX** command provides features for estimation with Box-Cox transformations. References are Zarembka [1974, Chapter 3], Greene [2003, Chapter 9.3.2; 2000, Chapter 10.5], Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 12.5], Magee [1988], White [1972] and Savin and White [1978]. It is possible to restrict the exact power transformation on any of the variables by using the **LAMBDA** command. Any variable in the regression with non-positive values will automatically be restricted to be untransformed.

The Classical Box-Cox Model

For a variable $Y = (Y_1\ Y_2 \ldots Y_N)'$ consider the transformation:

$$Y_t^{(\lambda)} = \frac{Y_t^{\lambda} - 1}{\lambda} \qquad \lambda \neq 0$$

$$= \ln Y_t \qquad \lambda = 0$$

The Box-Cox model (from Box and Cox [1964]) is written:

$$Y^{(\lambda)} = X\beta + \varepsilon$$

where X is a (N x K) matrix of the observations on the independent variables and ε is a (N x 1) vector of random disturbances with $E(\varepsilon\varepsilon') = \sigma^2 I_N$.

The log-likelihood function is given by:

$$L\left(\lambda, \beta, \sigma^2; Y, X\right) = -\frac{N}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}(Y^{(\lambda)} - X\beta)'(Y^{(\lambda)} - X\beta) + \ln J$$

where $$J = \det\left[\frac{\partial Y^{(\lambda)'}}{\partial Y}\right] = \prod_{t=1}^{N} Y_t^{\lambda-1}$$

is the Jacobian of the transformation on the dependent variable. Maximization of the above log-likelihood function with respect to σ^2 and β given λ gives the estimators:

$$\hat{\beta}(\lambda) = (X'X)^{-1}X'Y^{(\lambda)} \qquad \text{and} \qquad \hat{\sigma}^2(\lambda) = \frac{1}{N}(Y^{(\lambda)} - X\hat{\beta}(\lambda))'(Y^{(\lambda)} - X\hat{\beta}(\lambda)) .$$

Substitution gives the concentrated log-likelihood function:

$$L^*(\lambda; Y, X) = -\frac{N}{2}\{\ln(2\pi) + 1\} - \frac{N}{2}\ln \hat{\sigma}^2(\lambda) + (\lambda - 1)\sum_{t=1}^{N} \ln Y_t$$

The first step of the estimation procedure is to find an estimate of λ to maximize L^*. SHAZAM does this by an iterative algorithm. Then the estimate $\tilde{\lambda}$ is used to obtain an estimate of β as:

$$\tilde{\beta} = (X'X)^{-1}X'Y^{(\tilde{\lambda})}$$

and the estimate of σ^2 is: $$\tilde{\sigma}^2 = \frac{1}{N - K}(Y^{(\tilde{\lambda})} - X\tilde{\beta})'(Y^{(\tilde{\lambda})} - X\tilde{\beta})$$

When the **DN** option is used the divisor is N instead of N–K. An estimate of the covariance matrix of $\tilde{\beta}$ is:

$$\tilde{\sigma}^2 (X'X)^{-1}$$

Note that this covariance matrix gives conditional standard errors, conditional on $\lambda = \tilde{\lambda}$. For discussion on conditional vs. unconditional standard errors see Judge, Hill, Griffiths, Lütkepohl and Lee [1988, pp.558-9].

The elasticities evaluated at sample means for the Box-Cox model (see Savin and White [1978]) are estimated as:

$$E_k = \tilde{\beta}_k \bar{X}_k \Big/ \bar{Y}^{\tilde{\lambda}}$$

See Poirier and Melino [1978] for the derivation of the change in the expected value of Y with respect to a given regressor when a Box-Cox transformation is used.

The Extended Box-Cox Model

The Box-Cox model can be extended to applications in which both the dependent and the set of independent variables are transformed in the same way. This model is written:

$$Y^{(\lambda)} = X^{(\lambda)}\beta + \varepsilon$$

where the same value of λ is used to transform all the variables in the model. In this case, the log-likelihood function is as previously given except the regressors are transformed in addition to the dependent variable. As before the model estimation proceeds by implementing an iterative procedure to find a maximizing value $\tilde{\lambda}$. This is then used to find parameter estimates $\tilde{\beta}$. In this model, the elasticities evaluated at the variable means are computed as:

$$E_k = \tilde{\beta}_k \, \overline{X}_k^{\tilde{\lambda}} \Big/ \overline{Y}^{\tilde{\lambda}}$$

The Box-Tidwell Model

A variation of the Box-Cox model involves no transformation of the dependent variable but each of the independent variables is transformed by a different λ. This model is known as the Box-Tidwell model (see Box and Tidwell [1962]) and is given by:

$$Y = \beta_0 + \beta_1 X_1^{(\lambda_1)} + \beta_2 X_2^{(\lambda_2)} + \ldots + \beta_k X_k^{(\lambda_k)} + \varepsilon$$

Since there is no transformation on the dependent variable the log-likelihood function for the Box-Tidwell model has no Jacobian term. After model estimation the elasticities are computed as:

$$E_k = \tilde{\beta}_k \, \overline{X}_k^{\tilde{\lambda}_k} \Big/ \overline{Y}$$

The Combined Box-Cox and Box-Tidwell Model

The **FULL** option on the **BOX** command implements combined Box-Cox and Box-Tidwell estimation by allowing all variables, dependent and independent, to be transformed by a different value of λ.

The Box-Cox Autoregressive Model

The Box-Cox autoregressive model (described in Savin and White [1978]) is given by:

$$Y_t^{(\lambda)} = X_t'\beta + \varepsilon_t \qquad \text{with} \qquad \varepsilon_t = \rho\,\varepsilon_{t-1} + v_t\,, \qquad |\rho| < 1\,,$$

and $\quad E(v_t^2) = \sigma_v^2$

The log-likelihood function is:

$$L\left(\lambda, \rho, \beta, \sigma_v^2; Y, X\right) = -\frac{N}{2}\ln\left(2\pi\sigma_v^2\right) + \frac{1}{2}\ln\left(1-\rho^2\right) - \frac{1}{2\sigma_v^2}(Y^{(\lambda)} - X\beta)'\Omega^{-1}(Y^{(\lambda)} - X\beta) + \ln J$$

where $\quad \Omega^{-1} = E\left[\varepsilon\varepsilon'\right]^{-1} =$

$$\begin{bmatrix} 1 & -\rho & 0 & \cdot & \cdot & \cdot & & 0 \\ -\rho & 1+\rho^2 & -\rho & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & -\rho & 1+\rho^2 & -\rho & \cdot & \cdot & \cdot & 0 \\ \cdot & 0 & -\rho & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & & \cdot & & & \\ & \cdot & \cdot & & & 1+\rho^2 & -\rho & \\ 0 & 0 & 0 & & & -\rho & 1 & \end{bmatrix}$$

The Jacobian of the transformation on the dependent variable is:

$$J = \det\left[\frac{\partial Y^{(\lambda)'}}{\partial Y}\right]$$

where $\quad \left[\frac{\partial Y^{(\lambda)'}}{\partial Y}\right] =$

$$\begin{bmatrix} Y_1^{\lambda-1} & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \rho Y_2^{\lambda-1} & Y_2^{\lambda-1} & 0 & \cdot & \cdot & \cdot & 0 \\ \rho^2 Y_3^{\lambda-1} & \rho Y_3^{\lambda-1} & Y_3^{\lambda-1} & & & & \cdot \\ \cdot & \cdot & & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot & & \cdot \\ \cdot & \cdot & & & & \cdot & \\ \rho^{N-1}Y_N^{\lambda-1} & \rho^{N-2}Y_N^{\lambda-1} & \cdot & \cdot & \cdot & & Y_N^{\lambda-1} \end{bmatrix}$$

This is an N x N triangular matrix and the determinant of a triangular matrix is the product of the diagonal elements. Therefore:

$$J = \prod_{t=1}^{N} Y_t^{\lambda-1}$$

Maximizing the log-likelihood function with respect to β and σ_v^2 given both λ and ρ yields:

$$\hat{\beta}(\lambda,\rho) = \left(X'\Omega^{-1}X\right)^{-1}X'\Omega^{-1}Y^{(\lambda)} \qquad \text{and}$$

$$\hat{\sigma}_v^2(\lambda,\rho) = \frac{1}{N}[Y^{(\lambda)} - X\hat{\beta}(\lambda,\rho)]'\Omega^{-1}[Y^{(\lambda)} - X\hat{\beta}(\lambda,\rho)]$$

Substitution of these estimators into the log-likelihood function gives the concentrated log-likelihood function:

$$L^*(\lambda,\rho;Y,X) = -\frac{N}{2}\{\ln(2\pi)+1\} - \frac{N}{2}\ln\hat{\sigma}_v^2(\lambda,\rho) + \frac{1}{2}\ln\left(1-\rho^2\right) + (\lambda-1)\sum_{t=1}^{N}\ln Y_t$$

A grid search is used to find estimates $\tilde{\lambda}$ and $\tilde{\rho}$ that maximize L^*. Then an estimate for β is obtained as:

$$\tilde{\beta} = \left(X'\tilde{\Omega}^{-1}X\right)^{-1}X'\tilde{\Omega}^{-1}Y^{(\tilde{\lambda})}$$

and the estimated covariance matrix of coefficients conditional on λ is given by:

$$\tilde{\sigma}_v^2\left(X'\tilde{\Omega}^{-1}X\right)^{-1}$$

where $\tilde{\sigma}_v^2 = \frac{1}{N-K}(Y^{(\tilde{\lambda})} - X\tilde{\beta})'\tilde{\Omega}^{-1}(Y^{(\tilde{\lambda})} - X\tilde{\beta})$

When the **DN** option is used the divisor is N instead of N–K.

The Extended Box-Cox Autoregressive Model

The extended Box-Cox autoregressive model that transforms all variables by the same λ can be estimated in a similar fashion to the procedure described above. The extended Box-Cox autoregressive model requires the use of the **AUTO** and **ALL** options on the **BOX** command.

BOX *COMMAND OPTIONS*

In general, the format of the **BOX** command is:

BOX *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of desired options. Some of the available options on the **BOX** command are slightly different than those for the **OLS** command. In particular, the **NOCONSTANT** option produces a modified transformation (see details below), since the normal transformation requires an intercept in the regression. Options as defined for **OLS** that are available are:

ANOVA, **GF**, **LIST**, **MAX**, **PCOR**, **PCOV**, **RSTAT**, **BEG=**, **END=**, **COV=**, **PREDICT=** and **RESID=**

Additional options available for use with the **BOX** command are:

ACCUR Normally, SHAZAM will estimate λ to an **ACCUR**acy of 0.01. However, if this option is used, SHAZAM will iterate to an **ACCUR**acy of 0.001 at some increase in computation time.

ALL Used to extend the Box-Cox model so that **ALL** variables receive the same power transformation (unless restricted). Without this option only the dependent variable is transformed. The extended Box-Cox model is far more difficult to estimate than the classical model. For example, the classical model usually requires only four iterations. The **ALL** option often requires twenty iterations.

AUTO Simultaneously estimate λ and the first-order **AUTO**correlation parameter ρ. The method is that of Savin and White [1978]. This option is very slow as over 100 iterations are required. An accuracy of 0.01 is

obtained for λ and ρ (the **ACCUR** option is not available with this option). When using this option, the **DROP**, **NMISS=**, and **GAP=** options described in the chapter on *AUTOCORRELATION MODELS* may be used. The **RHO=** option described below can also be used. The λ range for the **AUTO** option is preset at (–2 , 3.5) and may not be altered. It is important to check for corner solutions.

DN Uses a divisor of N instead of N–K when estimating the error variance.

DUMP Used with the **AUTO** option to get some intermediate output on the iterations to be **DUMP**ed. This output is usually of little value, but sometimes contains useful information on the grid search.

FULL Attempts a combined Box-Cox and Box-Tidwell estimation so that all variables in the equation have different λs. This option can be rather slow as many iterations are required. The warning on Box-Tidwell regressions also applies for the **FULL** option. In fact, overflows are quite common with this option so it should be used with care. **RESTRICT** commands are not permitted. The **ALL**, **AUTO**, **LAMBDA=**, **LAMS=**, **LAME=** and **LAMI=** options must not be used with the **TIDWELL** or **FULL** options.

NOCONSTANT Estimates a model with no intercept. The user should be aware that the Box-Cox model is not well defined for models without an intercept, and the model is generally not scale-invariant. The transformation that is used on this option is the one suggested by Zarembka [1974] so that $X_t^{(\lambda)} = X_t^{\lambda} / \lambda$ for $\lambda \neq 0$. A further result is that the likelihood function is not continuous at λ = 0; SHAZAM will use a value of 0.01 instead of 0.0. A Golden Section search algorithm will be used.

RESTRICT Forces linear parameter restrictions. The restrictions are specified with **LAMBDA** and/or **RESTRICT** commands as shown later in this chapter. The **RESTRICT** option is not available with the **FULL** or **TIDWELL** options.

TIDWELL Does a Box-**TIDWELL** regression instead of a Box-Cox regression. Only the independent variables will be transformed and each variable will have a different λ. For details on the method see Box and Tidwell [1962]. All independent variables must be strictly positive for this technique. Users should be aware that, quite frequently, the Box-Tidwell method will not converge, thus causing the run to be terminated unsuccessfully. This happens most often in small samples where there is a high variance

in one of the parameters. If this happens, the run might be successful if the data is scaled to a lower range (for example, divide all variables by 100). **RESTRICT** commands are not permitted. The **ALL**, **AUTO**, **LAMBDA=**, **LAMS=**, **LAME=** and **LAMI=** options must not be used with the **TIDWELL** or **FULL** options.

UT **U**n**T**ransforms the observed and predicted dependent variables for purposes of computing and plotting the residuals. All values will then be in their original form. This is a useful option since the transformed residuals and predicted values usually are difficult to interpret. However, this option will raise computation time somewhat since an extra pass through the data is required. In addition, the untransformed residuals will no longer necessarily have a zero mean. Note that this option does not affect the regression results. Only the residual listing will be affected.

COEF= Saves the **COEF**ficients, the λs for each independent variable and the dependent variable, and ρ if the **AUTO** option is used.

LAMBDA= Used to specify the value of **LAMBDA** desired by the user. Expensive iterations are eliminated with this option.

LAMS=
LAME=
LAMI= These options are used to do a manual grid test for λ. The **S**tarting value of **LAM**bda (**LAMS=**), the **E**nding value of **LAM**bda (**LAME=**) and an **I**ncrement (**LAMI=**) should be specified. When the optimal λ has been determined it can be specified on the **LAMBDA=** option. The manual grid search is rarely needed. It may be necessary if the iterative procedure fails. The manual grid search is not available when using the **AUTO** option.

RHO= Specifies a fixed value for **RHO** to be used when the **AUTO** option is specified.

The available temporary variables on the **BOX** command are:

$ADR2, *$ANF*, *$DF*, *$DW*, *$ERR*, *$K*, *$LLF*, *$N*, *$R2*, *$R2OP*, *$RAW*, *$RHO*, *$SIG2*, *$SSE*, *$SSR*, *$SST*, *$ZANF*, *$ZDF*, *$ZSSR* and *$ZSST*.

For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

As noted above, SHAZAM uses iterative methods to estimate Box-Cox regressions. In some cases the program will fail during the iterations with a floating-point overflow. If this happens, the data could be scaled by dividing all variables by a constant such as 100 or 1000. This is usually not a problem, however, unless the magnitude of the data is initially very large. Since many iterations are necessary for Box-Cox regressions, the costs will rise substantially for large sample sizes. The user should also be aware that the maximum likelihood methods used here are not for small samples, and the use of these techniques with small samples may yield nonsense results.

EXAMPLE

The output that follows shows the use of the **BOX** command with the Theil textile data set.

```
 |_BOX CONSUME INCOME PRICE / DN ALL

DEPENDENT VARIABLE =CONSUME
DN OPTION IN EFFECT - DIVISOR IS N
BOX-COX REGRESSION        17 OBSERVATIONS

ITERATION LAMBDA   LOG-L.F.      GRADIENT   R-SQUARE     SSE         SSE/N
   1        .000  -46.5862       .466329E-01   .9744   .13613E-01   .80077E-03
   2       1.000  -51.6471      -5.06084       .9513   433.31       25.489
   3       -.618  -46.4049      -3.23983       .9762   .31739E-04   .18670E-05
   4      -1.000  -48.0606       4.33477       .9722   .92261E-06   .54271E-07
   5       -.382  -46.0401       3.26927       .9767   .30541E-03   .17965E-04
   6       -.236  -46.0919      -.354947       .9762   .12787E-02   .75217E-04
   7       -.472  -46.1138       .927319E-01   .9766   .12763E-03   .75075E-05
   8       -.326  -46.0353       .538015       .9766   .52622E-03   .30954E-04
   9       -.292  -46.0476      -.358561       .9765   .73786E-03   .43404E-04
  10       -.348  -46.0335      -.253926       .9766   .42730E-03   .25135E-04
  11       -.361  -46.0346       .861095E-01   .9766   .37579E-03   .22106E-04
  12       -.339  -46.0336       .457475E-01   .9766   .46264E-03   .27214E-04
  13       -.353  -46.0337       .536509E-02   .9766   .40683E-03   .23931E-04
  14       -.344  -46.0335       .302815E-01   .9766   .44046E-03   .25910E-04
  15       -.342  -46.0335      -.195690E-01   .9766   .44880E-03   .26400E-04
  16       -.346  -46.0335      -.136884E-01   .9766   .43538E-03   .25611E-04
  17       -.350  -46.0336       .235114E-01   .9766   .41708E-03   .24534E-04

BOX-COX REGRESSION FOR LAMBDA =    -.350000

 R-SQUARE =     .9766     R-SQUARE ADJUSTED =     .9733
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .24534E-04
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .49532E-02
SUM OF SQUARED ERRORS-SSE=   .41708E-03
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -46.0336

                          ASYMPTOTIC
                         CONDITIONAL                                  BOX-COX
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.1535      .1260       9.152      .000  .926      .3493     1.2665
PRICE     -.68995      .2606E-01  -26.48      .000 -.990    -1.0106     -.8413
```

CONSTANT	1.2292	.2810	4.375	.000	.760	.0000	6.8342

The **ALL** option is used and so SHAZAM transforms all the variables by the same λ. A value of λ = 1 gives a linear model and a value of λ = 0 gives a double log model.

Let $L^{(i)}$ be the value of the log-likelihood function and let $\tilde{\lambda}^{(i)}$ be the estimate of λ at iteration i. On the above estimation output the GRADIENT of the log-likelihood function with respect to λ is approximated as:

$$\left(L^{(i)} - L^{(i-1)}\right) / \left(\tilde{\lambda}^{(i)} - \tilde{\lambda}^{(i-1)}\right)$$

Likelihood ratio tests can be used to test hypotheses about the values of λ. For a test of the linear model the test statistic is:

$$2[L(\tilde{\lambda}) - L(\lambda = 1)]$$

This can be compared with a $\chi^2_{(1)}$ distribution. From the above output the test statistic is computed as:

$$2(-46.0336 + 51.647054) = 11.23$$

From statistical tables the 5% critical value is 3.84 and the 1% critical value is 6.64. The test statistic exceeds these critical values and, therefore, the linear model is rejected.

BOX-COX WITH RESTRICTIONS

The user may want to restrict some of the λs. When the **RESTRICT** option is used on the **BOX** command the **LAMBDA** command can be used to impose the restrictions. If the **ALL** option is not in effect, any of the right-hand side variables may be restricted to any λ. Those that are unrestricted will remain untransformed. If **RESTRICT** commands are used they must follow all **LAMBDA** commands (these are optional). For details on the **RESTRICT** command see the section *RESTRICTED LEAST SQUARES* in the chapter *ORDINARY LEAST SQUARES*. The **END** command should follow all the **LAMBDA** and **RESTRICT** commands. No λ may be restricted if the **TIDWELL** or **FULL** options are being used.

The general command format for restricted estimation is:

BOX *depvar indeps* / **RESTRICT** *options*
LAMBDA *var1=value1 var2=value2 . . .*

RESTRICT *equation*
END

The SHAZAM output that follows shows estimation results when the value of λ for *INCOME* is restricted to λ = 0.

```
|_BOX CONSUME INCOME PRICE / DN ALL RESTRICT
|_LAMBDA INCOME=0
|_END

DEPENDENT VARIABLE =CONSUME
DN OPTION IN EFFECT - DIVISOR IS N

LAMBDA RESTRICTIONS --- VARIABLE   LAMBDA
                         INCOME     .0000

BOX-COX REGRESSION        17 OBSERVATIONS

ITERATION LAMBDA   LOG-L.F.      GRADIENT   R-SQUARE     SSE         SSE/N
   1        .000  -46.5862       .466329E-01   .9744   .13613E-01   .80077E-03
   2       1.000  -51.5842      -4.99795       .9516   430.12       25.301
   3       -.618  -46.3158      -3.25604       .9764   .31408E-04   .18475E-05
   4      -1.000  -47.8558       4.03182       .9728   .90064E-06   .52979E-07
   5       -.382  -46.0048       2.99505       .9768   .30414E-03   .17891E-04
   6       -.236  -46.0778      -.500439       .9763   .12766E-02   .75092E-04
   7       -.472  -46.0605      -.731765E-01   .9768   .12683E-03   .74606E-05
   8       -.326  -46.0092       .351498       .9766   .52461E-03   .30859E-04
   9       -.416  -46.0168       .842899E-01   .9768   .21753E-03   .12796E-04
  10       -.361  -46.0030       .248213       .9767   .37440E-03   .22023E-04
  11       -.348  -46.0040      -.803312E-01   .9767   .42582E-03   .25048E-04
  12       -.369  -46.0032      -.417153E-01   .9767   .34581E-03   .20342E-04
  13       -.356  -46.0032      -.311991E-02   .9767   .39325E-03   .23133E-04
  14       -.364  -46.0030      -.270137E-01   .9767   .36321E-03   .21365E-04
  15       -.366  -46.0030       .207597E-01   .9767   .35646E-03   .20968E-04
  16       -.363  -46.0030       .151163E-01   .9767   .36744E-03   .21614E-04
  17       -.360  -46.0030      -.120099E-01   .9767   .37689E-03   .22170E-04

BOX-COX REGRESSION FOR LAMBDA =    -.360000

 R-SQUARE =     .9767     R-SQUARE ADJUSTED =     .9734
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .22170E-04
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .47085E-02
SUM OF SQUARED ERRORS-SSE=   .37689E-03
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -46.0030

                           ASYMPTOTIC
                          CONDITIONAL                                BOX-COX
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     .21745      .2365E-01   9.196      .000  .926      .3504     1.2697
PRICE     -.68670      .2587E-01  -26.54      .000 -.990    -1.0113     -.8421
CONSTANT   2.7933      .1107       25.23      .000  .989      .0000    16.3104
```

13. COINTEGRATION AND UNIT ROOT TESTS

"It is interesting that Humans try to find Meaningful Patterns in things that are essentially random."

Mr. Data
Star Trek, 1992

The **COINT** command implements tests for unit roots and cointegration including Dickey-Fuller unit root tests, Phillips-Perron unit root tests and tests on the residuals of a cointegrating regression. References include Davidson and MacKinnon [1993, Chapter 20], Maddala [1992, Chapter 14] and special issues of the *Oxford Bulletin of Economics and Statistics* [1986], the *Journal of Economic Dynamics and Control* [1988] and the *Journal of Applied Econometrics* [1991].

UNIT ROOT TESTS

Dickey-Fuller Unit Root Tests

The finding of a unit root in a time series indicates nonstationarity which has implications for economic theory and modelling. Test statistics can be based on the OLS estimation results from a suitably specified regression equation. For a time series Y_t two forms of the "augmented Dickey-Fuller" regression equations are:

$$(1) \qquad \Delta Y_t = \alpha_o + \alpha_1 Y_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta Y_{t-j} + \varepsilon_t$$

$$(2) \qquad \Delta Y_t = \alpha_o + \alpha_1 Y_{t-1} + \alpha_2 t + \sum_{j=1}^{p} \gamma_j \Delta Y_{t-j} + \varepsilon_t$$

where ε_t for $t = 1, \ldots, N$ is assumed to be Gaussian white noise. Equation (1) is with-constant, no-trend and (2) is with-constant, with-trend. The number of lagged terms p is chosen to ensure the errors are uncorrelated. The test statistics calculated are:

Null hypothesis	Test statistic	
$\alpha_1 = 0$ in (1)	(i) $N\hat{\alpha}_1$	z-test
$\alpha_1 = 0$ in (1)	(ii) t-ratio	τ-test
$\alpha_0 = \alpha_1 = 0$ in (1)	F-test Φ_1	Unit root test (zero drift)

$\alpha_1 = 0$ in (2)	(i) $N\hat{\alpha}_1$	z-test
$\alpha_1 = 0$ in (2)	(ii) t-ratio	τ-test
$\alpha_0 = \alpha_1 = \alpha_2 = 0$ in (2)	F-test Φ_2	Unit root test (zero drift)
$\alpha_1 = \alpha_2 = 0$ in (2)	F-test Φ_3	Unit root test (non-zero drift)

When $\alpha_1 = 0$ the time series Y_t is nonstationary so that standard asymptotic analysis cannot be used to obtain the distributions of the test statistics. Various researchers have designed Monte Carlo experiments to generate critical values that can be used for testing purposes (see Fuller [1976], Dickey and Fuller [1981], Guilkey and Schmidt [1989], and Davidson and MacKinnon [1993]). The SHAZAM output reports asymptotic critical values.

The z-test depends on p (see the remarks in Davidson and MacKinnon [1993]) and, therefore, the SHAZAM output gives the z-test only when p=0. As a practical consideration, when $p > 0$, the treatment of initial values will affect the parameter estimates. In SHAZAM, the initial observations are deleted. Another method, that does not affect the asymptotic results, is to set the initial values of the ΔY_{t-j} to zero. Some researchers use the Akaike (AIC) and the Schwarz (SC) information criteria as a guide for selection of p and these statistics are reported on the SHAZAM output.

Phillips-Perron Unit Root Tests

As an alternative to the inclusion of lag terms to allow for serial correlation the Phillips-Perron method is to use a non-parametric correction for serial correlation. The approach is to first calculate the above unit root tests from regression equations with p=0. The statistics are then transformed to remove the effects of serial correlation on the asymptotic distribution of the test statistic. The critical values are the same as those used for the Dickey-Fuller tests. The formula for the transformed test statistics are listed in Perron [1988, Table 1, p.308-9]. The Newey and West [1987] method is used to construct an estimate of the error variance from the estimated residuals $\hat{\varepsilon}_t$ as:

$$\frac{1}{N}\sum_{t=1}^{N}\hat{\varepsilon}_t^2 + \frac{2}{N}\sum_{s=1}^{l}\omega(s, l)\sum_{t=s+1}^{N}\hat{\varepsilon}_t\hat{\varepsilon}_{t-s}$$

where l is a truncation lag parameter and $\omega(s,l)$ is a window. SHAZAM uses a window choice of: $\omega(s,l) = 1 - s/(l+1)$

The selection of l is an important consideration and further discussion is available in Phillips [1987] and Perron [1988]. The **ARIMA** command (see the chapter *ARIMA MODELS*) can be used to inspect the time series properties of the variable to guide in the

choice of the lag order. By default SHAZAM sets the order as the highest significant lag order from either the autocorrelation function or the partial autocorrelation function of the first differenced series.

TESTS FOR COINTEGRATION

An approach to testing for cointegration (or evidence of a long run relationship between non-stationary variables) is to construct test statistics from the residuals of a cointegrating regression. With M time series $Y_{t1}, \ldots, Y_{tM}$ each of which is I(1) (integrated of order 1), two forms of the cointegrating regression equations are:

$$\text{(A)} \qquad Y_{t1} = \beta_0 + \sum_{j=2}^{M} \beta_j Y_{tj} + u_t$$

$$\text{(B)} \qquad Y_{t1} = \beta_0 + \beta_1 t + \sum_{j=2}^{M} \beta_j Y_{tj} + u_t$$

Equation (A) is no-trend and equation (B) is with-trend. The choice of regressand is arbitrary and different choices can be considered. A test for no cointegration is given by a test for a unit root in the estimated residuals $\hat{u}_t$. The augmented Dickey-Fuller regression equation is:

$$\Delta \hat{u}_t = \alpha_* \hat{u}_{t-1} + \sum_{j=1}^{p} \phi_j \Delta \hat{u}_{t-j} + v_t$$

Test statistics are (i) $N\hat{\alpha}_*$ (the z-test) and (ii) a t-ratio test for $\alpha_* = 0$ (the τ-test). Alternatively, Phillips unit root test statistics can be constructed that use a non-parametric correction for serial correlation. Sources of critical values include Phillips and Ouliaris [1990], MacKinnon [1991] and Davidson and MacKinnon [1993]. The SHAZAM output reports asymptotic critical values obtained from Davidson-MacKinnon. Significant negative test statistics suggest rejection of the unit root hypothesis and evidence for cointegration.

COINT *COMMAND OPTIONS*

In general, the format of the **COINT** command is:

COINT *vars* / *options*

where *vars* is a list of variable names and *options* is a list of desired options. The following options are available on the **COINT** command:

DN Uses a divisor of N, rather than N–K (where K is the number of regressors), when estimating the variance used in calculating the t-statistics. The F-tests are computed with the standard finite sample adjustments.

DUMP Gives output of interest to SHAZAM consultants.

LOG Takes logs of the data.

MAX Gives more detailed output including the correlogram of the first differenced series, the estimated lag coefficients of the augmented Dickey-Fuller regressions, and the parameter estimates of the cointegrating regressions.

BEG=, END= Specifies the **BEG**inning and **END**ing observations to be used in the calculations. This option overrides the **SAMPLE** command and defaults to the sample range in effect.

NDIFF= Specifies the order of differencing to transform the data.

NLAG= Specifies the number of lag terms in the augmented Dickey-Fuller regressions or the truncation lag parameter for the Phillips tests. If this is not specified then the order is set as the highest significant lag order (using an approximate 95% confidence interval) from either the autocorrelation function or the partial autocorrelation function of the first differenced series (up to a maximum lag order of $\sqrt{N}$).

RESID= Specifies a matrix to save the residuals from the regression equations used for the unit root tests. The first column is the residuals from the constant, no trend regression and the second column is the residuals from the constant, trend regression for the first series. The third and fourth column are the residuals for the second series (if requested). For example, to inspect the ACF of the residuals the following SHAZAM commands could be used.

```
sample 1 17
coint consume price / nlag=2 resid=emat

* Adjust the sample period
sample 4 17
do #=1,4
matrix e=emat(0,#)
arima e
endo
```

SIGLEVEL= Specifies the significance level for the critical values. The available choices are 1, 5 and 10. The default is **SIGLEVEL=10** for 10% critical values.

TESTSTAT= Saves the test statistics in the variable specified.

TYPE= Specifies the type of tests to calculate. The available options are:

DF for augmented Dickey-Fuller unit root tests. This is the default.

PP for Phillips-Perron unit root tests.

RESD for Dickey-Fuller tests on the residuals of cointegrating regressions.

RESP for Phillips tests on the residuals of cointegrating regressions.

For the cointegrating regressions the first variable given in the variable list for the **COINT** command is used as the regressand. Note that when the **TYPE=PP** or **TYPE=RESP** options are requested and the truncation lag order is set to 0 (**NLAG=0**) then the Dickey-Fuller unit root tests are calculated.

EXAMPLES

An example of Dickey-Fuller unit root tests for the Theil textile data is shown in the output:

```
|_COINT CONSUME PRICE

...NOTE..TEST LAG ORDER AUTOMATICALLY SET

 TOTAL NUMBER OF OBSERVATIONS =   17

 VARIABLE : CONSUME
 DICKEY-FULLER TESTS - NO.LAGS =   2   NO.OBS =   14
```

```
     NULL                TEST       ASY. CRITICAL
  HYPOTHESIS           STATISTIC      VALUE 10%
-------------------------------------------------------------------------
 CONSTANT, NO TREND
 A(1)=0  T-TEST        -1.4910      -2.57
 A(0)=A(1)=0            3.7439       3.78
                                              AIC =      5.143
                                               SC =      5.326
-------------------------------------------------------------------------
 CONSTANT, TREND
 A(1)=0  T-TEST        -1.4131      -3.13
 A(0)=A(1)=A(2)=0       2.7911       4.03
 A(1)=A(2)=0            1.5715       5.34
                                              AIC =      5.187
                                               SC =      5.416
-------------------------------------------------------------------------

 VARIABLE : PRICE
 DICKEY-FULLER TESTS - NO.LAGS =   0   NO.OBS =   16

     NULL                TEST       ASY. CRITICAL
  HYPOTHESIS           STATISTIC      VALUE 10%
-------------------------------------------------------------------------
 CONSTANT, NO TREND
 A(1)=0  Z-TEST        -1.6049      -11.2
 A(1)=0  T-TEST        -1.2120      -2.57
 A(0)=A(1)=0            2.4019       3.78
                                              AIC =      3.502
                                               SC =      3.599
-------------------------------------------------------------------------
 CONSTANT, TREND
 A(1)=0  Z-TEST        -6.3269      -18.2
 A(1)=0  T-TEST        -1.4458      -3.13
 A(0)=A(1)=A(2)=0       2.0596       4.03
 A(1)=A(2)=0            1.3886       5.34
                                              AIC =      3.533
                                               SC =      3.678
```

In the above example the lag order p for the augmented Dickey-Fuller regression is set automatically (the method is described in the **NLAG=** option). For the *CONSUME* variable 2 lags were included and for the *PRICE* variable no lags were included. Note that the z-test is only reported when p = 0. In all cases, the test statistic is not significant at the 10% level and therefore the null hypothesis is not rejected. For example, for the time series *CONSUME*, for the regression equation with constant and trend, the t-ratio test for $\alpha_1 = 0$ is –1.4131 which exceeds the critical value of –3.13. The listing shows that the tests for `A(0)=A(1)=A(2)=0` and `A(1)=A(2)=0` in the regression equation with constant and trend are not significant at the 10% level for any of the variables. Also, the Φ_1 test statistics for A(0)=A(1)=0 in the regression equation with constant and no trend do not exceed the critical values for the time series studied. The conclusion is that the null hypothesis of a unit root cannot be rejected for the *CONSUME* and *PRICE* time series.

The user should then verify that Dickey-Fuller tests on the first differences (not reported here) show stationarity so that the evidence is that the two time series are I(1). Unit root

tests on first differences can be obtained by using the **NDIFF=1** option on the **COINT** command. Also, the sensitivity of the test results to different choices for the **NLAG=** option may be useful to examine. On the SHAZAM output the AIC test is the Akaike information criteria and the SC test is the Schwarz criteria. Another thing to consider is that, for macroeconomic time series, the **LOG** option may be sensible to use.

To evaluate whether a linear combination of the variables is stationary cointegrating regressions can be estimated as:

```
|_COINT CONSUME PRICE / TYPE=RESD

 ...NOTE..TEST LAG ORDER AUTOMATICALLY SET

 COINTEGRATING REGRESSION - CONSTANT, NO TREND   NO.OBS =   17
 REGRESSAND : CONSUME

  R-SQUARE =  .8961         DURBIN-WATSON =  1.191

  DICKEY-FULLER TESTS ON RESIDUALS - NO.LAGS =  0   M =  2

                         TEST      ASY. CRITICAL
                       STATISTIC     VALUE 10%
 ---------------------------------------------------------------------------
  NO CONSTANT, NO TREND
          Z-TEST       -9.8313     -17.1
          T-TEST       -2.3828     -3.04
                                              AIC =     4.035
                                               SC =     4.083
 ---------------------------------------------------------------------------

 COINTEGRATING REGRESSION - CONSTANT, TREND      NO.OBS =   17
 REGRESSAND : CONSUME

  R-SQUARE =  .8963         DURBIN-WATSON =  1.214

  DICKEY-FULLER TESTS ON RESIDUALS - NO.LAGS =  0   M =  2


                         TEST      ASY. CRITICAL
                       STATISTIC     VALUE 10%
 ---------------------------------------------------------------------------
  NO CONSTANT, NO TREND
          Z-TEST       -9.9990     -23.4
          T-TEST       -2.4364     -3.50
                                              AIC =     4.040
                                               SC =     4.088
```

The output reports the R^2 and the Durbin-Watson test statistic from the cointegrating regressions. A high R^2 value and a low Durbin-Watson value is evidence of cointegration (for more discussion see Engle and Granger [1987]). The test statistics on the regression residuals can be compared with the critical values that are reported on the SHAZAM output. The results show that the null hypothesis of non-stationarity cannot be rejected. This suggests that the *CONSUME* and *PRICE* variables are not cointegrated.

14. DIAGNOSTIC TESTS

"That is a question which has puzzled many an expert, and why? Because there was no reliable test. Now we have the Sherlock Holmes test, and there will no longer be any difficulty."

Holmes to Watson in "A Study in Scarlet" by A. Conan Doyle

SHAZAM can perform a number of diagnostic tests after estimating a single-equation regression model including tests on recursive residuals, Goldfeld-Quandt tests, Chow tests, RESET specification error tests, and tests for autocorrelation and heteroskedasticity. The **DIAGNOS** command is used for these tests and many other statistics. Discussion and examples are available in good econometrics textbooks such as Greene [2003], Gujarati [2003] or Wooldridge [2006]. Other references are: Harvey [1990, Chapter 5]; Godfrey, McAleer, and McKenzie [1988]; Zarembka [1974, Chapter 1]; Breusch and Pagan [1979]; and Pagan and Hall [1983].

DIAGNOS *COMMAND OPTIONS*

The **DIAGNOS** command can be used following an **OLS** command. In general, the format of the **DIAGNOS** command is:

OLS *depvar indeps*
DIAGNOS / *options*

where *options* is a list of desired options. The options are summarized below and then are more fully described and demonstrated in the *EXAMPLES* section that follows.

The available options on the **DIAGNOS** command are:

ACF Prints the **A**uto**C**orrelation **F**unction of residuals and associated test statistics.

BACKWARD Used with the **RECUR** option to compute **BACKWARD**s recursive residuals.

BOOTLIST Used with the **BOOTSAMP=** option to print the entire list of Bootstrapped coefficients for every generated sample. This could generate a lot of output.

CHOWTEST Produces a set of sequential **CHOW TEST** statistics and sequential Goldfeld-Quandt Test statistics which split the sample of dependent and independent variables in 2 pieces at every possible point. Some users may wish to sort the data first as suggested by Goldfeld and Quandt [1972]. It also computes some recursive residuals test statistics. Also see the **CHOWONE=, GQOBS=** and **MHET=** options.

CTEST Produces Pinkse's [1996] C-test for serial independence of error terms and stores the result in the temporary variable *$CTES*. The C-test tests for independence of the errors instead of lack of correlation. In the SHAZAM implementation the null hypothesis is independence and the alternative hypothesis is serial dependence of order one. The test is consistent against higher order dependence structures also, as long as under the alternative consecutive elements of the time series are not independent.

GRAPH Prepares gnuplot plots of the recursive residuals when the **MAX** or **RECUR** options are specified. For more information on this option see the chapter *PLOTS AND GRAPHS*. With the **GRAPH** option the **APPEND**, **OUTPUT=**, **DEVICE=, PORT=** and **COMMFILE=** options are also available as described for the **GRAPH** command.

HANSEN Reports Hansen [1992] tests for parameter instability. Test statistics for the stability of each parameter (β, σ^2) individually and for the joint stability of all (K+1) parameters are calculated. Asymptotic critical values are listed in Table 1 of the Hansen paper. SHAZAM flags rejection of the null hypothesis of stability at the 1%, 5% and 10% significance levels. In contrast to the Chow test, the Hansen test does not require the specification of a breakpoint.

HET Runs a series of tests for **HET**eroskedasticity.

JACKKNIFE Runs a series of regressions, successively omitting a different observation to get **JACKKNIFE** coefficient estimates. An example of jackknife estimation is in the Appendix to Chapter 9 in the *Judge Handbook*.

LIST Prints a table of observed (Y) and predicted ($\hat{Y}$) values of the dependent variable, and regression residuals (e). This gives the same output as the **LIST** option on **OLS**.

MAX Equivalent to specifying the **LIST**, **RECUR**, **ACF**, **BACKWARD**, **CHOWTEST**, **HANSEN**, **RESET** and **HET** options. The computation of all these tests can be slow and can generate a lot of output.

RECEST
NORECEST **NORECEST** suppresses the printing of the **REC**ursive **EST**imated coefficients when the **RECUR** option is specified.

RECRESID
NORECRESID **NORECRESID** suppresses the printing of the **REC**ursive **RESID**uals when the **RECUR** option is specified.

RECUR Performs **RECUR**sive Estimation by running a series of regressions by adding one observation per regression. It is often used for tests of structural change. Recursive residuals and CUSUM tests are printed along with a recursive t-test and Harvey's recursive residuals exact heteroskedasticity test. Also see the **MHET=** and **SIGLEVEL=** options.

RESET Used to compute Ramsey [1969] **RESET** (regression specification error test) statistics and the DeBenedictis and Giles [1998, 2000] FRESET tests for mis-specification.

WHITE
NOWHITE With the **HET** option, the **NOWHITE** option excludes the computation of the White test statistics for heteroskedasticity. This is recommended when dummy variables are included in the list of explanatory variables.

WIDE
NOWIDE Reduces the width of output to 80 columns. The default value is explained in the chapter *SET AND DISPLAY*.

BOOTSAMP= Specifies the number of samples desired for **BOOTSAMP** experiments on the previous OLS regression. This option is not appropriate with models that include lagged dependent variables as regressors. Examples of this option can be found in the Appendix to Chapter 9 of the *Judge Handbook*. Also see the example *Bootstrapping Regression Coefficients* in the chapter *PROGRAMMING IN SHAZAM*. Initially, a regression is run and the K coefficients $\hat{\beta}$ and N residuals e are saved. Then, random samples

of size N of residuals are drawn with replacement and the residuals are normalized using: $e_t^* = e_t / \sqrt{1 - K/N}$.

Next, a new dependent variable is generated as: $Y^* = X\hat{\beta} + e^*$

The bootstrap estimates are computed as: $\hat{\beta}^* = (X'X)^{-1}X'Y^*$.

BOOTUNIT= Writes out the generated coefficients for each sample in a **BOOT**strap experiment on the **UNIT** specified. It is used in conjunction with the **BOOTSAMP=** option. A file should be assigned to the unit with the SHAZAM **FILE** command. Units 11-49 may be used.

CHOWONE= Specifies the number of observations in the first group for the Chow test and Goldfeld-Quandt test. (The **CHOWTEST** option reports test statistics at every breakpoint).

GQOBS= Used with the **CHOWTEST** option to specify the number of central observations to be omitted for the Goldfeld-Quandt Test. The default is zero.

MHET= Used with the **CHOWTEST** and **RECUR** options to specify M, the number of residuals to use in Harvey's recursive residuals exact heteroskedasticity test.

RECUNIT= Used with the **RECUR** option to write the **REC**ursive residuals and the CUSUM and CUSUMSQ of the recursive residuals on the **UNIT** specified. A file should be assigned to the unit with the SHAZAM **FILE** command. Units 11-49 may be used.

SIGLEVEL= Used with the **RECUR** option to specify the significance level desired for the CUSUM and CUSUMSQ tests. The available choices are 1, 5, and 10. The default is **SIGLEVEL=5**.

Warnings: After a regression with the **RESTRICT** option or a distributed lag model that incorporates restrictions the **DIAGNOS** command will not recognize these restrictions. That is, the **CHOWTEST** and **RECUR** options will work with the unrestricted model. After a weighted least squares regression with the **WEIGHT=** option the **DIAGNOS** command will compute test statistics with the untransformed residuals and so the **HET** option will not be appropriate for testing for heteroskedasticity in the transformed residuals.

EXAMPLES

For the regression equation $Y_t = X_t'\beta + \varepsilon_t$ with K coefficients β, denote the residuals by e_t and the predicted values by $\hat{Y}_t$ for t = 1, . . . , N . Using Theil's textile data a complete listing of diagnostic tests is obtained with the SHAZAM commands:

```
ols consume income price
diagnos / max
```

Users may want to be selective in the tests that they consider. The examples that follow show SHAZAM output for some of the **DIAGNOS** options from the above OLS regression.

Tests for Autocorrelation

The output produced with the **ACF** option is:

```
RESIDUAL CORRELOGRAM
  LM-TEST FOR HJ:RHO(J)=0, STATISTIC IS STANDARD NORMAL
  LAG      RHO         STD ERR      T-STAT      LM-STAT     DW-TEST BOX-PIERCE-LJUNG
    1     -.1455         .2425      -.5998       .7014      2.0185       .4272
    2     -.2231         .2425      -.9200      1.2257      2.0359      1.4994
    3      .1871         .2425       .7716       .9975      1.1956      2.3074
    4     -.3002         .2425     -1.2377      1.7388      2.0133      4.5463
 CHI-SQUARE WITH    4  D.F. IS      3.333
```

The residual autocorrelations (RHO) are calculated as:

$$\hat{\rho}_j = \frac{\sum_{t=j+1}^{N} e_t e_{t-j}}{\sum_{t=1}^{N} e_t^2} \qquad \text{for } j = 1, \ldots, p$$

Note that SHAZAM automatically sets the maximum lag order p based on the sample size of the data. In the above example p=4. If the autoregressive order is j the higher order autocorrelations are asymptotically normally distributed with zero mean and standard deviation $1/\sqrt{N}$ (given on the SHAZAM output in the column STD ERR). The t-statistics (T-STAT) are calculated as $\sqrt{N}\hat{\rho}_j$.

A Lagrange multiplier statistic for a test of H_0: $\rho_j = 0$ is discussed in Breusch and Pagan [1980, Section 3.2]. Denote e_{-j} as the (N x 1) vector containing e_{t-j} (with zeroes for initial values). The test statistic is:

$$LM = N^2\hat{\rho}_j^2\tilde{\sigma}^2 \Big/ [e_{-j}'e_{-j} - e_{-j}'X(X'X)^{-1}X'e_{-j}] \qquad \text{where} \qquad \tilde{\sigma}^2 = \frac{e'e}{N}$$

The value $\sqrt{LM}$ is reported as the LM-STAT statistic on the SHAZAM output. These test statistics have an asymptotic standard normal distribution. This test is appropriate when X contains lagged dependent variables.

A test for $H_0 : \rho_1 = \rho_2 = \ldots = \rho_J = 0$ is given by the Box-Pierce-Ljung statistic (also see the chapter *ARIMA MODELS*) computed as:

$$Q = N(N+2)\sum_{j=1}^{J}\frac{1}{N-j}\hat{\rho}_j^2 \qquad \text{for } J = 1,\ldots,p$$

This test is not appropriate when X includes lagged dependent variables. Under the null hypothesis Q has an asymptotic χ^2_J distribution. The Q statistic incorporates a small sample adjustment to give a modified form of the Box-Pierce statistic. The chi-square statistic reported at the end of the ACF output is the Box-Pierce statistic for a test that the residual autocorrelations are jointly zero when there are no lagged dependent variables. The statistic, that can be compared with a χ^2_p distribution, is given by:

$$N\sum_{j=1}^{p}\hat{\rho}_j^2$$

Tests for Heteroskedasticity

The output produced with the **HET** option is:

```
HETEROSKEDASTICITY TESTS
                              CHI-SQUARE     D.F.   P-VALUE
                            TEST STATISTIC
E**2 ON YHAT:                      2.495      1    0.11424
E**2 ON YHAT**2:                   2.658      1    0.10304
E**2 ON LOG(YHAT**2):              2.303      1    0.12911
E**2 ON LAG(E**2) ARCH TEST:       1.490      1    0.22221
LOG(E**2) ON X (HARVEY) TEST:      3.421      2    0.18081
ABS(E) ON X (GLEJSER) TEST:        4.202      2    0.12231
E**2 ON X                  TEST:
          KOENKER(R2):             4.900      2    0.08631
          B-P-G (SSR):             2.529      2    0.28241
E**2 ON X X**2    (WHITE) TEST:
          KOENKER(R2):             4.936      4    0.29390
          B-P-G (SSR):             2.548      4    0.63612
E**2 ON X X**2 XX (WHITE) TEST:
          KOENKER(R2):             4.951      5    0.42191
          B-P-G (SSR):             2.555      5    0.76816
```

The test statistics are obtained from the results of auxiliary regressions of the form:

	Regressand	Regressors	Test Statistic	D.F.
	e_t^2	$\hat{Y}_t$, constant	NR^2	1
	e_t^2	$\hat{Y}_t^2$, constant	NR^2	1
	e_t^2	$\log(\hat{Y}_t^2)$, constant	NR^2	1
ARCH	e_t^2	e_{t-1}^2, constant	NR^2	1
Harvey	$\log(e_t^2)$	X_t	SSR/4.9348	$K-1$
Glejser	$\lvert e_t \rvert$	X_t	$SSR/[(1-\frac{2}{\pi})\tilde{\sigma}_\varepsilon^2]$	$K-1$
Koenker	e_t^2	X_t	NR^2	$K-1$
B-P-G	e_t^2	X_t	$SSR/(2\tilde{\sigma}_\varepsilon^4)$	$K-1$
White	e_t^2	$X_{kt}, k=1,\ldots,K;$ $X_{kt}^2, k=1,\ldots,K-1$	NR^2	$2(K-1)$
	e_t^2	same as above	$SSR/(2\tilde{\sigma}_\varepsilon^4)$	$2(K-1)$
White	e_t^2	$X_{kt}, k=1,\ldots,K;$ $X_{kt}X_{jt}, k=1,\ldots,K-1,$ $j=1,\ldots,k$	NR^2	$\frac{K(K+1)}{2}-1$
	e_t^2	same as above	$SSR/(2\tilde{\sigma}_\varepsilon^4)$	$\frac{K(K+1)}{2}-1$

where $\tilde{\sigma}_\varepsilon^2 = e'e/N$ and R^2 and SSR are the multiple coefficient of determination and the regression sum of squares respectively from the auxiliary regression. Under the null hypothesis of homoskedasticity the test statistics can be compared with a χ^2 distribution with degrees of freedom as given in the D.F. column.

The ARCH test was introduced by Engle [1982]. The Harvey test is from Harvey [1976] and is discussed in Judge, Griffiths, Hill, Lutkepohl and Lee [1985, Equation 11.2.60]. The Glejser test is due to Glejser [1969] and is in Judge et al. [1985, Equation 11.2.29]. The B-P-G test is the Breusch-Pagan-Godfrey statistic (see Breusch and Pagan [1979] and Godfrey [1978]). Textbook discussion of this test is available in Griffiths, Hill and Judge [1993, pp. 495-496], Gujarati [2003, pp. 411-2] and Greene [2003, pp. 223-4]. The Koenker test is described in Greene [2003, p. 224] and Judge et al. [1985, p. 447, Equation 11.3.4]. White's

[1980] general heteroskedasticity test is discussed in Gujarati [1995, p. 379] and Greene [2003, p. 222].

Warning: The White general heteroskedasticity test is not defined when dummy variables are included in the list of explanatory variables. The **NOWHITE** option can be used to suppress the calculation of the White test statistics.

Recursive Residuals and the CUSUM and CUSUMSQ Tests

The output produced with the **RECUR** option is:

```
RECURSIVE COEFFICIENT ESTIMATES
   3   .58599       1.1338      -71.974
   4  -.54970E-01  -1.2802       233.31
   5  -.86357      -1.9976       384.60
   6   .92446E-01  -1.5186       242.54
   7   .60090      -1.2248       163.09
   8   .61319      -1.4191       180.66
   9   .36233      -1.6835       231.55
  10   .55817      -1.5320       197.07
  11   .59587      -1.5096       191.07
  12   .93135      -1.3893       145.05
  13   1.0961      -1.3266       122.08
  14   1.0210      -1.3876       135.31
  15   1.0177      -1.3746       134.45
  16   1.0213      -1.3458       131.41
  17   1.0617      -1.3830       130.71

RECURSIVE RESIDUALS - SIGNIFICANCE LEVEL = 5%
  OBS   REC-RES       CUSUM    BOUND            LOWER    CUSUMSQ  UPPER
    4    .94469       .16379   4.0538          -.32902   .00206   .47188
    5    2.2989       .56238   4.5605          -.25759   .01426   .54331
    6    2.7913      1.04635   5.0673          -.18616   .03224   .61474
    7    3.7605      1.69835   5.5740          -.11474   .06487   .68616
    8    5.2155      2.60264   6.0807          -.04331   .12765   .75759
    9    2.5217      3.03985   6.5875           .02812   .14232   .82902
   10   -3.5083      2.43157   7.0942           .09955   .17073   .90045
   11   -.95106      2.26667   7.6009           .17098   .17282   .97188
   12   -9.1586       .67873   8.1076           .24241   .36639  1.04331
   13   -8.5802      -.80893   8.6144           .31384   .53629  1.11474
   14    7.2539       .44877   9.1211           .38526   .65773  1.18616
   15   -2.4963       .01595   9.6278           .45669   .67211  1.25759
   16   -6.5587     -1.12122  10.1345           .52812   .77138  1.32902
   17    9.9530       .60447  10.6413           .59955  1.00000  1.40045

HARVEY-COLLIER  [1977] RECURSIVE T-TEST =           .1616  WITH   13 D.F.
HARVEY-PHILLIPS [1974] HETEROSKEDASTICITY TEST =  7.1480  WITH M =    4
```

A good discussion of recursive least squares is in Harvey [1990, Chapter 2.6]. The `RECURSIVE COEFFICIENT ESTIMATES` are OLS estimates based on the first t observations. Therefore, for t=N the recursive coefficients are identical to OLS. On the above output it can be verified that the coefficient estimates at 17 are the same as the coefficient estimates that

are produced with the **OLS** command. Denote the recursive coefficients by b_t for t = K, . . . , N and define the t x K matrix $X_{(t)}$ as the matrix containing X_1, X_2, . . . , X_t. The recursive residuals (in the column REC-RES on the SHAZAM output) are calculated as standardized prediction errors as:

$$v_t = (Y_t - X_t' b_{t-1}) \Big/ \sqrt{1 + X_t' \left(X_{(t-1)}' X_{(t-1)}\right)^{-1} X_t} \qquad \text{for } t = K+1, \ldots, N$$

The CUSUM (cumulative sum) of recursive residuals is:

$$W_t = \frac{1}{\hat{\sigma}} \sum_{j=K+1}^{t} v_j \qquad \text{for } t = K+1, \ldots, N, \quad \text{where} \quad \hat{\sigma}^2 = \frac{1}{N-K-1} \sum_{t=K+1}^{N} (v_t - \bar{v})^2$$

and $\bar{v}$ is the mean of the recursive residuals. The CUSUMSQ (cumulative sum of squares) is:

$$WW_t = \sum_{j=K+1}^{t} v_j^2 \Big/ \sum_{t=K+1}^{N} v_t^2 \qquad \text{for } t = K+1, \ldots, N$$

A test of misspecification can be based on the inspection of the CUSUM and CUSUMSQ of the recursive residuals. The construction of significance lines is developed in Brown, Durbin and Evans [1975]. The significance values for the CUSUMSQ test are tabulated in Durbin [1969] and reprinted in Harvey [1990, p. 366-7]. The SHAZAM output reports approximate 5% upper and lower bounds for the CUSUMSQ test. The **SIGLEVEL=** option can be used to request 1% or 10% bounds. Brown, Durbin and Evans [1975] recommend plots as a way of presenting these tests and they comment that "the significance tests suggested should be regarded as yardsticks for the interpretation of data rather than leading to hard and fast decisions". Plots are obtained with the **GRAPH** option. Note that these tests are not valid for models with lagged dependent variables.

Test statistics can be constructed to detect departures from randomness in the residuals. To test the hypothesis that the mean of the recursive residuals is zero the SHAZAM output reports the RECURSIVE T-TEST (from Harvey and Collier [1977] and given in Harvey [1990, Chapter 5, Equation 2.10]) calculated as:

$$\bar{v} \Big/ \sqrt{\hat{\sigma}^2 / (N-K)}$$

The statistic can be compared with a t-distribution with (N–K–1) degrees of freedom. The second test statistic that the SHAZAM output reports is a HETEROSKEDASTICITY TEST

proposed in Harvey and Phillips [1974] and given in Harvey [1990, Chapter 5, Equation 2.12]. This test statistic is based on the first set and last set of m recursive residuals and is calculated as:

$$\sum_{t=N-m+1}^{N} v_t^2 \Big/ \sum_{t=K+1}^{K+m} v_t^2$$

This has an $F_{(m,m)}$ distribution under the null hypothesis of homoskedasticity. SHAZAM sets m equal to (N–K)/3. Other choices of m can be set with the **MHET=** option.

The output produced with the **BACKWARD** and **RECUR** options is:

```
 BACKWARDS
RECURSIVE COEFFICIENT ESTIMATES
  15   6.2400       1.5400      -576.61
  14   17.460      -12.984      -853.75
  13   2.1880      -2.4999       83.360
  12   1.8767      -2.3500       106.17
  11   2.0353      -2.2604       85.733
  10   2.0944      -2.2117       77.007
   9   2.1966      -2.1140       60.963
   8   2.1916      -2.0003       54.550
   7   2.0917      -1.8616       56.131
   6   2.1017      -1.8725       55.789
   5   1.8743      -1.7343       70.398
   4   1.8319      -1.7090       73.130
   3   1.3919      -1.5101       105.21
   2   1.2061      -1.4356       119.36
   1   1.0617      -1.3830       130.71

BACKWARDS
RECURSIVE RESIDUALS - SIGNIFICANCE LEVEL = 5%
  OBS   REC-RES       CUSUM    BOUND            LOWER    CUSUMSQ  UPPER
   14    5.6889      1.68608   4.0538          -.32902   .07469   .47188
   13    11.008      4.94850   4.5605          -.25759   .35432   .54331
   12    3.2906      5.92376   5.0673          -.18616   .37930   .61474
   11    4.9015      7.37648   5.5740          -.11474   .43475   .68616
   10    .98227      7.66761   6.0807          -.04331   .43698   .75759
    9    2.1057      8.29170   6.5875           .02812   .44721   .82902
    8    1.7982      8.82465   7.0942           .09955   .45467   .90045
    7    3.0412      9.72602   7.6009           .17098   .47602   .97188
    6   -.45786      9.59031   8.1076           .24241   .47650  1.04331
    5    6.4177     11.49238   8.6144           .31384   .57155  1.11474
    4    1.3958     11.90606   9.1211           .38526   .57605  1.18616
    3    9.8188     14.81615   9.6278           .45669   .79854  1.25759
    2    6.4933     16.74062  10.1345           .52812   .89584  1.32902
    1    6.7182     18.73175  10.6413           .59955  1.00000  1.40045

BACKWARDS
HARVEY-COLLIER  [1977] RECURSIVE T-TEST =          5.0063  WITH   13 D.F.
HARVEY-PHILLIPS [1974] HETEROSKEDASTICITY TEST =     .9855  WITH M =    4
```

The backward recursive coefficients and recursive residuals estimates are calculated using the same formulas as given above, but the calculations are started at the end, instead of at the beginning, of the sample.

The CHOW Test and Goldfeld-Quandt Test

The output produced with the **CHOWTEST** option is:

```
 HARVEY-COLLIER  [1977] RECURSIVE T-TEST =            .1616  WITH   13 D.F.
HARVEY-PHILLIPS [1974] HETEROSKEDASTICITY TEST =    7.1480  WITH M =    4

BACKWARDS
HARVEY-COLLIER  [1977] RECURSIVE T-TEST =           5.0063  WITH   13 D.F.
HARVEY-PHILLIPS [1974] HETEROSKEDASTICITY TEST =     .9855  WITH M =    4

SEQUENTIAL CHOW AND GOLDFELD-QUANDT TESTS
  N1   N2    SSE1        SSE2       CHOW    PVALUE   G-Q        DF1  DF2 PVALUE
   4   13  .89245       247.66     2.7256    .095  .3604E-01    1   10  .147
   5   12  6.1774       206.47     3.8048    .043  .1346        2    9  .124
   6   11  13.969       206.26     3.5476    .051  .1806        3    8  .093
   7   10  28.110       197.02     3.3908    .058  .2497        4    7  .099
   8    9  55.312       193.78     2.7117    .096  .3425        5    6  .130
   9    8  61.671       189.35     2.6628    .100  .2714        6    5  .072
  10    7  73.979       188.38     2.3892    .124  .2244        7    4  .042
  11    6  74.883       164.36     2.9744    .078  .1709        8    3  .020
  12    5  158.76       153.53     1.4209    .289  .2298        9    2  .048
  13    4  232.38       32.364     2.3346    .130  .7180       10    1  .265

            CHOW TEST - F DISTRIBUTION WITH DF1=   3 AND DF2=  11
```

The Chow [1960] test gives a test for structural change. The SHAZAM output gives statistics that test for breaks at t = K+1,. . ., N–K–1. The Chow test statistic is calculated as:

$$\text{CHOW} = \frac{(\text{SSE} - \text{SSE}\,1 - \text{SSE}\,2)/\text{K}}{(\text{SSE}\,1 + \text{SSE}\,2)/(\text{N}_1 + \text{N}_2 - 2\text{K})}$$

where SSE1 and SSE2 are the sum of squared errors from the first and second parts of the split sample, N is the number of observations in the entire sample, K is the number of estimated parameters, N_1 and N_2 are the observations in the first and second part of the split sample respectively, and SSE is the residual sum of squares from the regression over the entire sample. If the test statistic is less than the critical value from an $F_{(K,N_1+N_2-2K)}$ distribution then there is no evidence for a structural break.

The Goldfeld-Quandt [1965, 1972] (G-Q) statistic provides a test for different error variance between two subsets of observations. Denote the variance in the first subset by σ_1^2 and the variance in the second subset by σ_2^2. The null hypothesis is $H_0 : \sigma_1^2 = \sigma_2^2$ and the alternative hypothesis is $H_1 : \sigma_1^2 > \sigma_2^2$ (that is, the second subset of observations has smaller variance

than the first subset). Note that many authors present the alternative as larger variance in the second subset of observations. Goldfeld and Quandt recommend ordering of the observations by the values of one of the regressors. This can be done with the **SORT** command. The **DESC** option on the **SORT** command should be used if it is assumed that the variance is positively related to the value of the sort variable. The test can be implemented by omitting r central observations. The value for r can be specified with the **GQOBS=** option and the default is r = 0. The sample is split into two groups with N_1 and N_2 observations such that $N_1+N_2 = N$ and the test statistic is calculated as:

$$GQ = \frac{RSSE\ 1 / DF\ 1}{RSSE\ 2 / DF\ 2}$$

where RSSE1 and RSSE2 are the sum of squared errors from the first $N_1–r/2$ and the last $N_2–r/2$ observations respectively and DF1 = $N_1–K–r/2$ and DF2 = $N_2–K–r/2$. The statistic can be compared with an $F_{(DF\ 1, DF\ 2)}$ distribution.

If the GQ test statistic is less than 1 then the p-value reported in the final column of the SHAZAM output is for a test of the null hypothesis of equal variance against the alternative hypothesis of $\sigma_1^2 < \sigma_2^2$ (larger variance in the second group). Therefore, there is evidence for *smaller* variance in the second group if GQ > 1 and the p-value is less than 0.05 (or some selected significance level). There is evidence for *larger* variance in the second group if GQ < 1 and the p-value is less than 0.05.

Hansen Tests

The output produced with the **HANSEN** option is:

```
HANSEN INSTABILITY TEST
   COEFFICIENT   TEST-STAT
   INCOME        0.59145E-01   STABLE:10% CRITICAL VALUE IS 0.353
   PRICE         0.91351E-01   STABLE:10% CRITICAL VALUE IS 0.353
   CONSTANT      0.61507E-01   STABLE:10% CRITICAL VALUE IS 0.353
   VARIANCE      0.63016     UNSTABLE: 5% CRITICAL VALUE IS 0.470
    _JOINT_       1.1880     UNSTABLE:10% CRITICAL VALUE IS 1.070
```

A test statistic that exceeds the critical value gives evidence for rejecting the null hypothesis of parameter stability. The joint stability test includes the variance. Discussion of the Hansen test is available in Greene [2003, p. 134].

RESET Tests

The output produced with the **RESET** option is:

```
RAMSEY RESET SPECIFICATION TESTS USING POWERS OF YHAT
  RESET(2)=   11.787     - F WITH DF1=   1 AND DF2=  13 P-VALUE= 0.004
  RESET(3)=   5.4877     - F WITH DF1=   2 AND DF2=  12 P-VALUE= 0.020
  RESET(4)=   3.6049     - F WITH DF1=   3 AND DF2=  11 P-VALUE= 0.049

DEBENEDICTIS-GILES FRESET SPECIFICATION TESTS USING FRESETL
 FRESET(1)=   3.8735     - F WITH DF1=   2 AND DF2=  12 P-VALUE= 0.050
 FRESET(2)=   1.9191     - F WITH DF1=   4 AND DF2=  10 P-VALUE= 0.184
 FRESET(3)=   1.0519     - F WITH DF1=   6 AND DF2=   8 P-VALUE= 0.460

DEBENEDICTIS-GILES FRESET SPECIFICATION TESTS USING FRESETS
 FRESET(1)=   2.0201     - F WITH DF1=   2 AND DF2=  12 P-VALUE= 0.175
 FRESET(2)=  0.88559     - F WITH DF1=   4 AND DF2=  10 P-VALUE= 0.507
 FRESET(3)=   2.4366     - F WITH DF1=   6 AND DF2=   8 P-VALUE= 0.121
```

The Ramsey [1969] RESET tests (REgression Specification Error Test) are computed by introducing test variables constructed as powers of the predicted values $\hat{Y}$ as additional regressors. The RESET test is an F test that tests whether the coefficients on the new regressors are zero.

The DeBenedictis and Giles [1998, 2000] FRESET tests for mis-specification consider a Fourier approximation for the mis-specified part of the regression equation. They propose constructing:

$$w_t = \pi \cdot \frac{2\hat{Y}_t - \left(\hat{Y}_{max} + \hat{Y}_{min}\right)}{\hat{Y}_{max} - \hat{Y}_{min}}$$

An alternative is: $w_t^* = 2\pi \sin^2\left(\hat{Y}_t\right) - \pi$

The test statistics for the various mis-specification tests are calculated from auxiliary regressions that include m additional regressors as given in the table below.

Test	m	Test Variables
RESET(2)	1	$\hat{Y}_t^2$
RESET(3)	2	$\hat{Y}_t^2$, $\hat{Y}_t^3$
RESET(4)	3	$\hat{Y}_t^2$, $\hat{Y}_t^3$, $\hat{Y}_t^4$

FRESETL(1)	2	$\sin(w_t), \cos(w_t)$
FRESETL(2)	4	$\sin(w_t), \cos(w_t), \sin(2w_t), \cos(2w_t)$
FRESETL(3)	6	$\sin(w_t), \cos(w_t), \sin(2w_t), \cos(2w_t), \sin(3w_t), \cos(3w_t)$
FRESETS(1)	2	$\sin(w_t^*), \cos(w_t^*)$
FRESETS(2)	4	$\sin(w_t^*), \cos(w_t^*), \sin(2w_t^*), \cos(2w_t^*)$
FRESETS(3)	6	$\sin(w_t^*), \cos(w_t^*), \sin(2w_t^*), \cos(2w_t^*), \sin(3w_t^*), \cos(3w_t^*)$

Denote the multiple coefficient of determination from the initial regression by R_0^2 and the multiple coefficient of determination from the auxiliary regression by R^2. The F-test statistic that can compared with an F-distribution with (m, N–K–m) degrees of freedom is:

$$\frac{(R^2 - R_0^2)/m}{(1 - R^2)/(N - K - m)}$$

The Jackknife Estimator

The output produced with the **JACKKNIFE** option is:

```
JACKKNIFE COEFFICIENTS

COEFFICIENT    AVERAGE      ST.ERR
   INCOME       1.0259       .27080
   PRICE       -1.3694       .91245E-01
   CONSTANT     133.48       28.928
```

With this option SHAZAM runs a series of regressions, successively omitting a different observation to get jackknife coefficient estimates (see, for example, Judge et al. [1988, Section 9.A.2]). The jackknife coefficients are:

$$\hat{\beta}_{(t)} = \hat{\beta} - (X'X)^{-1}X_t e_t^+ \qquad \text{where} \qquad e_t^+ = e_t/(1 - K_{tt})$$

and K_{tt} is the t^{th} diagonal element of the matrix $X(X'X)^{-1}X'$. A total of N (Kx1) coefficient vectors are generated each corresponding to a separate regression with the t^{th} observation dropped. The average of the N (Kx1) coefficient vectors is calculated and reported on the SHAZAM output. The jackknife estimator of the covariance matrix of the parameter estimates is given in Judge et al. [1988, Section 9.A.2].

15. DISTRIBUTED-LAG MODELS

"Gentlemen, you have come sixty days too late. The depression is over."
Herbert Hoover
U.S. President, June 1930

Models that include lagged variables as explanatory variables can be specified with a special form of notation available only with the **OLS**, **AUTO**, **BOX**, **GLS** and **POOL** commands. This is ***not*** available with the **SYSTEM**, **MLE**, **NL**, **PROBIT**, **LOGIT**, **TOBIT**, **ROBUST**, **ARIMA** or any other estimation command. An extension is available for the estimation of Almon polynomial distributed lags. With the **OLS** command, the **PIL** option is available for estimation with the Mitchell-Speaker [1986] polynomial inverse lags.

Discussion on the use of lagged variables in regression analysis is available in many econometrics textbooks. For example, see Gujarati [2003, Chapter 17]; Greene [2003, Chapter 19]; Griffiths, Hill and Judge [1993, Chapter 21]; Maddala [1977, pp. 355-359]; Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 9.3]; Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 17]; Pindyck and Rubinfeld [1998, Chapter 9]; and Johnston [1984]. The distributed lag model has the form:

$$Y_t = \gamma + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + \beta_S X_{t-S} + \varepsilon_t$$

where S is the lag length.

A distributed lag for any explanatory variable on the estimation command (**OLS**, **AUTO**, **BOX**, **GLS** or **POOL**) can be specified using the special form:

indep(first.last)

where *indep* is the name of an independent variable. The numbers in parentheses specify the *first* and *last* periods to use for lags. For example, (0.3) means to use the current period (0) and lags t–1, t–2, and t–3. Each explanatory variable may have a different lag structure.

An example, from Griffiths, Hill and Judge [1993, p. 683], is the response of quarterly capital expenditures (*Y*) to capital appropriations (*X*) in manufacturing. Assuming a lag length of 8 periods the SHAZAM commands to estimate the model are:

```
sample 1 88
read (table21.1) time y x
ols y x(0.8)
```

SHAZAM will automatically delete the necessary number of observations corresponding to any undefined lagged variables at the beginning of the data, so this should not be done with the **SAMPLE** command. In the above example the estimation sample period will start at observation 9. However, if you wish to use the **SAMPLE** command and delete the observations yourself, you should use the **SET NODELETE** command. For example, the distributed lag model for manufacturing expenditure can be estimated with the following commands.

```
sample 1 88
read (table21.1) time y x
sample 9 88
set nodelete
ols y x(0.8)
```

The SHAZAM output that corresponds to Table 21.2 of Griffiths et al. [1993, p. 685] is:

```
|_OLS Y X(0.8)
 LAG FOR X       RANGE =  0   8 ORDER= 0 ENDCON=0
 OLS ESTIMATION
      80 OBSERVATIONS     DEPENDENT VARIABLE = Y
...NOTE..SAMPLE RANGE SET TO:    9,   88

 R-SQUARE =     .9934     R-SQUARE ADJUSTED =     .9926
VARIANCE OF THE ESTIMATE-SIGMA**2 =   35214.
STANDARD ERROR OF THE ESTIMATE-SIGMA =   187.65
SUM OF SQUARED ERRORS-SSE=   .24650E+07
MEAN OF DEPENDENT VARIABLE =   4532.5
LOG OF THE LIKELIHOOD FUNCTION = -526.942

                  TESTS ON LAGGED COEFFICIENTS
VARIABLE  SUM(COEFS)  STD ERROR  T-RATIO  P-VALUE | MEAN LAG| JOINT-F  P-VALUE
X         0.93923      0.117E-01   80.0     0.000 |   3.917 |  0.117E+04 0.000

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      70 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
X          .38379E-01  .3467E-01   1.107      .272  .131      .0537      .0454
X          .67204E-01  .6851E-01   .9809      .330  .116      .0911      .0776
X          .18124      .8936E-01   2.028      .046  .236      .2243      .2019
X          .19443      .9254E-01   2.101      .039  .244      .2208      .2095
X          .16989      .9312E-01   1.824      .072  .213      .1814      .1779
X          .52360E-01  .9177E-01   .5706      .570  .068      .0526      .0534
X          .52461E-01  .9385E-01   .5590      .578  .067      .0494      .0521
X          .56178E-01  .9415E-01   .5967      .553  .071      .0499      .0544
X          .12708      .5983E-01   2.124      .037  .246      .1086      .1204
CONSTANT   33.415      53.71       .6221      .536  .074      .0000      .0074
```

The parameter estimates $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_8$ and $\hat{\alpha}$ are listed in the ESTIMATED COEFFICIENT column. The estimate $\hat{\beta}_0$ has an interpretation as an impact or short-run multiplier. The

long-run or total distributed lag multiplier is estimated as the sum of the lag coefficients (SUM(COEFS)):

$$\hat{\beta}^{m} = \sum_{i=0}^{S} \hat{\beta}_i$$

The standard error (STD ERROR) of the long run multiplier is calculated as:

$$SE(\hat{\beta}^{m}) = \left[\sum_{i=0}^{S} V(\hat{\beta}_i) + 2 \sum\sum_{i<j} Cov\left(\hat{\beta}_i, \hat{\beta}_j\right) \right]^{1/2}$$

A T-RATIO is computed as $\hat{\beta}^{m} / SE(\hat{\beta}^{m})$.

The mean lag gives a measure of the speed of adjustment and is estimated as the weighted average:

$$\sum_{i=0}^{S} i \cdot \hat{\beta}_i \Big/ \sum_{i=0}^{S} \hat{\beta}_i$$

The mean lag may only have a useful interpretation if the lag coefficients are all positive.

The SHAZAM output also reports a JOINT-F statistic for a test of the null hypothesis that all coefficients associated with the distributed lag variable are simultaneously equal to zero.

In the above example the estimate of the long-run impact is 0.93923 and the t-statistic of 80.028 indicates that it is statistically significant. The estimate of the short-run impact is 0.038379 and this does not appear significantly different from zero. However, Griffiths et al. [1993, p. 683] discuss that multicollinearity may be a problem with unrestricted estimation of distributed lag models.

The special notation for lagged variables can be used to specify equations that include lagged dependent variables as regressors. For example, the next SHAZAM command implements OLS estimation of a model that includes a lagged dependent variable. The explanatory variables also include the current value of *INCOME* and the current and 2 lagged values of the variable *PRICE*.

```
ols consume consume(1.1) income price(0.2)
```

Almon Polynomial Distributed Lag Models

The Almon [1965] method imposes restrictions on the coefficients of the distributed lag model. The coefficients are restricted to lie on a polynomial of degree r. The form of the regression equation is:

$$Y_t = \gamma + \sum_{i=a}^{b} \beta_i X_{t-i} + \varepsilon_t \qquad \text{where} \qquad \beta_i = \sum_{j=0}^{r} \alpha_j (i)^j$$

SHAZAM allows the user to specify a lag length, order of polynomial and endpoint constraints on any independent variable in the model. Each independent variable may have different order and endpoint constraints. An Almon polynomial distributed lag for any explanatory variable on the estimation command (**OLS**, **AUTO**, **BOX**, **GLS**, or **POOL**) can be specified using the special form:

indep(first.last,order,endcon)

where *indep* is the name of an independent variable. Each independent variable may have up to 3 parameters in parentheses which specify the form of the polynomial lag. The first parameter contains two numbers separated by a dot (.). These numbers specify the *first* and *last* periods to use for lags. The *order* parameter specifies the order of the Almon lag scheme. The *endcon* parameter specifies the endpoint restrictions as follows:

0 = No Endpoint restrictions;
1 = Endpoint restrictions on the left side of the polynomial;
2 = Endpoint restrictions on the right side of the polynomial;
3 = Endpoint restrictions on both left and right sides.

If *order* and *endcon* are not specified an unrestricted lag is used.

The Almon method imposes restrictions on the coefficients. In this case the **DWPVALUE**, **METHOD=HH**, and *STEPWISE REGRESSION* may not be used (see the chapter *ORDINARY LEAST SQUARES*). It should also be noted that some statistics on the **DIAGNOS** command like the Chow, Goldfeld-Quandt and recursive residuals tests will not incorporate the restrictions (see the chapter *DIAGNOSTIC TESTS*).

Note that other computer packages may use a different method to specify the degree of the polynomial. In particular the order may be equal to the SHAZAM definition plus 1.

The number of degrees of freedom is increased by the number of restrictions. Suppose J explanatory variables are specified and variable j has first and last lag period a_j, b_j with polynomial order r_j and number of endpoint restrictions m_j. When a constant term is included the number of variables in the regression is:

$$K = 1 + \sum_{j=1}^{J} (b_j - a_j + 1)$$

The number of restrictions is: $q = (K - 1) - \sum_{j=1}^{J} (r_j - m_j + 1)$

The number of degrees of freedom is then N–K+q.

An estimation method is to construct variables as linear combinations of the X variables and apply unrestricted estimation using the constructed variables as regressors. The parameter estimates $\hat{\beta}_i$ are then recovered from linear combinations of the estimated coefficients. This method is described in Gujarati [2003] and other econometrics textbooks. The SHAZAM estimation method is to cast the problem in the framework of restricted estimation. That is, restrictions are imposed as with the **RESTRICT** option on the estimation command.

The restrictions on the parameter vector β due to the specification of the degree of the polynomial, and/or due to the specification of end-point constraints on the polynomial, may be found using Pascal's triangle (for discussion see Johnston [1984] or Greene [1993, p. 523]). The restrictions have the form $R\beta = 0$ where the elements in the matrix R are obtained from Pascal's triangle given by:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & -1 & & \\ & 1 & & -2 & & 1 & \\ 1 & & -3 & & 3 & & -1 \\ 1 & -4 & & 6 & & -4 & 1 \end{array} \qquad \begin{array}{l} \\ \\ \\ \text{Polynomial of degree 2} \\ \text{Polynomial of degree 3} \end{array}$$

This can be illustrated with the manufacturing data set used previously. Assuming a lag length of 8 periods and a polynomial of degree 2 the parameter restrictions are:

$$\beta_0 - 3\beta_1 + 3\beta_2 - \beta_3 = 0$$
$$\beta_1 - 3\beta_2 + 3\beta_3 - \beta_4 = 0$$
$$\beta_2 - 3\beta_3 + 3\beta_4 - \beta_5 = 0$$
$$\beta_3 - 3\beta_4 + 3\beta_5 - \beta_6 = 0$$
$$\beta_4 - 3\beta_5 + 3\beta_6 - \beta_7 = 0$$
$$\beta_5 - 3\beta_6 + 3\beta_7 - \beta_8 = 0$$

The next output shows the estimation results that correspond to Table 21.5 of Griffiths, Hill and Judge [1993, p. 687].

```
|_OLS Y X(0.8,2)
 LAG FOR X       RANGE =  0  8 ORDER= 2 ENDCON=0
 OLS ESTIMATION
      80 OBSERVATIONS     DEPENDENT VARIABLE = Y
...NOTE..SAMPLE RANGE SET TO:    9,   88

F TEST ON RESTRICTIONS=   1.1500     WITH    6 AND   70 DF  P-VALUE=  .34309

 R-SQUARE =    .9928     R-SQUARE ADJUSTED =    .9925
VARIANCE OF THE ESTIMATE-SIGMA**2 =   35631.
STANDARD ERROR OF THE ESTIMATE-SIGMA =   188.76
SUM OF SQUARED ERRORS-SSE=   .27079E+07
MEAN OF DEPENDENT VARIABLE =   4532.5
LOG OF THE LIKELIHOOD FUNCTION = -530.702

                  TESTS ON LAGGED COEFFICIENTS
VARIABLE  SUM(COEFS)  STD ERROR  T-RATIO  P-VALUE | MEAN LAG| JOINT-F  P-VALUE
X         0.93296      0.115E-01   81.3     0.000 |   3.817 |  0.348E+04 0.000

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      76 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
X          .67168E-01  .1523E-01   4.411      .000  .451      .0941      .0794
X          .10022      .5114E-02   19.60      .000  .914      .1359      .1157
X          .12302      .5410E-02   22.74      .000  .934      .1522      .1370
X          .13556      .9413E-02   14.40      .000  .855      .1539      .1461
X          .13785      .1072E-01   12.86      .000  .828      .1472      .1444
X          .12988      .9079E-02   14.31      .000  .854      .1304      .1324
X          .11165      .5337E-02   20.92      .000  .923      .1051      .1109
X          .83175E-01  .7346E-02   11.32      .000  .792      .0739      .0806
X          .44442E-01  .1797E-01   2.473      .016  .273      .0380      .0421
CONSTANT   51.573      53.16       .9701      .335  .111      .0000      .0114
```

A test of the validity of the restrictions is given by testing the null hypothesis $H_0 : R\beta = 0$. The F-statistic is calculated as:

$$F = \frac{1}{q}\left(R\hat{\beta}\right)' \left[R\left(\hat{\sigma}^2 (X'X)^{-1}\right)R'\right]^{-1}\left(R\hat{\beta}\right)$$

where q is the number of restrictions. Under the null hypothesis the F-statistic is distributed as $F_{(q,N-K)}$. N is the number of observations used in the estimation and K is the total number of coefficients estimated.

On the above output the F test on the restrictions has the value 1.1500. The p-value is reported as 0.34309 and so the null hypothesis of valid restrictions is not rejected. For the t-ratios the degrees of freedom is adjusted by the number of restrictions so that the degrees of freedom is N–K+q = 80–10+6 = 76.

Endpoint restrictions as described in Pindyck and Rubinfeld [1998, p. 240] may be specified for any polynomial. The use of endpoint restrictions increases the number of restrictions imposed in the model. For example, for the manufacturing data with a lag length of 8 periods and a second degree polynomial, an endpoint restriction on the left side can be incorporated with the command:

```
OLS Y X(0.8,2,1)
```

The additional restriction imposed is:

$$-3\beta_0 + 3\beta_1 - \beta_2 = 0$$

An endpoint restriction on the right side gives the added restriction:

$$\beta_6 - 3\beta_7 + 3\beta_8 = 0$$

Granger Causality

A test for noncausality is obtained by testing for zero restrictions on the coefficients of a bivariate VAR(p) process. With the distributed lag notation, the `JOINT-F` statistic reported on the SHAZAM output gives a test statistic. The example below uses quarterly data on consumption (*Y1*) and income (*Y2*) from Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Table 18.1]. A VAR(4) model is assumed to follow the example in Judge et al. [1988, p. 770].

```
|_SAMPLE 1 75
|_READ  Y1 Y2
   2 VARIABLES AND       75 OBSERVATIONS STARTING AT OBS       1

|_* Consider a VAR(4) model
|_SAMPLE 1 71
|_* Test H0: Y2 does not Granger-cause Y1
|_* The test statistic is the JOINT-F on Y2
|_OLS Y1 Y1(1.4) Y2(1.4)
 LAG FOR Y1      RANGE =  1  4 ORDER= 0 ENDCON=0
 LAG FOR Y2      RANGE =  1  4 ORDER= 0 ENDCON=0

 OLS ESTIMATION
      67 OBSERVATIONS     DEPENDENT VARIABLE = Y1
...NOTE..SAMPLE RANGE SET TO:      5,     71

 R-SQUARE =   0.3418     R-SQUARE ADJUSTED =   0.2510
VARIANCE OF THE ESTIMATE-SIGMA**2 =   283.86
STANDARD ERROR OF THE ESTIMATE-SIGMA =   16.848
SUM OF SQUARED ERRORS-SSE=   16464.
MEAN OF DEPENDENT VARIABLE =   14.836
LOG OF THE LIKELIHOOD FUNCTION = -279.460

                   TESTS ON LAGGED COEFFICIENTS
VARIABLE  SUM(COEFS)  STD ERROR  T-RATIO  P-VALUE | MEAN LAG| JOINT-F  P-VALUE
Y1        0.11389E-01  0.340      0.335E-01 0.973 | -14.976 |   1.07     0.379
Y2        0.38335      0.284       1.35     0.182 |   0.635 |   4.78     0.002

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      58 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
Y1       -0.68107E-01 0.1511     -0.4507     0.654-0.059    -0.0685    -0.0673
Y1        0.15389     0.1654      0.9305     0.356 0.121     0.1507     0.1435
Y1        0.11265     0.1669      0.6750     0.502 0.088     0.1097     0.1026
Y1       -0.18705     0.1244      -1.503     0.138-0.194    -0.1959    -0.1487
Y2        0.50283     0.1248       4.028     0.000 0.468     0.5678     0.5362
Y2       -0.29019E-01 0.1489     -0.1948     0.846-0.026    -0.0329    -0.0308
Y2       -0.16032     0.1492      -1.075     0.287-0.140    -0.1802    -0.1632
Y2        0.69857E-01 0.1279      0.5461     0.587 0.072     0.0789     0.0715
CONSTANT   8.2510      3.137       2.630     0.011 0.326     0.0000     0.5562

|_* Test H0: Y1 does not Granger-cause Y2
|_* The test statistic is the JOINT-F on Y1
|_OLS Y2 Y1(1.4) Y2(1.4)
 LAG FOR Y1      RANGE =  1  4 ORDER= 0 ENDCON=0
 LAG FOR Y2      RANGE =  1  4 ORDER= 0 ENDCON=0

 OLS ESTIMATION
      67 OBSERVATIONS     DEPENDENT VARIABLE = Y2
...NOTE..SAMPLE RANGE SET TO:      5,     71

 R-SQUARE =   0.2037     R-SQUARE ADJUSTED =   0.0939
VARIANCE OF THE ESTIMATE-SIGMA**2 =   427.75
STANDARD ERROR OF THE ESTIMATE-SIGMA =   20.682
SUM OF SQUARED ERRORS-SSE=   24810.
MEAN OF DEPENDENT VARIABLE =   16.134
LOG OF THE LIKELIHOOD FUNCTION = -293.198

                   TESTS ON LAGGED COEFFICIENTS
VARIABLE  SUM(COEFS)  STD ERROR  T-RATIO  P-VALUE | MEAN LAG| JOINT-F  P-VALUE
Y1        0.32388      0.417      0.776     0.441 |   2.025 |  0.640     0.636
Y2        0.16905      0.348      0.485     0.629 |  -0.552 |   1.23     0.306
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 58 DF	PARTIAL P-VALUE	STANDARDIZED CORR. COEFFICIENT	ELASTICITY AT MEANS
Y1	0.23290	0.1855	1.256	0.214	0.163	0.2099	0.2116
Y1	-0.47743E-01	0.2030	-0.2352	0.815	-0.031	-0.0419	-0.0409
Y1	0.36336E-01	0.2049	0.1774	0.860	0.023	0.0317	0.0304
Y1	0.10239	0.1528	0.6703	0.505	0.088	0.0961	0.0748
Y2	0.32560	0.1532	2.125	0.038	0.269	0.3294	0.3193
Y2	-0.11014	0.1828	-0.6024	0.549	-0.079	-0.1118	-0.1076
Y2	0.13080E-01	0.1831	0.7142E-01	0.943	0.009	0.0132	0.0122
Y2	-0.59482E-01	0.1570	-0.3788	0.706	-0.050	-0.0602	-0.0560
CONSTANT	8.9742	3.851	2.330	0.023	0.293	0.0000	0.5562

The first OLS estimation shows an F-test statistic of 4.78. The p-value of 0.002 suggests that the null hypothesis that there is no causality from income (*Y2*) to consumption (*Y1*) can be rejected. The second OLS estimation reports an F-test statistic of 0.640 to indicate that the null hypothesis that there is no causality from consumption to income is not rejected.

Polynomial Inverse Lag Models

The polynomial inverse lag model proposed by Mitchell and Speaker [1986] assumes an infinite distributed lag so that no lag length need be specified. The regression equation is:

$$Y_t = \gamma + \sum_{i=0}^{\infty} \beta_i X_{t-i} + \varepsilon_t \qquad \text{where} \qquad \beta_i = \sum_{j=2}^{r} \alpha_j \left(\frac{1}{i+1} \right)^j$$

The degree of polynomial r must be specified (see the discussion in Mitchell and Speaker [1986, p. 331]). With the **OLS** command, a polynomial inverse lag for any explanatory variable can be specified using the general command format:

OLS *depvar indep(first.last,order)* / **PIL** *options*

where *first* and *last* specifies the range of lag coefficients to list on the SHAZAM output and *order* gives the degree of polynomial r. No endpoint restrictions are allowed.

16. FORECASTING

"The 1976 Olympics could no more lose money than I could have a baby."
Mr. Jean Drapeau
Mayor of Montreal, 1973

Forecasting can be implemented in SHAZAM with the **FC** command. Forecasts can be generated for models estimated with the **AUTO**, **BOX**, **LOGIT**, **OLS**, **POOL**, **PROBIT** and **TOBIT** commands. The **FC** command can use the estimated coefficients from the previous regression and then generate predicted values over any chosen set of observations. It is also possible to specify the coefficients with the **COEF=** option. The calculation of the forecast standard errors does not make any adjustments if lagged dependent variables are present. References for forecasting are Salkever [1976], Pagan and Nichols [1984] and Harvey [1990].

The standard linear regression equation is written:

$$Y_t = X_t'\beta + \varepsilon_t \qquad t = 1, 2, \ldots, N$$

where Y_t is the dependent variable, X_t is a vector of explanatory variables, β is a Kx1 parameter vector and ε_t is a random error with zero mean and variance σ^2. Suppose that estimation with the **OLS** command yields the estimates $\hat{\beta}$ and $\hat{\sigma}^2$. For an origin date N_0, predictions of Y_{N_0+t} are obtained as:

$$\hat{Y}_{N_0+t} = X'_{N_0+t}\hat{\beta} \qquad t = 1, 2, \ldots, M$$

Forecast Standard Errors

There are two approaches to calculating forecast standard errors. One approach is to view the problem as predicting an individual value for Y for a given value of X. The forecast error variance is estimated as:

$$V(Y_{N_0+t} - \hat{Y}_{N_0+t}) = \hat{\sigma}^2\left(1 + X'_{N_0+t}(X'X)^{-1}X_{N_0+t}\right)$$

where X is the NxK matrix of regressors. The square root of the above gives the forecast standard error.

An alternative approach, implemented with the **MEANPRED** option on the **FC** command, is to consider predicting the conditional mean of Y for a given value of X. In this case, the forecast error variance is estimated as:

$$V(\hat{Y}_{N_0+t}) = \hat{\sigma}^2 X'_{N_0+t}(X'X)^{-1} X_{N_0+t}$$

Forecast Evaluation

The mean squared error is computed as: $$MSE = \frac{1}{M}\sum_{t=1}^{M}(Y_{N_0+t} - \hat{Y}_{N_0+t})^2$$

and the root mean squared error is $\sqrt{MSE}$. The mean absolute error is computed as:

$$MAE = \frac{1}{M}\sum_{t=1}^{M}\left|Y_{N_0+t} - \hat{Y}_{N_0+t}\right|$$

The mean squared percentage error is computed as:

$$MSPE = \frac{1}{M}\sum_{t=1}^{M}\left[100\,\frac{Y_{N_0+t} - \hat{Y}_{N_0+t}}{Y_{N_0+t}}\right]^2$$

(when $Y_{N_0+t} = 0$ the observation is excluded from the formula).

Consider $\overline{Y}$, $\overline{Y}^p$, s_a , and s_p as the means and standard deviations of the series Y_{N_0+t} and $\hat{Y}_{N_0+t}$ respectively for t = 1, . . . , M. The calculation of the standard deviations uses a divisor of M. Denote r as the correlation between the observed and predicted values. SHAZAM reports two decompositions of the mean square error such that:

$$U^B + U^V + U^C = U^B + U^R + U^D = 1 \qquad \text{where}$$

$U^B = \left(\overline{Y} - \overline{Y}^p\right)^2 / MSE$ is the proportion due to Bias

$U^V = (s_a - s_p)^2 / MSE$ is the proportion due to Variance

$U^C = 2(1 - r)s_p s_a / MSE$ is the proportion due to Covariance

$U^R = (s_p - rs_a)^2 / MSE$ is the proportion due to the Regression

$$U^D = \left(1 - r^2\right) s_a^2 / MSE \qquad \text{is the disturbance proportion}$$

Theil [1966, Chapter 2] and Maddala [1977, Section 15-6] give discussion on the interpretation of the results of the decomposition.

The Theil inequality coefficient (described in Theil [1966, p. 28]) is calculated as:

$$U = \left[\frac{\sum_{t=2}^{M} (Y_{N_0+t} - \hat{Y}_{N_0+t})^2}{\sum_{t=2}^{M} (Y_{N_0+t} - Y_{N_0+t-1})^2} \right]^{1/2}$$

For a perfect forecast $\hat{Y}_{N_0+t} = Y_{N_0+t}$ for all t and U = 0. A value of U = 1 results from a naive model where $\hat{Y}_{N_0+t} = Y_{N_0+t-1}$. A value of U > 1 results from a model that forecasts less precisely compared to a naive model.

Prediction for the Model with AR(1) Errors

The AR(1) error model has disturbances of the form: $\varepsilon_t = \rho \varepsilon_{t-1} + v_t$

where ρ is the autoregressive parameter, v_t is a random error with zero mean and variance σ_v^2 and $E(\varepsilon\varepsilon') = \sigma_v^2 \Omega$. Suppose that estimation with the **AUTO** command yields the estimates $\hat{\beta}$, $\hat{\rho}$, $\hat{\sigma}_v^2$ and $\hat{\Omega}$. For further details see the chapter *AUTOCORRELATION MODELS*. The BLUP (best linear unbiased predictor) is given by:

$$\hat{Y}_{N+t} = X'_{N+t}\hat{\beta} + \hat{\rho}^t \left(Y_N - X'_N \hat{\beta}\right) \qquad t = 1, 2, \ldots, M$$

where Y_N and X_N are the final observations of the estimation sample period. The IBLUP (predictor) uses information from the immediately preceding observation to obtain a series of one-step ahead predictions as:

$$\hat{Y}_{N+t} = X'_{N+t}\hat{\beta} + \hat{\rho}\left(Y_{N+t-1} - X'_{N+t-1}\hat{\beta}\right) \qquad t = 1, 2, \ldots, M$$

The mean-square-error of prediction (see Judge et al. [1985, Equation 8.3.13 p. 318] and Harvey [1990, Equation 7.10, p. 215]) for t = 1, 2, . . . , M is estimated as:

$$MSE\left(\hat{Y}_{N+t}\right) = \hat{\sigma}_v^2 \left(\frac{1 - \hat{\rho}^{2t}}{1 - \hat{\rho}^2}\right) + \hat{\sigma}_v^2 \left(X_{N+t} - \hat{\rho}^t X_N\right)' \left(X'\hat{\Omega}^{-1}X\right)^{-1} \left(X_{N+t} - \hat{\rho}^t X_N\right)$$

The asymptotic MSE of the predictor based on Baillie [1979] (see Judge et al. [1985, Equation 8.3.14, p. 318] and Harvey [1990, Equation 7.11, p. 215]) can be estimated as:

$$\text{AMSE}\left(\hat{Y}_{N+t}\right) \approx \text{MSE}\left(\hat{Y}_{N+t}\right) + \hat{\sigma}_v^2 t^2 \hat{\rho}^{2(t-1)} / N$$

where the last term is the MSE contribution due to estimation of ρ. If the **AFCSE** option is specified with the **BLUP** or **IBLUP** option on the **FC** command then the asymptotic forecast standard error is reported as the square root of the above.

FC *COMMAND OPTIONS*

In general, the format of the **FC** command, when the estimated coefficients from the immediately preceding regression are being used, is:

estimation command
FC / *options*

where *estimation command* can be **AUTO**, **BOX**, **LOGIT**, **OLS**, **POOL**, **PROBIT** or **TOBIT**. Note that the reported forecast standard errors are not valid when the **HETCOV** or **AUTCOV=** options are specified on the **OLS** command. The *estimation command* can also be **GLS**, **MLE**, or **2SLS**. For the latter cases the forecast standard errors are not available. The format when reading in all the coefficients is:

FC *depvar indeps* / *options* **COEF=**

where *depvar* is the variable name of the dependent variable, *indeps* is a list of variable names of the independent variables, **COEF=** is a required option used to specify the name of the variable in which the coefficients are stored, and *options* is a list of desired options.

When the coefficients from the previous regression are being used the variables must not be specified since SHAZAM has automatically saved them. However, when a new set of coefficients is being specified SHAZAM must be told which variables to use and the variable in which the coefficients have been saved must be specified on the **COEF=** option. The **FC** command is similar to the **OLS** command except no estimation is done. If the **PREDICT=** or **RESID=** options are used, the variable used for the results must have been previously defined with the **DIM** command. For a description of the **DIM** command see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.

Options as described for **OLS** that are available are:

GF, **LIST**, **BEG=**, **END=**, **PREDICT=** and **RESID=**

Additional options are:

AFCSE Used with the **BLUP** or **IBLUP** options to get **A**symptotic **F**orecast **S**tandard **E**rrors as described above.

BLUP, **IBLUP** Used in autoregressive models when the user wishes the predicted values to be adjusted with the lagged residual, to give the **B**est **L**inear **U**nbiased **P**redictions. **BLUP** only uses information from the first observation specified while **IBLUP** uses information from the **I**mmediately preceding observation. Note, that correct forecast standard errors are only available for the first-order autoregressive model as described above.

DYNAMIC Performs **DYNAMIC** forecasts for models with a lagged dependent variable. It is assumed that the lag variable is the first independent variable listed.

FIXED Performs forecasts for **FIXED** effects **POOL** models. Also, see the **CSNUM=** and **NCROSS=** options.

MAX This option is equivalent to the **LIST** and **GF** options.

MEANPRED Calculate the forecast standard errors for the case of mean prediction. The default method is to calculate forecast standard errors for the case of individual prediction.

PERCENT This option adds one line to every observation listing to display the **PERCENT**age change in the actual and predicted values (measured as the ratio of the current year to the previous year). In addition, it reports the ratio of the residual to the actual value. These ratios are printed in parentheses below the usual listing of actual, predicted, and residual values.

COEF= Gives an input vector of model coefficients. If this is not specified then the estimated coefficients from the previous SHAZAM command are used.

CSNUM= Specifies which cross-section to use on a Pooled Cross-Section Time-Series model for **POOL** models.

ESTEND= Specifies the last observation of the estimation for **AUTO** and **POOL** models.

FCSE= Saves the **F**ore**C**ast **S**tandard **E**rrors in the variable specified. The option is only available when the coefficients from the previous regression are used for forecasting. The variable to be used for the forecast standard errors must be defined before the estimation with the **DIM** command. Note that if the **HETCOV** or **AUTCOV=** options were used on the previous **OLS** command, the forecast standard errors are incorrect.

NCROSS= Specifies the **N**umber of **CROSS**-sections in a Pooled Cross-Section Time-Series model for **POOL** models.

When the **COEF=** option is specified the following options may also be included:

NOCONSTANT No intercept is included in the regression. In this case, a value for the intercept should not be included.

UPPER Used with **MODEL=TOBIT** (see the chapter *TOBIT REGRESSION*).

LIMIT= Used with **MODEL=TOBIT** (see the chapter *TOBIT REGRESSION*).

MODEL= Specifies the type of **MODEL**. The available models are:

AUTO When **MODEL=AUTO** is used the autoregressive order is specified with the **ORDER=** option and the default is AR(1). The **COEF=** vector must contain K values for β (the intercept coefficient is last) followed by the values for the autoregressive parameters.

BOX When **MODEL=BOX** is used the **COEF=** vector must contain K values for β followed by K values for λ of each of the independent variables followed by the λ value on the dependent variable.

LOGIT Use Coefficients from **LOGIT** command estimation.

OLS The default is **MODEL=OLS.**

POOL When **MODEL=POOL** is used the forecast must be done for only one Cross-Section. The relative position of the Cross-Section is specified with the **CSNUM=** and **NC=** options. The **COEF=** vector must contain K values for β, then a ρ value for each cross-section, and finally a standard error estimate for each cross-section. (See the chapter *POOLED CROSS-SECTION TIME-SERIES*).

PROBIT Use coefficients from **PROBIT** command estimation.

TOBIT When **MODEL=TOBIT** is used the *normalized* coefficients (including the one on the dependent variable) must be read in.

ORDER= Specifies the **ORDER** of the model for **MODEL=AUTO**. This option is only available for **ORDER=1** or **ORDER=2**.

POOLSE= Specifies the standard error of the cross-section in a Pooled Cross-Section Time-Series model when **MODEL=POOL**.

RHO= Specifies a value of the first-order autoregressive parameter for the regression when **MODEL=AUTO** or **MODEL=BOX** or **MODEL=POOL** is specified.

SRHO= Used with **MODEL=AUTO** and **ORDER=2** and the **RHO=** option to specify a value of the **S**econd-order **RHO**.

EXAMPLES

An example of the use of the **FC** command after an **OLS** estimation is:

```
sample 1 17
ols consume income price / list
fc / list
```

Another example is:

```
sample 1 10
ols consume income price / coef=beta
test income = -price
fc consume income price / beg=1 end=17 coef=beta
```

In the above example, the **OLS** command saves the coefficient vector with the **COEF=** option. This is necessary since the **FC** command does not immediately follow the estimation command. On the **FC** command the dependent variable and the regressors are supplied along with the vector containing the estimated coefficients.

Another example of the use of the **FC** command is:

```
sample 1 10
ols consume income price
fc / list beg=11 end=17
```

The output from the final example is:

```
|_SAMPLE 1 10
|_OLS CONSUME INCOME PRICE
 OLS ESTIMATION
      10 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   10

 R-SQUARE =     .9810     R-SQUARE ADJUSTED =     .9755
VARIANCE OF THE ESTIMATE-SIGMA**2 =   10.568
STANDARD ERROR OF THE ESTIMATE-SIGMA =   3.2509
SUM OF SQUARED ERRORS-SSE=   73.979
MEAN OF DEPENDENT VARIABLE =   121.45
LOG OF THE LIKELIHOOD FUNCTION = -24.1954

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR       7 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME      .55817      .2456       2.273      .057  .652      .1462      .4834
PRICE     -1.5320       .1098      -13.95      .000 -.982     -.8976    -1.1060
CONSTANT   197.07       32.44       6.075      .001  .917      .0000     1.6226

|_FC / LIST BEG=11 END=17
DEPENDENT VARIABLE = CONSUME          7 OBSERVATIONS
REGRESSION COEFFICIENTS
   .558165798286      -1.53203247506       197.070728749

    OBS.   OBSERVED     PREDICTED   CALCULATED  STD. ERROR
     NO.    VALUE        VALUE       RESIDUAL
     11   158.50       159.92      -1.4226        4.863              *I
     12   140.60       154.57      -13.968        5.729        *      I
     13   136.20       153.44      -17.241        5.472       *       I
     14   168.00       170.96      -2.9628        6.191              *I
     15   154.30       162.76      -8.4646        4.898           *   I
     16   149.00       162.62      -13.624        5.016         *     I
     17   165.50       161.09       4.4053        4.613               I *

SUM OF ABSOLUTE ERRORS=   62.088
R-SQUARE BETWEEN OBSERVED AND PREDICTED =  .6833
RUNS TEST:    2 RUNS,    1 POS,    0 ZERO,    6 NEG  NORMAL STATISTIC = -1.5811
MEAN ERROR =  -7.6111
SUM-SQUARED ERRORS =   779.82
MEAN SQUARE ERROR =   111.40
MEAN ABSOLUTE ERROR=   8.8697
ROOT MEAN SQUARE ERROR =   10.555
```

```
MEAN SQUARED PERCENTAGE ERROR=   54.804
THEIL INEQUALITY COEFFICIENT U =  .650
  DECOMPOSITION
     PROPORTION DUE TO BIAS =   .51999
     PROPORTION DUE TO VARIANCE =   .29305
     PROPORTION DUE TO COVARIANCE =   .18696
  DECOMPOSITION
     PROPORTION DUE TO BIAS =   .51999
     PROPORTION DUE TO REGRESSION =   .12868
     PROPORTION DUE TO DISTURBANCE =   .35133
```

The next example shows forecasting with a model with AR(1) errors. The **AUTO** command is used for estimation and the AR(1) parameter is estimated using the **ML** option. The first 13 observations are used for estimation. Forecasts are then made for observation 13 to 17 using the **BLUP** option on the **FC** command. In order to get correct estimates of the forecast standard error for the 14th through the 17th predictions, the forecast must begin at the 13th observation and end at the 17th using the **BEG=** and **END=** options in the **FC** command. This is necessary since each forecast must be adjusted by the lagged residual to get correct forecast standard errors. Note carefully the information given in the output:

```
. . . FORECAST STD. ERRORS USE JUDGE (1985, EQ. 8.3.13)
IGNORE FORECASTS AND STD. ERRORS BEFORE OBSERVATION 14.
```

The forecasts and standard errors have been saved using the **PREDICT=** and **FCSE=** options on the **FC** command. Note that the **DIM** command is needed to allocate space for these variables. These variables could then be used as input for graphical display of the forecast. The **LIST** option on the **FC** command is used to obtain a listing of the predictions and prediction errors. The SHAZAM commands for the estimation and forecasting are:

```
sample 1 13
dim fc 17 se 17
auto consume income price / ml
fc / blup list beg=13 end=17 model=auto predict=fc fcse=se
```

The SHAZAM output is:

```
|_SAMPLE 1 13
|_DIM FC 17 SE 17
|_AUTO CONSUME INCOME PRICE / ML
DEPENDENT VARIABLE =  CONSUME
..NOTE..R-SQUARE,ANOVA,RESIDUALS DONE ON ORIGINAL VARS
DN OPTION IN EFFECT - DIVISOR IS N
MAXIMUM LIKELIHOOD ESTIMATION             13 OBSERVATIONS
BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE =  .00100
    ITERATION          RHO               LOG L.F.            SSE
        1             .00000         -37.1885             232.38
        2             .34495         -36.5653             209.09
```

```
        3              .34847        -36.5653              209.04
        4              .34853        -36.5653              209.04

 LOG L.F. =   -36.5653       AT RHO =      .34853

                    ASYMPTOTIC  ASYMPTOTIC  ASYMPTOTIC
          ESTIMATE    VARIANCE    ST.ERROR     T-RATIO
RHO         .34853      .06758      .25996     1.34070

 R-SQUARE =    .9617     R-SQUARE ADJUSTED =    .9541
VARIANCE OF THE ESTIMATE-SIGMA**2 =   16.080
STANDARD ERROR OF THE ESTIMATE-SIGMA =   4.0100
SUM OF SQUARED ERRORS-SSE=   209.04
MEAN OF DEPENDENT VARIABLE =   126.91
LOG OF THE LIKELIHOOD FUNCTION = -36.5653

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.0867      .2580       4.212      .000  .800      .2994      .8861
PRICE     -1.3087      .1044      -12.54      .000 -.970     -.9380     -.8442
CONSTANT   121.50      27.43       4.429      .000  .814      .0000      .9574

|_FC / BLUP LIST BEG=13 END=17 MODEL=AUTO PREDICT=FC FCSE=SE
..ASSUMING ESTIMATION ENDED AT OBSERVATION    13
DEPENDENT VARIABLE = CONSUME          5 OBSERVATIONS
REGRESSION COEFFICIENTS
   1.08666458513      -1.30867753239       121.501461408
AUTOCORRELATION RHO
   .3485269810734
USER SPECIFIED RHO=  .34853
USER SPECIFIED SRHO=  .00000

..FORECAST STD. ERRORS USE JUDGE(1985, EQ. 8.3.13)
IGNORE FORECASTS AND STD. ERRORS BEFORE OBS.   14

    OBS.   OBSERVED     PREDICTED   CALCULATED  STD. ERROR
     NO.    VALUE        VALUE       RESIDUAL
     13   136.20       143.02      -6.8240        4.879           *   I
     14   168.00       156.35       11.655        4.830               I      *
     15   154.30       153.82       .48106        4.947               *
     16   149.00       153.75      -4.7514        5.082            *  I
     17   165.50       153.97       11.525        5.053               I      *

BLUP IS ACTIVE - PREDICTED VALUES ADJUSTED WITH LAGGED RESIDUALS

SUM OF ABSOLUTE ERRORS=   35.237
R-SQUARE BETWEEN OBSERVED AND PREDICTED =  .7513
RUNS TEST:    4 RUNS,    3 POS,    0 ZERO,    2 NEG  NORMAL STATISTIC =   .6547
MEAN ERROR =   2.4172
SUM-SQUARED ERRORS =   338.04
MEAN SQUARE ERROR =   67.609
MEAN ABSOLUTE ERROR=   7.0473
ROOT MEAN SQUARE ERROR =   8.2225
MEAN SQUARED PERCENTAGE ERROR=   26.399
THEIL INEQUALITY COEFFICIENT U =  .441
  DECOMPOSITION
     PROPORTION DUE TO BIAS =   .86420E-01
     PROPORTION DUE TO VARIANCE =   .70030
     PROPORTION DUE TO COVARIANCE =   .21328
  DECOMPOSITION
```

```
     PROPORTION DUE TO BIAS =    .86420E-01
     PROPORTION DUE TO REGRESSION =    .42188
     PROPORTION DUE TO DISTURBANCE =    .49170
```

17. FUZZY SET MODELS

"I know millions of things that won't work. I've certainly learned a lot."
Thomas A. Edison

The **FUZZY** command facilitates the construction of fuzzy-set econometric models. Although the use of fuzzy sets dates from Zadeh [1965], the particular methodology described here is very recent. It is discussed more fully and applied to economic data by Lindström [1998] in the case of aggregate investment behaviour, and by Draeseke and Giles [1999, 2000] in the case of the underground economy. Fuzzy set econometric modelling provides a means of using the information associated with economic concepts and linguistic economic variables to predict the value of another economic variable. A fuzzy set maps from a regular set to [0,1]. Membership of a fuzzy set is not crisp, but involves a numerical *degree of membership* that indicates the level of subjective association that is placed on the given concept. Degrees of membership need not sum to unity. In fuzzy set theory the union operator is replaced by the *max* operator; intersection is replaced by *min*; and complement is replaced by subtraction from unity.

Suppose that we wish to use two input (or causal) variables to explain (predict) the value of an output (dependent) variable of interest. The dependent variable is usually unobservable, and is not needed for the fuzzy modelling. We first define fuzzy sets associated with the values of the two causal variables, and then we assign association values with the subjective levels for each variable in each period. Finally, decision rules are used to establish a level in [0,1] for the indicator (or index). The decision rules incorporate the use of fuzzy operators. The prediction for the dependent variable is in the form of an index that can then be scaled according to some suitable metric.

More specifically, the steps involved are:

(i) For each input variable, assign 5 break-points, based on a moving-average of the sample values. Once *normal* values have been established for each variable in each period, calculate the other break-points by taking one and two sample standard deviations around the *normal* value. The regions between break-points are termed *membership levels*.

(ii) Associate each value for each input variable with one or two membership levels, using a triangular *membership function*.

(iii) Create decision rules that will determine how particular levels of association for each of the input variables are combined to establish the (5) levels of association for the dependent variable. The 25 decision rules are represented in a (5x5) matrix. A row of this matrix corresponds to a membership level for the first causal variable. A column corresponds to a membership level for the second causal variable.

(iv) Provide *degrees of association* to be used with these decision rules. The 25 degrees of association are represented in a (5x5) matrix. A row of this matrix corresponds to a membership level for the first causal variable. A column corresponds to a membership level for the second causal variable.

(v) Weights are assigned to each of the 5 levels of association for the dependent variable.

(vi) The decision rules, the degrees of association, the weights, and the fuzzy operators *max* and *min* are used with each combination of levels for the two causal variables to generate an index value for the dependent variable.

The **FUZZY** command is currently limited to handle only two input (causal) variables.

FUZZY *COMMAND OPTIONS*

In general, the format of the **FUZZY** command is:

FUZZY *x1 x2* / **RULES**=*rulemat* **DEGREES**=*degreemat options*

where *x1* and *x2* are input variables and *options* is a list of desired options. *rulemat* and *degreemat* are 5 x 5 matrices of decision rules and degrees, respectively.

Options that start with **GRAPH** prepare graphs with the GNUPLOT software as described in the *PLOTS AND GRAPHS* chapter.

Options as defined for the **OLS** command that are available are:

DUMP, **BEG=** and **END=**

Additional options available on the **FUZZY** command are:

GRAPHDATA Provides a graphics plot of the data.

GRAPHRULE Provides a graphics plot of the rule surface.

MEDIAN Specifies to center the data around the median instead of the mean.

NOLIST Suppresses the List of computed index values and rules.

NOPMATRIX Suppresses printing of the rules and degrees matrices.

NOSTANDARD Assumes that the input data is standardized.

PASSOC Prints the table of Association values.

PBREAK Prints the table of Breakpoints.

CMA= Specifies the number of periods to start the Cumulative Moving Average. If this is used CMA–1 observations will be lost.

PREDICT= Saves the calculated index in the variable specified.

RMA= Specifies the number of periods for the Regular Moving Average. If this is used RMA–1 observations will be lost.

WEIGHT= Specifies a 5 element vector of weights to replace the default of (0, 0.25, 0.5, 0.75, 1.0).

EXAMPLES

Draeseke and Giles [1999, 2000] set the task of measuring the size of the New Zealand underground economy (*UE*) on an annual basis. The two causal variables are the effective tax rate (*TR*) and an index of the degree of regulation in New Zealand (*REGS*). The SHAZAM output below shows the use of the **FUZZY** command to generate a time series index for the underground economy.

```
|_TIME 1955 1
|_SAMPLE 1955.0 1994.0
|_READ (TREGS.DAT) YEAR TR REGS
UNIT 88 IS NOW ASSIGNED TO: TREGS.DAT
   3 VARIABLES AND       40 OBSERVATIONS STARTING AT OBS       1

|_* Matrix of decision rules
|_READ R / ROWS=5 COLS=5
      5 ROWS AND           5 COLUMNS, BEGINNING AT ROW       1
|_* Matrix of degrees
|_READ D / ROWS=5 COLS=5
      5 ROWS AND           5 COLUMNS, BEGINNING AT ROW       1
```

```
|_SAMPLE 1962.0 1994.0
|_FUZZY TR REGS / RMA=7 RULES=R DEGREES=D
FUZZY ECONOMETRICS
WEIGHTS:
 0.0000 0.2500 0.5000 0.7500 1.0000
RULES MATRIX:
   1.00   1.00   2.00   2.00   3.00
   1.00   2.00   2.00   3.00   4.00
   2.00   2.00   3.00   4.00   4.00
   2.00   3.00   4.00   4.00   5.00
   3.00   2.00   2.00   5.00   5.00
DEGREES MATRIX:
   1.00   0.80   1.00   0.80   0.80
   1.00   1.00   0.80   1.00   1.00
   1.00   0.80   1.00   0.80   1.00
   1.00   1.00   0.80   1.00   1.00
   0.80   0.80   1.00   0.80   1.00

REGULAR MOVING AVERAGE=   7 OBSERVATIONS LOST=   6

ACTIVE RULES AND FUZZY INDEX
 OBS     0.000    0.250    0.500    0.750    1.000   INDEX
    7   0.0000   0.5871   0.2661   0.1033   0.0000   0.3735
    8   0.0000   0.0000   0.4133   0.4693   0.0000   0.6329
    9   0.0000   0.1778   0.0000   0.5705   0.1422   0.6901
   10   0.0000   0.0000   0.0000   0.8107   0.1893   0.7973
   11   0.0000   0.0000   0.0000   0.4668   0.0700   0.7826
   12   0.0000   0.0000   0.3048   0.5453   0.0000   0.6604
   13   0.0000   0.0000   0.0000   0.5875   0.1788   0.8083
   14   0.0000   0.0000   0.0000   0.1055   0.8237   0.9716
   15   0.0000   0.0851   0.0000   0.2598   0.5921   0.8626
   16   0.0000   0.3196   0.0000   0.6614   0.2709   0.6764
   17   0.0000   0.0000   0.0000   0.5831   0.3268   0.8398
   18   0.0000   0.3871   0.5484   0.0000   0.0000   0.3965
   19   0.0000   0.0081   0.9914   0.0000   0.0000   0.4980
   20   0.0000   0.1125   0.0000   0.8204   0.1437   0.7311
   21   0.0000   0.0000   0.3739   0.5008   0.0000   0.6431
   22   0.0000   0.0000   0.3163   0.6003   0.0000   0.6637
   23   0.0000   0.5912   0.3008   0.0000   0.0000   0.3343
   24   0.1930   0.5436   0.0000   0.0000   0.0000   0.1845
   25   0.0000   0.2258   0.4685   0.4252   0.0000   0.5445
   26   0.0000   0.0621   0.0000   0.4845   0.0497   0.7188
   27   0.0000   0.0000   0.0434   0.8562   0.0000   0.7379
   28   0.0000   0.0000   0.0000   0.3656   0.5075   0.8953
   29   0.0000   0.0000   0.4311   0.5689   0.0000   0.6422
   30   0.0000   0.1941   0.7574   0.0954   0.0000   0.4764
   31   0.0000   0.0000   0.3274   0.5381   0.0000   0.6554
   32   0.0000   0.3721   0.0000   0.5023   0.1149   0.5910
   33   0.0000   0.6942   0.1322   0.0024   0.0000   0.2913
```

The results show that, for 1968 (corresponding to observation 7), the *UE* index value is 0.3735.

18. GENERALIZED ENTROPY

"Many economic-statistical models are ill-posed or under-determined. Consequently, it is important that we learn to reason in these logically indeterminate situations."

Ed Jaynes, 1985

Given a general linear model of the form: $Y = X\beta + \varepsilon$

SHAZAM will recover estimates of β and ε according to the generalized maximum entropy (GME) and generalized cross-entropy (GCE) methods described in Golan, Judge and Miller [1996]. GME and GCE employ limited prior information, and the methods are robust alternatives to OLS and other estimation procedures. Users should be familiar with generalized entropy methods before attempting this procedure.

The unknowns are reparameterized as:

$$\beta = Zp = \begin{bmatrix} z_1' & 0 & . & 0 \\ 0 & z_2' & . & 0 \\ . & . & . & . \\ 0 & 0 & . & z_K' \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ . \\ p_K \end{bmatrix} \quad \text{and} \quad \varepsilon = Vw = \begin{bmatrix} v_1' & 0 & . & 0 \\ 0 & v_2' & . & 0 \\ . & . & . & . \\ 0 & 0 & . & v_N' \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ . \\ w_N \end{bmatrix}$$

where z_k is a vector of M ($2 \le M$) support points for β_k and these are specified with the **ZENTROPY=** option on the **GME** command. Z is a K x KM matrix and p is a KM x 1 vector of weights. v_t is a vector of J ($2 \le J$) support points for ε_t and these are specified with the **VENTROPY=** option.

The general linear model is rewritten as: $Y = X\beta + \varepsilon = XZp + Vw$

The generalized maximum entropy (GME) problem is to choose p and w to maximize an entropy measure. If prior distributions are non-uniform the problem can be extended to employ a generalized cross-entropy (GCE) criterion. Let q be the KM x 1 vector of prior weights for β and let u be the NJ x 1 vector of prior weights on ε. Values for q and u can be specified with the **QPRIOR=** and **UPRIOR=** options respectively. The GCE problem is to select p and w to minimize:

$$I(p,q,w,u) = \sum_{k=1}^{K}\sum_{i=1}^{M} p_{ki}\ \ln(\ p_{ki}\ /\ q_{ki}\ ) + \sum_{t=1}^{N}\sum_{j=1}^{J} w_{tj}\ \ln(\ w_{tj}\ /\ u_{tj}\ )$$

subject to the set of data consistency constraints: $Y = XZp + Vw$

and normalization constraints on p and w. This is a constrained minimization problem. A Lagrangian function can be stated with λ as the N x 1 vector of Lagrange multipliers on the data consistency constraints. The GCE solutions $\tilde{p}$ and $\tilde{w}$ are functions of $\tilde{\lambda}$. A solution method, that is implemented in SHAZAM, is to specify a dual version of the GCE problem. Numerical optimisation methods are then used to choose λ to maximize the objective function. Further discussion on the numerical solution procedure is in Golan, Judge and Miller [1996, Chapter 17].

Normalized entropy measures for the recovered coefficients are calculated as:

$$\sum_{i=1}^{M} \tilde{p}_{ki}\ \ln(\ \tilde{p}_{ki}\ ) \Big/ \sum_{i=1}^{M} q_{ki}\ \ln(\ q_{ki}\ ) \qquad \text{for} \quad k = 1, \ldots, K$$

The normalized entropy measure for p is:

$$\sum_{k=1}^{K}\sum_{i=1}^{M} \tilde{p}_{ki}\ \ln(\ \tilde{p}_{ki}\ ) \Big/ \sum_{k=1}^{K}\sum_{i=1}^{M} q_{ki}\ \ln(\ q_{ki}\ )$$

GME *COMMAND OPTIONS*

In general, the format of the **GME** command is:

GME *depvar indeps / options*

where *depvar* is a vector of N observations. If the set of unknown model parameters, β, is a probability distribution, then the support of the distribution is specified in *indeps*. For a regression model, *indeps* should include the K vectors of explanatory variables, each containing N observations. Unlike the **OLS** command, **GME** does not require N > K.

Options as defined for the **OLS** command that are available are:

LININV, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **NOCONSTANT**, **PCOV**, **RSTAT**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=** and **TRATIO=**

Options as defined for the **NL** command that are available are:

CONV=, **ITER=**, **PITER=** and **START=**

Additional options available on the **GME** command are:

DEVIATION Specifies that the estimation is to use variables measured as deviations from their mean. This option automatically turns on the **NOCONSTANT** option.

LOGEPS= Specifies a small constant to be included in each logarithmic term of the objective function in order to avoid numerical underflow. The default value is 1E–8.

QPRIOR= Specifies a (K x M) matrix of prior probability distributions on the supports specified in **ZENTROPY=**. If this option is not used, the priors are assumed to be discrete uniform.

UPRIOR= Specifies a (N x J) matrix of prior probability distributions on the supports specified in **VENTROPY=**. The matrix must be adjusted appropriately if the data set contains missing values. That is, N must be set to the number of observed values. If this option is not used, the priors are assumed to be discrete uniform.

VENTROPY= Specifies a (N x J) matrix containing the J support points for each of the N unknown disturbances. The matrix must be adjusted appropriately if the data set contains missing values. That is, N must be set to the number of observed values. If this option is not used, the disturbance term, ε, is ignored.

ZENTROPY= Specifies a (K x M) matrix containing the M support points for each of the K unknown parameters. If this option is not used, SHAZAM recovers a probability distribution for the K elements in *indeps*.

EXAMPLES

This example shows the estimation of a demand equation for beer using a data set from Griffiths, Hill and Judge [1993, Table 11.1, p. 372]. The variables are quantity demanded (*Q*), price of beer (*PB*), price of other liquor (*PL*), price of other goods (*PR*) and income (*Y*).

The SHAZAM commands that follow first estimate a log-linear equation by OLS. The GME method is then applied.

```
* Beer demand data set
sample 1 30
read (beer.dat) q pb pl pr y

* Consider a log-linear model
genr lq=log(q)
genr c=1
genr lpb=log(pb)
genr lpl=log(pl)
genr lpr=log(pr)
genr ly=log(y)

* OLS estimation
ols lq c lpb lpl lpr ly / noconstant loglog

* Specify the parameter and error support matrices Z and V
dim z 5 5 v 30 3
sample 1 5
genr z:1=-5
genr z:2=-2.5
genr z:3=0
genr z:4=2.5
genr z:5=5
sample 1 30
genr v:1=-1
genr v:2=0
genr v:3=1

* Solve the dual problem and get the optimal Lagrange multipliers
gme lq c lpb lpl lpr ly / noconstant zentropy=z ventropy=v loglog
```

The OLS estimation results are:

```
|_OLS LQ C LPB LPL LPR LY / NOCONSTANT LOGLOG

 OLS ESTIMATION
      30 OBSERVATIONS     DEPENDENT VARIABLE = LQ
...NOTE..SAMPLE RANGE SET TO:    1,   30
...WARNING...VARIABLE C        IS A CONSTANT

 R-SQUARE =    .8254     R-SQUARE ADJUSTED =    .7975
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .35968E-02
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .59973E-01
SUM OF SQUARED ERRORS-SSE=   .89920E-01
MEAN OF DEPENDENT VARIABLE =   4.0185
LOG OF THE LIKELIHOOD FUNCTION(IF DEPVAR LOG) = -75.9736
RAW MOMENT R-SQUARE =    .9998
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 25 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
C	-3.2432	3.743	-.8665	.394	-.171	.0000	-3.2432
LPB	-1.0204	.2390	-4.269	.000	-.649	-1.6871	-1.0204
LPL	-.58293	.5602	-1.041	.308	-.204	-.4095	-.5829
LPR	.20954	.7969E-01	2.629	.014	.465	.3971	.2095
LY	.92286	.4155	2.221	.036	.406	.9737	.9229

The GME solution follows.

```
|_GME LQ C LPB LPL LPR LY / NOCONSTANT ZENTROPY=Z VENTROPY=V LOGLOG
...NOTE..SAMPLE RANGE SET TO:     1,    30
        30 OBSERVATIONS

INITIAL STATISTICS :

TIME =    5.220 SEC.   ITER. NO.     1 FUNCTION EVALUATIONS     1
GENERALIZED MAXIMUM ENTROPY
FUNCTION VALUE=   -41.00556
COEFFICIENTS
   .0000000       .0000000       .0000000       .0000000       .0000000
   .0000000       .0000000       .0000000       .0000000       .0000000
   .0000000       .0000000       .0000000       .0000000       .0000000
   .0000000       .0000000       .0000000       .0000000       .0000000
   .0000000       .0000000       .0000000       .0000000       .0000000
   .0000000       .0000000       .0000000       .0000000       .0000000
GRADIENT
   4.403054       4.041295       4.160444       4.180522       4.160444
   4.062166       4.122284       4.178992       4.056989       4.151040
   4.188138       3.877432       4.018183       3.869116       4.043051
   3.943522       3.992681       3.945458       4.023564       3.953165
   3.960813       3.790985       4.055257       3.943522       3.985273
   3.912023       3.835142       3.845883       3.945458       3.910021

FINAL STATISTICS :

TIME =    7.810 SEC.   ITER. NO.     8 FUNCTION EVALUATIONS    16
GENERALIZED MAXIMUM ENTROPY
FUNCTION VALUE=   -40.88953
COEFFICIENTS
  -.7471853E-01   .7047584E-01  -.3980811E-01  -.2088085E-01   .1718525E-01
   .6260548E-01  -.4277361E-01  -.4776353E-01   .4011995E-02  -.9758509E-01
  -.1127975       .1687698      -.7855024E-01   .8392667E-01  -.1466195E-01
   .6197439E-01   .1182172E-01   .2583825E-01  -.3993996E-01  -.1753328E-02
  -.7051842E-01   .2650401      -.1504612      -.5927065E-01  -.7903290E-01
  -.3510743E-01   .1288732       .8452928E-01  -.3989216E-01   .1387945E-01
GRADIENT
   .1093738E-04   .1449848E-04   .1710085E-04   .1049372E-04   .1772534E-04
   .2659682E-04   .8632597E-05   .9548913E-05   .6241976E-05   .1157758E-04
   .4525873E-05   .1523205E-04   .9083578E-05   .2201034E-04   .2532559E-04
   .1897241E-04   .8911556E-05   .1447326E-04   .1181824E-04   .1539831E-04
   .1122714E-04  -.4600764E-05   .1419110E-04   .7498713E-05   .9664092E-05
   .9152785E-05   .2343221E-04   .2228759E-04   .1035147E-04   .1755680E-04

NORMALIZED ENTROPY FOR COEFFICIENT    1   .99983
NORMALIZED ENTROPY FOR COEFFICIENT    2   .98237
NORMALIZED ENTROPY FOR COEFFICIENT    3   .99906
NORMALIZED ENTROPY FOR COEFFICIENT    4   .99938
NORMALIZED ENTROPY FOR COEFFICIENT    5   .99365
```

```
NORMALIZED ENTROPY FOR P=   .99486
NORMALIZED ENTROPY FOR W=   .99774

R-SQUARE =    .8072     R-SQUARE ADJUSTED =    .7764
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .39710E-02
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .63016E-01
SUM OF SQUARED ERRORS-SSE=   .99274E-01
MEAN OF DEPENDENT VARIABLE =   4.0185
LOG OF THE LIKELIHOOD FUNCTION(IF DEPVAR LOG) = -79.6664
RAW MOMENT R-SQUARE =    .9935

                                    PSEUDO
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      25 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
C          .82291E-01  .5140E-03   160.1      .000 1.000      .0000      .0823
LPB       -.83969      .7717E-01  -10.88      .000 -.909    -1.3883     -.8397
LPL       -.19429      .9870E-01  -1.969      .060 -.366     -.1365     -.1943
LPR        .15767      .6495E-01   2.428      .023  .437      .2988      .1577
LY         .50497      .1945E-01   25.96      .000  .982      .2217      .5050
```

For the beer demand model, the parameter estimates may be interpreted as elasticities. Given that GME is a shrinkage estimator, the GME solution gives elasticities that are typically smaller in absolute value relative to the OLS estimates. That is, the parameter vector recovered by GME is a "shrunken" version of the OLS estimated parameter vector.

Note that the approximate standard errors and t-ratios are only valid in some settings. For further details see Golan, Judge and Miller [1996, Chapter 6].

19. GENERALIZED LEAST SQUARES

"50 years hence...we shall escape the absurdity of growing a whole chicken in order to eat the breast or wing, by growing these parts separately under a suitable medium."

Winston Churchill
Member of British Parliament, 1932

The **GLS** command performs **G**eneralized **L**east **S**quares regressions. The method of generalized least squares is discussed in Greene [2003, Chapter 10; 2000, Chapter 11], Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 8] and other econometrics textbooks.

Consider the linear model $Y = X\beta + \varepsilon$ where Y is an N x 1 vector of observations on the dependent variable, X is an N x K matrix of explanatory variables, β is a vector of unknown parameters and ε is a random error vector with zero mean and covariance matrix:

$$E(\varepsilon\varepsilon') = \sigma^2 \Omega$$

The N x N matrix Ω is a known positive definite symmetric matrix that allows for a general error covariance structure and σ^2 is an unknown scalar. The GLS estimate is:

$$\hat{\beta} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}Y$$

Another way of approaching the analysis is to express the Ω^{-1} matrix as $\Omega^{-1} = P'P$ where P is a non-singular N x N lower triangular matrix. The matrix P is used to transform the model to the form:

$$PY = PX\beta + P\varepsilon$$

Then OLS can be applied to the transformed model to get the GLS estimate. The transformed residuals are obtained as:

$$\hat{v} = PY - PX\hat{\beta}$$

and the untransformed residuals (available with the **UT** option) are:

$$e = Y - X\hat{\beta}$$

The estimate of σ^2 is: $\hat{\sigma}^2 = \frac{1}{N-K}(\hat{v}'\hat{v}) = \frac{1}{N-K}\left(e'\Omega^{-1}e\right)$

When the **DN** option is used the divisor for $\hat{\sigma}^2$ is N instead of N–K. The estimated covariance matrix for $\hat{\beta}$ is:

$$\hat{\sigma}^2\left(X'\Omega^{-1}X\right)^{-1}$$

With the assumption of normality the log-likelihood function can be written as:

$$-\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln\sigma^2 - \frac{1}{2}\ln|\Omega| - \frac{1}{2\sigma^2}(Y - X\beta)'\Omega^{-1}(Y - X\beta)$$

This is evaluated as:

$$-\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln\tilde{\sigma}^2 + \sum_{t=1}^{N}\ln(P_{tt}) - \frac{N}{2} \qquad \text{where} \qquad \tilde{\sigma}^2 = \frac{1}{N}(\hat{v}'\hat{v})$$

and P_{tt} is the t^{th} diagonal element of P.

The Buse R^2 (see Buse [1973]) reported on the SHAZAM output gives a measure of goodness-of-fit and is calculated as:

$$R^2 = 1 - \frac{e'\Omega^{-1}e}{(Y - DY)'\Omega^{-1}(Y - DY)} \qquad \text{with} \qquad D = \frac{jj'\Omega^{-1}}{j'\Omega^{-1}j}$$

where j is an N x 1 vector of ones. The expression Y–DY transforms the observations to deviations from a weighted mean. When the **NOCONSTANT** option is specified the raw moment R^2 (described in Theil [1961, p. 221]) is calculated by replacing Y–DY with Y. The raw moment R^2 is meaningful when the equation does not have an intercept.

The **GLS** command allows for general error covariance structures. It is useful to note that some special cases are implemented in other SHAZAM commands. For example, the **WEIGHT=** option on the **OLS** command (see the chapter *ORDINARY LEAST SQUARES*) provides weighted least squares estimation and models with autoregressive errors (see the chapter *AUTOCORRELATION MODELS)* can be estimated with the **AUTO** command. Another case of generalized least squares estimation is implemented with the **POOL** command (see the chapter *POOLED CROSS-SECTION AND TIME-SERIES*).

GLS *COMMAND OPTIONS*

In general, the format of the **GLS** command is:

GLS *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of the independent variables, and *options* is a list of desired options. One of **OMEGA=**, **OMINV=** or **PMATRIX=** is required to tell SHAZAM which matrix to use for estimation. For example, to give SHAZAM the Ω matrix the **OMEGA=** option is used.

Options described for the **OLS** command that may be used are:

ANOVA, **DLAG**, **DN**, **GF**, **HETCOV**, **LININV**, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **MAX**, **NOCONSTANT**, **PCOR**, **PCOV**, **RESTRICT**, **RSTAT**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT=**, **STDERR=** and **TRATIO=**

Other options are:

BLUP The predicted values are adjusted using information obtained from previous period residuals according to the transformation specified by the P matrix. This option is not effective in forecasts using the **GLS** coefficients.

DUMP Prints out the Ω, Ω^{-1} and P matrices. It is important to be aware that each of these matrices is of the order N x N and in large samples could require many pages of printout. The **DUMP** option is useful for checking to see if the input of the matrix has been done correctly.

FULLMAT If the **FULLMAT** option is not specified, SHAZAM assumes that the specified matrix contains the diagonals of Ω (**OMEGA=**), Ω^{-1} (**OMINV=**) or the P matrix (**PMATRIX=**).

NOMULSIGSQ Uses $\left(X'\Omega^{-1}X\right)^{-1}$ as the estimate of the covariance matrix of the parameter estimates. It is **NO**t **MUL**tiplied by $\hat{\sigma}^2$.

UT The estimated coefficients will be used with the original data to compute predicted values and residuals that are **U**n**T**ransformed. Without this option, the residual output and predicted values given are transformed.

OMEGA=
OMINV=
PMATRIX=

Specifies the matrix to be used for estimation as Ω (**OMEGA=**), Ω^{-1} (**OMINV=**) or the P matrix (**PMATRIX=**). One of these options *must* be specified on each **GLS** command. The **FULLMAT** option must be used if the complete matrix, rather than just the diagonals of the matrix, is given. When just the diagonals are given the matrix must be set-up as follows. The main diagonal is entered in the first column in rows 1 to N. The first lower diagonal (if required) is entered in the second column in rows 1 to N–1 (the element in row N is ignored). The second lower diagonal (if required) is entered in the third column in rows 1 to N–2 (the elements in rows N–1 and N are ignored), etc.

RESID=

Saves the **RESID**uals in the variable specified. For details on which residuals are saved see the **UT** and **BLUP** options.

The available temporary variables on the **GLS** command are:

$ADR2, *$ANF*, *$DF*, *$DW*, *$ERR*, *$K*, *$LLF*, *$N*, *$R2*, *$R2OP*, *$RAW*, *$RHO*, *$SIG2*, *$SSE*, *$SSR*, *$SST*, *$ZANF*, *$ZDF*, *$ZSSR* and *$ZSST*.

For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

EXAMPLE

The **GLS** command can be demonstrated by estimating a model with first-order autoregressive errors for the Theil Textile data set. (Note that most SHAZAM users would use the **AUTO** command to estimate an AR(1) error model.) Consider an autoregressive parameter with a value of ρ = 0.8. The form of the P matrix (see Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Equation 9.5.29, p.390]) is:

$$P = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & . & 0 \\ -\rho & 1 & 0 & . & 0 \\ 0 & -\rho & 1 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & -\rho & 1 \end{bmatrix}$$

The SHAZAM commands for the GLS estimation are:

```
* Set-up the P matrix
dim p 17 2
sample 2 17
genr p:1=1
gen1 p:1=sqrt(1-.8*.8)
sample 1 17
genr p:2=-.8

* Print the P matrix so that it can be checked.
print p

* Get the GLS estimator
gls consume income price / pmatrix=p

* Now verify the results with the AUTO command
auto consume income price / rho=.8
```

The **DIM** command is used to dimension the matrix P to be the size of the **PMATRIX** for the problem. (The **DIM** command is explained in the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.) Next, the **GENR** command is used to assign a value of 1 to observations 2 to 17 of the first column of the matrix P. Note that on the **GENR** command the first column of P is referred to as *P:1*. The first observation of this column is given a value of $\sqrt{1 - 0.8 \cdot 0.8}$. Finally, the second column is assigned a value of –0.8.

The SHAZAM results from the GLS estimation follow.

```
 |_* Set-up the P matrix
 |_DIM P 17 2
 |_SAMPLE 2 17
 |_GENR P:1=1
 |_GEN1 P:1=SQRT(1-.8*.8)
 |_SAMPLE 1 17
 |_GENR P:2=-.8
 |_* Print the P matrix so that it can be checked.
 |_PRINT P
     P
    17 BY     2 MATRIX
    .6000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000
    1.000000      -.8000000

etc
```

```
   1.000000      -.8000000
   1.000000      -.8000000
   1.000000      -.8000000
   1.000000      -.8000000
   1.000000      -.8000000
   1.000000      -.8000000
   1.000000      -.8000000
   1.000000      -.8000000
|_* Get the GLS estimator
|_GLS CONSUME INCOME PRICE / PMATRIX=P
...WARNING..ASSUMING P        CONTAINS DIAGONALS
GLS ESTIMATION
     17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO    1,   17

 R-SQUARE DEFINITIONS BASED ON BUSE, AMSTAT(1973)
 R-SQUARE =    .7652     R-SQUARE ADJUSTED =    .7316
VARIANCE OF THE ESTIMATE-SIGMA**2 =   48.091
STANDARD ERROR OF THE ESTIMATE-SIGMA =   6.9348
SUM OF SQUARED ERRORS-SSE=   673.27
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -55.9037

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.1240      .5595       2.009      .064  .473      .2527      .8606
PRICE     -1.6577      .2525      -6.565      .000 -.869    -1.1859     -.9405
CONSTANT   148.12      59.60       2.485      .026  .553      .0000     1.1012
```

20. HETEROSKEDASTIC MODELS

"The 'state of the world' is a serially correlated thing; hence, we find ARCH."
Francis Diebold and Marc Nerlove, 1989
Journal of Applied Econometrics

The **HET** command implements maximum likelihood estimation of models which require corrections for heteroskedastic errors. The model to consider is:

$$Y_t = X'_t\beta + \varepsilon_t$$

where Y_t is the dependent variable, X_t are the independent variables, β are unknown parameters and ε_t is a zero mean, serially uncorrelated process with variance given by the function h_t. A survey of approaches to the specification of h_t is available in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 11].

Forms of Heteroskedasticity

A form that has been applied to cross-section studies of household expenditure (for example, Prais and Houthakker [1955] and Theil [1971]) is "dependent variable heteroskedasticity" with

$$h_t = (X'_t\beta)^2\,\alpha^2$$

where α is a scalar parameter. Further flexibility is obtained with a variance specification that is a function of exogenous variables, say Z_t, as in

$$h_t = Z'_t\alpha \qquad \text{or} \qquad h_t = (Z'_t\alpha)^2$$

where α is a vector of unknown parameters. Another form is the "multiplicative heteroskedasticity" model described by Harvey [1976, 1990] with

$$h_t = \exp(Z'_t\alpha)$$

ARCH Models

When modelling heteroskedasticity in time series data the ARCH (autoregressive conditional heteroskedasticity) process developed by Engle [1982] is of interest. ARCH models recognize the presence of successive periods of relative volatility and stability and allow the conditional variance to evolve over time as a function of past errors. The variance conditional on the past is given by the equation:

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2$$

where the α_j are unknown parameters and q is the order of the ARCH process.

Various extensions to the Engle ARCH model have been proposed in the literature. For example, ARMA models with ARCH errors are analyzed by Weiss [1984]. Bollerslev [1986] specifies a conditional variance equation, GARCH, that allows for a parsimonious parameterisation of the lag structure. Time varying risk premia can be considered by including some function of the conditional variance as an additional regressor in the mean equation. This gives the ARCH-M (ARCH-in-Mean) model discussed by Engle, Lilien and Robins [1987]. To incorporate these extensions consider the model:

$$Y_t = X_t'\beta + \gamma g(h_t) + \varepsilon_t + \sum_{j=1}^{r} \theta_j \varepsilon_{t-j}$$

where the ε_t is a GARCH(p,q) process with conditional variance function given by:

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \phi_j h_{t-j}$$

and the unknown parameters are β, γ, θ, α, and ϕ. The function $g(h_t)$ in the ARCH-M model may be specified by the practitioner and possible choices are $\log(h_t)$ or $\sqrt{h_t}$. ARCH processes assume constant unconditional variance and the parameter restrictions for stationarity are:

$$\alpha_0 > 0, \ \alpha_j \geq 0 \quad \text{for all j}, \quad \phi_j \geq 0 \quad \text{for all j}, \quad \text{and} \quad \sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \phi_j < 1$$

Also, the ARCH variance equation can be extended to include exogenous variables.

Maximum Likelihood Estimation

Maximum likelihood estimates are obtained based on the assumption that the errors are conditionally Gaussian. With the assumption of normality the log-density for observation t is:

$$l_t = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(h_t) - \frac{1}{2}\varepsilon_t^2 / h_t \qquad \text{for} \qquad t = 1, \ldots, N$$

The log-likelihood function is: $$L = \sum_{t=1}^{N} l_t$$

First derivatives of the log-likelihood function with respect to the mean equation parameters and the variance equation parameters can be derived and these are utilized by SHAZAM (with the exception of ARCH-in-Mean models where numerical derivatives are used). The nonlinear optimization method employed by SHAZAM is a quasi-Newton method and alternative algorithms can be selected with the **METHOD=** option as described for the **NL** command in the chapter *NONLINEAR REGRESSION*. For the **HET** command the optimization algorithm constructs an initial Hessian estimate from the outer-product of the gradient (with the exception of ARCH-in-Mean models where the model estimation uses numerical derivatives).

At model convergence there are a number of ways to compute an estimate of the variance-covariance matrix of the parameter estimates. The **HET** command computes the information matrix inverse (with the exception of ARCH-in-Mean models where the covariance matrix estimate is computed from a Hessian approximation). For ARCH models and the exogenous heteroskedasticity models the information matrix is block diagonal. For the dependent variable heteroskedasticity model the assumption of Gaussian errors leads to an information matrix that is not block diagonal. Analytic expressions for the information matrix are available in Harvey [1990, Chapter 3.4] (for the dependent variable heteroskedasticity and the multiplicative heteroskedasticity models), Engle [1982] (for ARCH models) and Bollerslev [1986] (for GARCH models). Alternative covariance matrix estimates are obtained by using the **NUMCOV**, or **OPGCOV** options as described for the **NL** command in the chapter *NONLINEAR REGRESSION*.

HET *COMMAND OPTIONS*

In general, the format of the **HET** command is:

HET *depvar indeps (exogs) / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, *exogs* is an optional list of exogenous variables in the variance equation and *options* is a list of desired options. For models with exogenous heteroskedasticity an intercept term is always included in the variance equation and therefore should not be specified in the *exogs* variable list. For models with exogenous variables in the variance equation if the *exogs* option is not specified then SHAZAM will set the variance equation exogenous variables to be identical to the regression equation exogenous variables. However, users should be warned that this may not be the best choice. For example, it may be appropriate to consider some transformation of the regression variables in the variance equation.

The options as defined for **OLS** that are available are:

DUMP, **LININV**, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **MAX**, **NOCONSTANT**, **PCOR**, **PCOV**, **RSTAT**, **BEG=/END=**, **COEF=**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=** and **TRATIO=**.

The options as defined for the nonlinear regression command **NL** that are available are:

NUMCOV, **NUMERIC**, **OPGCOV**, **CONV=**, **ITER=**, **METHOD=**, **PITER=** and **STEPSIZE=**.

The following additional options are available on the **HET** command:

PRESAMP The estimation of ARCH models requires pre-sample estimates of ε_t^2. The algorithm sets the pre-sample values as an additional parameter and then obtains an estimated value that maximizes the value of the likelihood function. Use the **PRESAMP** option to ensure that an initial value is fixed for the pre-sample data in all iterations. If this option is specified then the pre-sample values are set to the average of the squared errors evaluated at the starting parameter values or the square of the value specified in the **START=** option.

ARCH= Specifies the order q of the ARCH process.

ARCHM= Specifies the functional form for the ARCH-in-Mean term. With **ARCHM=*x*** the term h_t^x is included as an additional regressor. Set **ARCHM=0** for $\log(h_t)$ in the ARCH-M model. Set **ARCHM=0.5** for $\sqrt{h_t}$ in the ARCH-M model. When this option is used the model estimation uses numerical derivatives. The estimation algorithm provides a Hessian approximation that is used to compute the covariance matrix estimate. This may give unreliable standard errors. Therefore, the **NUMCOV** option (as described for the **NL** command) should also be considered with the **ARCHM=** option.

GARCH= Specifies the order p of the lagged conditional variances in the GARCH process. If the **ARCH=** option is not set then the process estimated is GARCH(p,1). In applied work the GARCH process that is often considered practical is the GARCH(1,1) process. This is obtained by specifying **GARCH=1**.

GMATRIX= Specifies an NxK matrix to use to store the derivatives of the log-likelihood function at each observation. This option is not available with the **NUMERIC** option. An example of the use of this option is given at the end of this chapter.

MACH= Specifies the order of the moving average process in models with ARCH errors.

MODEL= Specifies the form of heteroskedasticity. The available options are:

ARCH for ARCH models. When this option is requested the default for the ARCH order is **ARCH=1**.

DEPVAR for **DEP**endent **VAR**iable heteroskedasticity.

MULT for **MULT**iplicative heteroskedasticity.

STDLIN for $h_t = (Z_t'\alpha)^2$ (i.e. the **ST**andard **D**eviation is a **LIN**ear function of exogenous variables).

VARLIN for $h_t = Z_t'\alpha$ (i.e. the **VAR**iance is a **LIN**ear function of exogenous variables).

If the **ARCH=**, **ARCHM=**, **GARCH=** or **MACH=** options are requested then **MODEL=ARCH** is assumed. Otherwise, the default is **MODEL=DEPVAR**.

START= Specifies a vector of starting values for the estimation. The values must be in the same order as on the SHAZAM output. In addition, for ARCH

models, the final parameter may be the starting value for the standard deviation of the pre-sample innovations. If this option is not specified then the starting parameter values are chosen by default. The default starting estimate for β are obtained from an OLS regression. For ARCH models the default starting values for the α are obtained from a regression of the OLS squared residuals on a constant and q lags.

STDRESID= Saves the standardized residuals in the variable specified. The standardized residuals are computed from the estimated parameters as $\hat{\varepsilon}_t / \sqrt{\hat{h}_t}$.

Following model estimation the available temporary variables as described for the **OLS** command are:

$ERR , $K, $LLF, $N, and *$R2OP*.

A practical difficulty with **MODEL=VARLIN** is that the parameter values may wander into regions that give negative variance. If this is encountered the algorithm resets the negative variance to a small positive number. The ARCH models may also generate negative variance and overflows and the algorithm checks for this. Warning messages for negative variance corrections are given when the **DUMP** option is used. If the model does not converge then different starting values can be tried with the **START=** option.

For ARCH models, when the estimation results are displayed the variable name GAMMA_ gives the ARCH-in-Mean parameter, the variable names THETA_ give the moving average parameters, the variable names ALPHA_ give the ARCH parameters (the first is the constant term in the variance equation), and the PHI_ give the parameter estimates on the lagged variances in the GARCH conditional variance equation. Finally the variable name DELTA_ gives the estimate of the standard deviation of the pre-sample innovations.

EXAMPLES

Testing for Heteroskedasticity

To test for heteroskedasticity an approach is to first run an OLS regression and then apply Lagrange multiplier tests. The **DIAGNOS / HET** command reports a number of useful tests including a test for ARCH(1) errors. A test for ARCH(q) can be constructed by running a regression of the OLS squared residuals on a constant and q lags and comparing the $N \cdot R^2$ value with a χ^2 distribution with q degrees of freedom. The following SHAZAM commands generate tests for heteroskedasticity and ARCH(2) errors.

```
ols consume income price / resid=e
diagnos / het
genr e2=e**2
ols e2 e2(1.2)
gen1 lm=$n*$r2
print lm
```

By examining the test statistics the user can verify that the null hypothesis of homoskedasticity is not rejected for this simple example. However, the examples that follow serve to illustrate the **HET** command.

Dependent Variable Heteroskedasticity

The estimation results for a model with "dependent variable heteroskedasticity" are given below:

```
|_HET CONSUME INCOME PRICE
 ...NOTE..SAMPLE RANGE SET TO:     1,    17
 DEPVAR   HETEROSKEDASTICITY MODEL      17 OBSERVATIONS
          ANALYTIC DERIVATIVES

   QUASI-NEWTON METHOD USING BFGS UPDATE FORMULA

 INITIAL STATISTICS :
 TIME =    1.750 SEC.   ITER. NO.     1 FUNCTION EVALUATIONS     1
 LOG-LIKELIHOOD FUNCTION=   -51.00134
 COEFFICIENTS
    1.061709      -1.382986       130.7066       .3700365E-01
 GRADIENT
   -13.82284      -21.68072      -.1674465       7.936759

 FINAL STATISTICS :
 TIME =    1.970 SEC.   ITER. NO.     6 FUNCTION EVALUATIONS     6
 LOG-LIKELIHOOD FUNCTION=   -50.58788
 COEFFICIENTS
    .9084240      -1.351451       144.0436       .3580546E-01
 GRADIENT
    .6045146E-01   .6405245E-01   .5909730E-03   .5394427

 SQUARED CORR. COEF. BETWEEN OBSERVED AND PREDICTED    .95040

 ASY. COVARIANCE MATRIX OF PARAMETER ESTIMATES IS ESTIMATED USING
 THE INFORMATION MATRIX

 LOG OF THE LIKELIHOOD FUNCTION = -50.5879

                              ASYMPTOTIC
 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
         MEAN EQUATION:
 INCOME      .90842      .2140       4.246      .000  .762      .2042      .6955
 PRICE     -1.3515      .6706E-01  -20.15      .000 -.984     -.9668     -.7667
 CONSTANT   144.04      22.78       6.324      .000  .869      .0000     1.0709
```

```
          VARIANCE EQUATION:
ALPHA_      .35805E-01  .6148E-02   5.823      .000  .85
```

The coefficient ALPHA_ above can be interpreted as the estimate of the standard deviation of the ratio $\varepsilon_t / (X_t'\beta)$.

ARCH(1)

Output for the estimation of a first-order ARCH model is:

```
|_HET CONSUME INCOME PRICE / ARCH=1
 ...NOTE..SAMPLE RANGE SET TO:     1,    17
ARCH      HETEROSKEDASTICITY MODEL      17 OBSERVATIONS
          ANALYTIC DERIVATIVES

  QUASI-NEWTON METHOD USING BFGS UPDATE FORMULA

INITIAL STATISTICS :
TIME =     .340 SEC.   ITER. NO.      1 FUNCTION EVALUATIONS     1
LOG-LIKELIHOOD FUNCTION=   -51.39268
COEFFICIENTS
   1.061709      -1.382986       130.7066       16.95086       .3812538
   5.048663
GRADIENT
  -2.568027       2.385548      -.4618551E-01  -.1685253E-01   .2616805
  -.9919629E-02

FINAL STATISTICS :
TIME =     .870 SEC.   ITER. NO.      8 FUNCTION EVALUATIONS    11
LOG-LIKELIHOOD FUNCTION=   -51.20214
COEFFICIENTS
   .9432237      -1.409199       145.0000       15.23096       .4810183
   5.203280
GRADIENT
  -.7149841E-01  -.7095763E-01  -.7350608E-03  -.4425543E-04  -.6301406E-03
  -.1273054E-03

SQUARED CORR. COEF. BETWEEN OBSERVED AND PREDICTED    .95035

ASY. COVARIANCE MATRIX OF PARAMETER ESTIMATES IS ESTIMATED USING
THE INFORMATION MATRIX

LOG OF THE LIKELIHOOD FUNCTION = -51.2021

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
          MEAN EQUATION:
INCOME      .94322      .2019       4.672      .000  .815      .2121      .7222
PRICE     -1.4092      .6708E-01  -21.01      .000 -.988    -1.0081     -.7995
CONSTANT   145.00      20.63       7.029      .000  .904      .0000     1.0780
          VARIANCE EQUATION:
ALPHA_      15.231      9.221       1.652      .099  .44
ALPHA_      .48102      .5340       .9007      .368  .26
DELTA_      5.2033      8.321       .6253      .532  .18
```

The starting values listed in iteration 1 are obtained from OLS and the model converges in 8 iterations. Note that the estimated value for α_1 is 0.48102 which satisfies the stationarity constraints (although it is not statistically significant). The coefficient DELTA_ is the estimate of the standard deviation of the pre-sample innovations. If the **PRESAMP** option is used then the pre-sample values are fixed. More model estimation output is obtained with the **DUMP** and **PITER=1** options.

Multiplicative Heteroskedasticity

Consider the model: $$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \varepsilon_t$$

with an error variance function:

$$h_t = \alpha_1 X_{2t}^{\alpha_2} X_{3t}^{\alpha_3} = \exp\left\{\log(\alpha_1) + \alpha_2 \log(X_{2t}) + \alpha_3 \log(X_{3t})\right\}$$

This can be viewed as a "multiplicative heteroskedasticity" model. An example of SHAZAM commands for estimation of the model parameters is:

```
genr linc=log(income)
genr lprice=log(price)
het consume income price (linc lprice) / model=mult
```

Robust Standard Errors

Some applied workers report robust standard errors obtained from the covariance matrix calculation $A^{-1}BA^{-1}$ where A is the information matrix and B is the outer-product of the gradient (see White [1982], Weiss [1986] and Bollerslev [1986]). The standard errors are robust in the sense that conditional normality of the errors is not assumed. The next set of SHAZAM commands estimates a GARCH(1,1) model and computes robust standard errors. Starting values for the estimation are specified in the variable *INPUT*.

```
read(filename) y x1 x2
dim input 6

* Starting values for the mean equation
gen1 input:1=1
gen1 input:2=1
gen1 input:3=100

* Starting values for the GARCH(1,1) variance equation
gen1 input:4=.5
gen1 input:5=.2
```

```
gen1 input:6=.7

* Estimate a GARCH(1,1) model
het y x1 x2 / start=input arch=1 garch=1 coef=beta cov=ainv &
              gmatrix=g

* Compute robust standard errors
matrix v=ainv*(g'g)*ainv
matrix se=sqrt(diag(v))
matrix t=beta/se

* Print parameter estimates with robust standard errors and t-
ratios.
print beta se t
```

21. MAXIMUM LIKELIHOOD ESTIMATION OF NON-NORMAL MODELS

"It's the work of a madman."

Ambroise Vollard
French art dealer, 1907
(viewing a Picasso painting)

In regression applications where the dependent variable is strictly positive the assumption of normally distributed errors may be inappropriate. The **MLE** command provides maximum likelihood estimation of linear regression models for a range of distributional assumptions. If regressions with multivariate-t errors are desired see the chapter in this manual titled *ROBUST ESTIMATION*. If the desired form of the model is not listed with the **TYPE=** option described below see the **LOGDEN** option in the chapter *NONLINEAR REGRESSION* where a user-specified density function may be estimated. This command should only be used if this type of regression model is fully understood. The derivations in this chapter were derived by Trudy Cameron. For a further description of these models see Cameron and White [1990].

Consider the class of linear regression models such that: $E(Y_t \mid X_t) = X_t'\beta$

Assume Y_t are independent random variables from a distribution with probability density function $f(Y_t; X_t, \beta, \theta)$ where θ is a vector of parameters that describe the shape of the distribution. The log-likelihood function is obtained by summing the log densities over all observations:

$$L = \sum_{t=1}^{N} \log \{ f(Y_t; X_t, \beta, \theta) \}$$

The maximum likelihood estimates of β and θ are found by a nonlinear algorithm as those that maximize the value of the log-likelihood function. The estimated covariance matrix of the coefficient estimates is found by computing the inverse of the matrix of second derivatives of the log-likelihood function and evaluating this matrix at the maximum likelihood estimates.

Exponential Regression

The exponential distribution is the default distribution for the **MLE** command and is used if the **TYPE=** option is not specified. The conditional density function for Y_t can be expressed as:

$$f(Y_t; X_t, \beta) = (1/X_t'\beta)\exp(-Y_t/X_t'\beta)$$

Taking logs and summing over all N observations gives the log-likelihood function:

$$-\sum_{t=1}^{N}\log(X_t'\beta) - \sum_{t=1}^{N}\left(\frac{Y_t}{X_t'\beta}\right)$$

Generalized Gamma Regression

The generalized gamma distribution is a flexible distribution that contains the simple gamma, Weibull, exponential and lognormal distributions as special cases. The log-likelihood function for regression using the generalized gamma distribution is:

$$N\log(c) - N\log\Gamma(k) + N\cdot c\cdot k\cdot\log\left(\frac{\Gamma(k+(1/c))}{\Gamma(k)}\right)$$
$$-\sum_{t=1}^{N}\log(Y_t) + c\cdot k\sum_{t=1}^{N}\log\left(\frac{Y_t}{X_t'\beta}\right) - \sum_{t=1}^{N}\left(\left(\frac{\Gamma(k+(1/c))}{\Gamma(k)}\right)\frac{Y_t}{X_t'\beta}\right)^c$$

where c and k are shape parameters. When $c = k = 1$ the exponential distribution is obtained. When $k = 1$ the generalized gamma distribution reduces to the Weibull distribution and when $c = 1$ the generalized gamma distribution reduces to the gamma distribution.

Model Discrimination

Since the exponential distribution is a special case of the gamma distribution and also of the Weibull distribution it is easy to compute Lagrange Multiplier (LM) statistics for testing the hypothesis of exponential versus the more general distributions. These test statistics are reported with the **LM** option on the **MLE** command. A LM statistic to test the exponential model against the gamma model is computed as:

$$LM = g'H^{-1}g$$

where g and H are the first (gradient) and second (Hessian) derivatives respectively of the gamma log-likelihood function evaluated at the point $k = 1$. Under the null hypothesis that the exponential distribution is correct this statistic has an asymptotic chi-square distribution with 1 degree of freedom. A similar procedure is used to test the exponential model against the Weibull model.

An LM test of the Weibull or gamma model against the generalized gamma model can also be constructed. For nested models a likelihood ratio test statistic provides another basis for comparison (see the discussion in McDonald [1984] and Cameron and White [1990]). Wald test statistics (computed with the **TEST** command) can also be applied to aid in model selection.

Lognormal Regression

It can be shown (with difficulty) that the lognormal distribution becomes a special case of the generalized gamma distribution as the parameter k approaches infinity. For regression using the lognormal distribution the log-likelihood function is:

$$- N \log(\sqrt{2\pi}\sigma) - \sum_{t=1}^{N} \log(Y_t) - \frac{1}{2\sigma^2} \sum_{t=1}^{N} \left[\log\left(\frac{Y_t}{X_t'\beta}\right) + \frac{\sigma^2}{2} \right]^2$$

Beta Regression

The beta distibution can be used for modeling dependent variables that are proportions. The beta density function has two parameters: p and q and the mean of a beta distribution is equal to p/(p+q). One way to derive the model is to make one of the parameters a function of the independent variables. When the **TYPE=BETA** option is specified the beta regression model where the parameter q is conditional on X is:

$$E(Y_t \mid X_t) = X_t'\beta = \frac{p}{p + q(X_t)} \qquad \text{where} \qquad q(X_t) = \frac{p}{X_t'\beta} - p$$

The conditional beta density function becomes:

$$f(Y_t; X_t, \beta, p) = Y_t^{(p-1)} (1 - Y_t)^{(q(X_t)-1)} \Big/ \left\{ \frac{\Gamma(p)\Gamma(q(X_t))}{\Gamma(p + q(X_t))} \right\}$$

The beta regression log-likelihood function is:

$$-N\log\Gamma(p)+\sum_{t=1}^{N}\log\left[\frac{\Gamma(p+q(X_t))}{\Gamma(q(X_t))}\right]+(p-1)\sum_{t=1}^{N}\log(Y_t)+\sum_{t=1}^{N}\left(\frac{p}{X_t'\beta}-p-1\right)\log(1-Y_t)$$

When the **TYPE=EBETA** option is specified the model assumptions are:

$$p(X_t)=\exp(X_t'\alpha) \qquad \text{and} \qquad q(X_t)=\exp(X_t'\gamma)$$

This model is given in Paolino [2001, p. 327]. The parameters α and γ are estimated by maximizing the log-likelihood function. An alternative approach to beta estimation is proposed in Paolino [2001, p. 336] and implemented with the **TYPE=MBETA** option. The specification is:

$$E(Y_t\,|\,X_t)=\frac{\exp(X_t'\alpha)}{1+\exp(X_t'\alpha)}$$

The variance of the dependent variable depends on the function: $\exp(Z_t'\gamma)$

The X and Z are (potentially distinct) independent variables and the parameters α and γ give the effects of X and Z on the expected value and dispersion of the dependent variable.

Log-linear Models

An alternative model assumption is: $E(\log(Y_t)\,|\,X_t)=X_t'\beta$

This form is obtained when **TYPE=EGAMMA, EGG, EWEIBULL** or **EXTREMEV** is specified on the **MLE** command. The dependent variable must first be transformed to log form. Define $Z_t=\log(Y_t)$. For the log-linear generalized gamma regression model (**TYPE=EGG**) the conditional density function for the logarithmically transformed variable Z_t is:

$$f(Z_t;X_t,\beta,\sigma,k)=\frac{1}{\sigma\Gamma(k)}\exp\left[k(Z_t-X_t'\beta)/\sigma-\exp\left((Z_t-X_t'\beta)/\sigma\right)\right]$$

where $\sigma=1/c$. The log-likelihood function (see Cameron and White [1990, Equation 9]) can then be written as:

$$-N\log(\sigma)-N\log\Gamma(k)+k\sum_{t=1}^{N}(Z_t-X_t'\beta)/\sigma-\sum_{t=1}^{N}\exp\left[(Z_t-X_t'\beta)/\sigma\right]$$

The shape parameters reported on the SHAZAM output are σ and k. Note that for comparison purposes with the linear model the value of the log-likelihood function computed for the log-linear model must be adjusted to include the appropriate Jacobian transformation. This is implemented with the **LOGLIN** option as described for the **OLS** command. Also note that the log-linear lognormal model is simply OLS with $\log(Y_t)$ as the dependent variable.

Poisson Regression

Poisson regression can be applied for models where the dependent variable has integer values such as count data. This can be implemented with the **TYPE=EPOISSON** or **POISSON** options on the **MLE** command. With **TYPE=EPOISSON** the probability function is:

$$\Pr(Y_t) = \exp(-\lambda_t)\lambda_t^{Y_t}/Y_t! \qquad \text{for} \quad Y_t = 0, 1, 2, \ldots \quad \text{and} \quad \lambda_t = \exp(X_t'\beta)$$

The conditional mean is: $E(Y_t \mid X_t) = \lambda_t$

The Poisson regression log-likelihood function is:

$$\sum_{t=1}^{N} [-\exp(X_t'\beta) + Y_t X_t'\beta - \log(Y_t!)]$$

Goodness of fit measures are discussed in Greene [2003, Chapter 21, pp. 741-2]. SHAZAM reports two alternative R^2 measures for the Poisson regression model. Another fit measure is the sum of the deviances defined as:

$$G^2 = 2\sum_{t=1}^{N} Y_t \log\left(Y_t / \exp\left(X_t'\hat{\beta}\right)\right)$$

This is reported as the `POISSON G-SQUARE` on the SHAZAM output. For this statistic, a relatively smaller value indicates a better fit.

The **TYPE=POISSON** option on the **MLE** command gives a linear version where

$$E(Y_t \mid X_t) = \lambda_t = X_t'\beta$$

In this case, the log-likelihood function is:

$$\sum_{t=1}^{N} \left[- X_t'\beta + Y_t \log(X_t'\beta) - \log(Y_t!) \right]$$

Note that this is not defined for values of $X_t'\beta < 0$. For this reason, the model used by **TYPE=EPOISSON** may be preferred to **POISSON**.

MLE *COMMAND OPTIONS*

In general, the format of the **MLE** command is:

MLE *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of desired options. It is possible to specify the distribution of the errors when using the **MLE** command with the **TYPE=** option. When **TYPE=MBETA** is used the command format is:

MLE *depvar indeps (indeps2)* / **TYPE=MBETA NUMERIC** *options*

where *indeps2* is a list of independent variables for the variance function.

Options as described for the **OLS** command that are available are:

ANOVA, **DUMP**, **GF**, **LININV**, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **MAX**, **NOCONSTANT**, **NONORM**, **PCOR**, **PCOV**, **RSTAT**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=**, **TRATIO=** and **WEIGHT=**.

When the **WEIGHT=** option is specified the method explained under the **REPLICATE** option in **OLS** is used.

Options as described for the **NL** command (see the chapter *NONLINEAR REGRESSION*) that are available are:

NUMERIC, **CONV=**, **IN=**, **ITER=**, **OUT=** and **PITER=**.

In addition, the following options are available:

LM Performs a **L**agrange **M**ultiplier test of some models against a less restricted model. If **TYPE=EXP** is used, two **LM** Tests for the **GAMMA** or **WEIBULL** models will be done. If **TYPE=WEIBULL** or **TYPE=GAMMA** is used the **LM** test of a Generalized Gamma distribution is performed.

METHOD= Specifies the nonlinear algorithm to use. The choices are **BFGS** (the default) or **DFP**. These **METHOD**s are described in the chapter *NONLINEAR REGRESSION*.

TYPE= Specifies the **TYPE** of distribution to be assumed for the errors. The available **TYPE**s are **WEIBULL, EWEIBULL, GAMMA, EGAMMA, GG** (Generalized Gamma), **EGG**, **LOGNORM** (Lognormal), **BETA, EBETA, EXP** (Exponential), **EXTREMEV** (Extreme Value Distribution), **MBETA, POISSON** and **EPOISSON**. The default is **TYPE=EXP**. The types **EWEIBULL, EGAMMA, EGG** and **EXTREMEV** are used when the dependent variable is in log form. They correspond to the **WEIBULL, GAMMA, GG,** and **EXP** forms respectively.

The available temporary variables on the **MLE** command are:

$ADR2, $DF, $DW, $ERR, $K, $LLF, $N, $R2, $R2OP, $RAW, $RHO, $SIG2, $SSE, $SSR, $SST, $ZDF, $ZSSR and *$ZSST*.

For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

EXAMPLES

An example of **MLE** output using Theil's textile data and assuming an exponential distribution is:

```
|_MLE CONSUME INCOME PRICE / LM
EXP      REGRESSION      17 OBSERVATIONS
SUM OF DEPVAR=   2286.6     SUM OF LOG DEPVAR=   83.069

  QUASI-NEWTON METHOD USING BFGS UPDATE FORMULA

INITIAL STATISTICS :
TIME =     .440 SEC.   ITER. NO.     1 FUNCTION EVALUATIONS     1
LOG-LIKELIHOOD FUNCTION=    -100.0802
```

```
COEFFICIENTS
   1.061709      -1.382986       130.7066
GRADIENT
  -.1963727E-01  -.3026843E-01  -.2327428E-03

FINAL STATISTICS :
TIME =     .470 SEC.   ITER. NO.     2 FUNCTION EVALUATIONS     2
LOG-LIKELIHOOD FUNCTION=    -100.0797
COEFFICIENTS
   .9188548      -1.348463       142.7181
GRADIENT
  -.1911043E-02  -.1817679E-02  -.1911542E-04

**** LM TEST OF EXP      AGAINST WEIBULL  ****
  LM GRADIENT
  -.1911043E-02  -.1817679E-02  -.1911542E-04  -16.97903
  LM SECOND DERIVATIVES
   10.898
   8.6995       7.2588
   .10593       .84822E-01   .10323E-02
   5.6534       4.3493       .54957E-01   20.051
  CHI-SQUARE =   16.931     WITH 1 D.F.

**** LM TEST OF EXP      AGAINST GAMMA    ****
  LM GRADIENT
  -.1911043E-02  -.1817679E-02  -.1911542E-04  -9.801741
  LM SECOND DERIVATIVES
   10.898
   8.6995       7.2588
   .10593       .84822E-01   .10323E-02
  -.19110E-02  -.18177E-02  -.19115E-04   10.964
  CHI-SQUARE =   8.7628     WITH 1 D.F.

SQUARED CORR. COEF. BETWEEN OBSERVED AND PREDICTED    .95056

 R-SQUARE =    .9499     R-SQUARE ADJUSTED =    .9428
VARIANCE OF THE ESTIMATE-SIGMA**2 =   26.204
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.1190
SUM OF SQUARED ERRORS-SSE=   445.46
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION =  100.080

                           ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     .91885      5.996       .1532      .878  .041      .2066      .7035
PRICE     -1.3485      1.862      -.7241      .469 -.190     -.9646     -.7650
CONSTANT   142.72      642.6       .2221      .824  .059      .0000     1.0611
```

Note that if one simply wished to estimate the parameters of a univariate distribution rather than a full regression model it could be easily done by excluding all independent variables so that only the *CONSTANT* would be estimated as in:

```
mle consume / lm
```

22. NONLINEAR REGRESSION

"It may be safely asserted...that population, when unchecked, increases in geometrical progression of such a nature to double itself every twenty-five years."

Thomas Malthus
British Economist, 1830

The **NL** command provides general features for the estimation of nonlinear models. The model specification can be a single equation or a system of equations and estimation with autoregressive errors is available. A system of nonlinear simultaneous equations can also be estimated by Nonlinear Three Stage Least Squares (N3SLS) or by Generalized Method of Moments (GMM).

The **NL** command also has options for the estimation of a general nonlinear function. For example, the **LOGDEN** option can be used for estimation with non-normal errors (also see the chapter *MAXIMUM LIKELIHOOD ESTIMATION OF NON-NORMAL MODELS)*. The **MINFUNC** and **MAXFUNC** options can be used to minimize or maximize simple functions and the **SOLVE** option can be used to solve a set of nonlinear simultaneous equations.

The estimation of nonlinear models requires the use of a numerical optimization algorithm. SHAZAM uses a quasi-Newton method also known as a variable metric method (see Judge et al. [1985, pp.958-960]). Each updating step of the algorithm requires a gradient (first derivative) estimate and SHAZAM provides for exact evaluation of the gradient. If exact derivatives cannot be computed then SHAZAM will use a numerical approximation to obtain the gradient and a message will indicate that numerical derivatives are used. Each updating step also requires an approximation of the Hessian (second derivatives). The quasi-Newton family of algorithms obtains a Hessian inverse approximation in each iteration by an updating scheme that involves adding a correction matrix. At model convergence this approximation is then used as the covariance matrix estimate of the estimated parameters.

NOTE: Users should be familiar with nonlinear estimation before attempting this procedure. The *Nonlinear Least Squares by the Rank One Correction Method* example in the *PROGRAMMING IN SHAZAM* chapter shows a simple updating algorithm. Some basic information can be found in Maddala [1977]. A more rigorous treatment of nonlinear estimation can be found in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 6, Appendix B]; Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 12]; and in Amemiya

[1983] and Gallant [1987]. A procedure for testing for autocorrelation in nonlinear models is described in White [1992].

Users experienced with nonlinear estimation will be aware that there is no guarantee that the model will converge. If it does, convergence to a local rather than a global maximum is likely. For this reason, the model should always be re-estimated with different starting values to verify that the global maximum has probably been achieved. Since the computational time required for nonlinear estimation can be extremely high it is often useful to attempt to get good starting values by first estimating a linear simplification of the model.

Simulated Annealing

In addition to the traditional gradient methods offered by NL the SHAZAM also offers simulated annealing as a method to find global minima for nonlinear regression problems. This approach to regression is a type of Monte-Carlo method, which can be useful for finding solutions to problems where more traditional methods fail to find global minima. Simulated annealing can be used in SHAZAM as a standalone method and also as a starting point finder as hybrid approach to get a 'reasonable' set of starting conditions for the more traditional gradient methods.

Simulated annealing was originally developed by analogy to the physical process of annealing, whereby a heated piece of metal can have very different physical properties depending on the rate of cooling. As such a key variable involved in simulated annealing algorithm is the temperature variable. A difficulty with some simulated annealing algorithms is estimating a good starting temperature. SHAZAM automatically estimate the starting temperature using the approach outline by Goffe et al [1994].

There are a number of benefits to using simulated annealing for certain classes of problem, which the traditional methods can struggle to find appropriate solutions. That said, the approach is by no means a panacea. Like any optimization routine it is not guaranteed to find a global convergent solution to a given problem. Moreover, even for problems where a solution is found, the method can be significantly more computationally expensive, than a traditional gradient method approach.

As with other forms of the nonlinear optimization, users should be familiar with simulated annealing before attempting this procedure, Goffe et al [1994] give an extensive investigation of the algorithm implemented in SHAZAM.

NONLINEAR MODEL SPECIFICATION

To set up a nonlinear model in SHAZAM, it is necessary to tell SHAZAM some basic information as well as give **EQ** commands indicating the form of each equation in the model. It is very important for users to give SHAZAM good starting values of the coefficients. The desired starting values should be specified on **COEF** commands or placed in a vector and specified with the **START=** option on the **NL** command.

In general, the format for nonlinear estimation is:

NL *neq* / **NCOEF=** *options*
EQ *equation*
. . .
EQ *equation*
COEF *coef1 value1 coef2 value2* . . .
END

where *neq* is the number of equations and *options* is a list of desired options.

For estimation by Nonlinear Two-Stage Least Squares (N2SLS) or Nonlinear Three-Stage Least Squares (N3SLS) the **NL** command has the general form:

NL *neq exogs* / **NCOEF=** *options*

where *exogs* is a list of instrumental variables. SHAZAM automatically includes a constant in the list of instrumental variables unless the **NOCONEXOG** option is used.

For generalized method of moments (GMM) estimation the **NL** command has the general form:

NL *neq exogs* / **NCOEF=** **GMM=** *options*

One **EQ** command is required for every equation in the model. The **EQ** command can be continued on additional lines if there is an ampersand (&) typed at the end of the line to be continued. An equation with continuation lines may contain a total of 16,384 characters. **EQ** commands are similar to **GENR** commands. If functions like LOG( ) and EXP( ) are used then SHAZAM is able to compute exact derivatives. However, for some other

functions SHAZAM will automatically impose the **NUMERIC** option. If the equation has square root terms then to enable analytic derivatives write the terms in the form (*expression*)**.5 and not SQRT(*expression*). SHAZAM will assume that anything in the equation that has not already been defined as a variable will be a coefficient to estimate. Coefficients that appear in one equation may also appear in other equations.

RESTRICT commands are not permitted. Parameter restrictions can be incorporated directly in the **EQ** command. No forecasting options are available. **TEST** commands may follow the **END** command. For a discussion about linear and non-linear hypothesis testing see the chapter *HYPOTHESIS TESTING*.

NL *Command Options*

Options as defined for the **OLS** command that are available are:

LIST, **PCOV**, **RSTAT**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=**, and **TRATIO=**

Additional options are:

ACROSS Estimates the seemingly unrelated regressions model with vector autoregressive errors. This admits autocorrelation **ACROSS** equations as well as within a single equation. This option can be computationally slow. More details are in the section *ESTIMATION WITH AUTOREGRESSIVE ERRORS* later in this chapter.

AUTO Estimates the model with **AUTO**regressive errors. The order is specified with the **ORDER=** option and the default is first order autoregressive (AR(1)) errors. More details are in the section *ESTIMATION WITH AUTOREGRESSIVE ERRORS* later in this chapter.

DN Estimates the error variance $\hat{\sigma}^2$ by **D**ividing the residual sum of squares by **N** instead of N–K. This option is the default. Use **NODN** to divide by N-K.

DRHO Normally, when the **AUTO** option is specified SHAZAM gives the same value of ρ to each equation. With the **DRHO** option a **D**ifferent value of **RHO** is given to each equation. When the order of autocorrelation is specified as higher than one on the **ORDER=** option the **DRHO** option will also give more than one ρ for each equation.

DUMP **DUMP**s the internal code that SHAZAM has generated for the **EQ** commands. This option is only useful for SHAZAM consultants. If **DUMP** is specified with **EVAL** there will be a large amount of output.

EVAL **EVAL**uates the likelihood function for the starting values and prints out the answer. If **ITER=0** is also specified no estimation will be done. This is useful for experimentation purposes. If **EVAL** and **DUMP** are specified all the data in the nonlinear system is dumped along with the computed residuals and derivatives of the function with respect to all parameters. First, the data for each observation will be printed. Then the residuals for each equation and the derivatives of each equation with respect to each parameter will be printed, and finally, the derivatives for the equations will be printed consecutively. This option may not be used with the **NUMERIC** option.

GENRVAR Takes the vector of coefficients and generates a set of scalar variables using the same names as those used for the coefficients on the **EQ** command. These scalar variables can then be used for the rest of the SHAZAM run. This is an alternative to the **COEF=** method for saving the coefficients.
NOTE: A large number of variables may need to be generated if the model is large. The coefficient names used may not be used on any **EQ** command later in the same run. Since the coefficients are now variables **TEST** commands will no longer work.

LOGDEN Used to tell SHAZAM that the equation given on the **EQ** command is the **LOG-DEN**sity for a single observation rather than a regression equation. SHAZAM will then compute a complete likelihood function by summing the log-densities. This option allows maximum likelihood estimation of a large variety of functions. An example (available at the SHAZAM website) is in Chapter 12.3 of the *Judge Handbook.* Another example is in *Multinomial Logit Models* in the chapter *PROGRAMMING IN SHAZAM.* With the **LOGDEN** option, the **RESID=** option saves the log-densities evaluated at the final parameter estimates in the variable specified.

MAXFUNC Used to tell SHAZAM that the equation given on the **EQ** command is a function (such as a log-likelihood function) to be maximized rather than a regression equation. SHAZAM will find the values of the parameters that maximize the function. The **SAMPLE** command should be set to

include only one observation. An example of the use of this option is given later in this chapter.

MINFUNC Used to tell SHAZAM that the equation given on the **EQ** command is a function to be minimized rather than a regression equation. SHAZAM will then find the values of the parameters that minimize the function. The **SAMPLE** command should be set to include only one observation.

NOCONEXOG If a list of exogenous variables is included for either Nonlinear Two or Three Stage Least Squares SHAZAM will automatically add a *CONSTANT* to the list. If you do not want SHAZAM to automatically include a constant in the list of exogenous variables, specify the **NOCONEXOG** option.

NOPSIGMA Suppresses printing of the sigma matrix from systems estimation.

NUMCOV Uses numeric differences to compute the covariance matrix after estimation. If this option is NOT specified SHAZAM uses a method based on the Davidon-Fletcher-Powell algorithm which builds up the covariance matrix after many iterations. This method may not be accurate if the model only runs for a small number of iterations. The numeric method is more expensive and also may not necessarily be accurate. The differential to be used in numeric differences can be controlled with the **STEPSIZE=** option.

NUMERIC Uses the **NUMERIC** difference method to compute derivatives in the algorithm. SHAZAM normally computes analytic derivatives which are more accurate. However, in some models with many equations and parameters, considerable savings in required memory will result if the **NUMERIC** option is used to compute numeric derivatives. In some cases the **NUMERIC** option may even be faster. For large models SHAZAM may automatically switch to numeric derivatives. If this happens then analytic derivatives can be forced with the **NONUMERIC** option (this will not be effective if functions other than LOG() and EXP() are used in the model).

OPGCOV Uses the outer-product of the Gradient method to compute the covariance matrix. It is not valid with the **NUMERIC** option.

PCOV **P**rints an estimate of the **COV**ariance matrix of coefficients after convergence. This estimate is based on an estimate of the Hessian which SHAZAM computes internally. SHAZAM estimates the Hessian by building it up after repeated iterations. Therefore, if the model converges immediately, SHAZAM will have a very poor estimate of the Hessian or none at all. In this case, the covariance matrix will just be an identity matrix. If the **NUMCOV** or **OPGCOV** options are used the estimated covariance matrix is computed using alternate methods.

SAME Runs the previous **NL** regression without repeating the **EQ** commands. This should only be used in TALK mode at a terminal.

SOLVE Used to tell SHAZAM that the equations given on the **EQ** commands are to be solved as a set of nonlinear simultaneous equations. There should be one equation for each coefficient as specified by the **NCOEF=** option. An example is shown in *Solving Nonlinear Sets of Equations* in the chapter *PROGRAMMING IN SHAZAM*. The **SAMPLE** should be set to include only one observation.

AUTCOV= Specifies the lag length to be used in computing the weighting matrix for the **GMM=** option. If this option is not specified automatic formulas are used.

CONV= Specifies the **CONV**ergence criterion for the coefficients. This value will be multiplied by each coefficient starting value to compute the convergence condition for each coefficient. The default is **CONV=.00001**. Let $\tilde{\beta}^{(i)}$ be the parameter estimates at iteration i and let δ be the value set with **CONV=**. The iterations stop when:

$$\left|\tilde{\beta}_k^{(i)} - \tilde{\beta}_k^{(i-1)}\right| < \alpha_k \cdot \delta \quad \text{for all k;} \quad \text{where } \alpha_k = \begin{cases} \left|\tilde{\beta}_k^{(0)}\right| & \text{for } \tilde{\beta}_k^{(0)} \neq 0 \\ 0.1 & \text{for } \tilde{\beta}_k^{(0)} = 0 \end{cases}$$

GMM= Specifies the weighting matrix to use for Generalized Method of Moments Estimation. If a matrix is provided it should be a symmetric matrix conforming to the dimensions in the equations described in the section *GENERALIZED METHOD OF MOMENTS ESTIMATION* in this chapter. Alternatively, SHAZAM will automatically compute the matrix corresponding to the keywords **HETCOV**, **BARTLETT**, **TRUNC**, **QS**, **PARZEN** or **TUKEY**. The option **GMM=IDENTITY** gives estimation by

N2SLS or N3SLS. See also the **AUTCOV=** option when using the **GMM=** option. If you don't know what you are doing and want to use GMM anyway, you should probably use either **HETCOV** or **BARTLETT**. The identity matrix is likely not a good choice for the weighting matrix.

GMMOUT= Saves the weighting matrix in the variable specified. This option is available with **GMM=HETCOV**, **BARTLETT**, **TRUNC**, **QS**, **PARZEN** or **TUKEY**.

IN=*unit* Reads back the values of the coefficients and log-likelihood function that were saved with the **OUT=** option. This option is only useful when there is something to **IN**put from a previous run. This option may be combined with the **OUT=** option to insure that the **IN=** file always contains the values of the coefficients from the most recent iteration. The **COEF** command should only be used with the **IN=** option if the starting values of some of the coefficients are to be modified. When both **OUT=** and **IN=** are used the same unit number is usually used. A binary file should be assigned to the unit with the SHAZAM **FILE** command or an operating system command.

ITER= Specifies the maximum number of **ITER**ations. The default is 100.

METHOD= Specifies the nonlinear algorithm to use for estimation. The default is a Davidon-Fletcher-Powell algorithm. An alternative **METHOD=BFGS**, Broyden-Fletcher-Goldfarb-Shanno (BFGS), is described in Belsley [1980]. Another alternative is a slightly different D-F-P algorithm which can be obtained with **METHOD=DFP**. Simulated Annealing is specified via **METHOD=SA**, which is described in Goffe et al [1994]

HYBRID Specifies that Simulated Annealing should be used to estimate a set of starting points and subsequently the method specified in **METHOD=** should be used to find the solution to the problem.

SAITER= Specifies the maximum number of function trials the Simulated Annealing algorithm will evaluate before returning. The default value of this parameter is 1,000,000 and usually the solution is found in less iterations than this. Requires **METHOD=SA** or **HYBRID.**

SACONV= Specifies the convergence criterion of the Simulated Annealing algorithm. The default value of this parameter is 0.0001 for pure

Simulated Annealing and 0.1 for hybrid options. Requires **METHOD=SA** or **HYBRID.**

SAUPPER= A vector used to specify upper limits on the range of allowed coefficients. The default value is a vector with all elements equal to 10. Requires **METHOD=SA** or **HYBRID.**

SALOWER= A vector used to specify lower limits on the range of allowed coefficients. The default value is a vector with all elements equal to -10. Requires **METHOD=SA** or **HYBRID.**

SANEPS= Specifies how many function evaluations should be used to determine convergence of when performing Simulated Annealing. The default value is 4. If SANEPS evaluations lie within SACONV of each other the Simulated Annealing algorithm will terminate. Requires **METHOD=SA** or **HYBRID.**

SANS= Determines the number of cycles the Simulated Annealing algorithm iterates through. The algorithm adjusts an internal vector so that approximately half of the function evaluations lie within range. This calculation is done after NCOEF*SANS evaluations. The default value is 20. Requires **METHOD=SA** or **HYBRID.**

SANT= Determines how many evaluations that should be used by Simulated Annealing before reducing the, internal, temperature variable. After NCOEF*SANS*SANT function evaluations the temperature is reduced. The default value is 100. Requires **METHOD=SA** or **HYBRID.**

SATRF= A positive value, between 0 and 1, that determines how slowly simulated annealing reduces the internal temperature variable. The default is 0.85. Requires **METHOD=SA** or **HYBRID.**

SAUPFAC= A factor that can be used to specify the upper bounds array in terms of a factor applied to the initial conditions vector. This factor is over ridden by the use of an SAUPPER vector. The default value is 10. Requires **METHOD=SA** or **HYBRID.**

SALOWFAC= A factor that is can be used to specify the lower bounds array in terms of a factor applied to the initial conditions vector. This factor is over

ridden by the use of an SALOWER vector. The default is -10. Requires **METHOD=SA or HYBRID.**

NCOEF= Specifies the **N**umber of different **COEF**ficients to be estimated. This option is required.

ORDER= Specifies the **ORDER** of autocorrelation to be corrected when the **AUTO** option is used. The default is **ORDER=1**.

OUT=*unit* Writes **OUT** on the unit specified the values of the coefficients and log-likelihood function after each iteration. This is quite useful for restarting the model in another run with the **IN=** option described above. When this option is used, a file must be assigned to the output unit as described in the chapter *DATA INPUT AND OUTPUT*. The values will be written in double precision (binary) all on one line. Units 11-49 are available for use.

PITER= Specifies the frequency with which **ITER**ations will be **P**rinted in the output. The default **PITER=15** indicates that one out of every 15 iterations will be printed.

SIGMA= Saves the sigma matrix from systems estimation in the variable specified. For single equation estimation the estimate of σ^2 is saved.

START= Uses the values in the specified variable as starting values for the estimation. The order of the parameters should be the same as normally printed by the SHAZAM **NL** command, namely, the order that they appear on the **EQ** commands (followed by autocorrelation coefficients when the **AUTO** option is used). In some cases this may be an easier way to input starting values than by using the **COEF** command. Be careful to make sure that the length of the **START=** vector is equal to the number of coefficients specified with the **NCOEF=** option (plus the number of autocorrelation coefficients if any).

STEPSIZE= Specifies the stepsize to use with the **NUMCOV** and **NUMERIC** options to control the differential in numeric derivatives. The default is **STEPSIZE=1E-4**. The calculated covariance matrix may be very sensitive to this value.

ZMATRIX= Specifies a matrix to create and use to store the derivatives of the nonlinear function with respect to each parameter. This option should be used only when there is only one equation that is estimated. The option is not valid if the **NUMERIC** option is also used. The Z matrix is described in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Equation 6.2.3].

After estimation the temporary variables as described for the **OLS** command are *$ERR, $K, $LLF,* and *$N*. For the **NL** command, the temporary variable *$LLF* is used to store the function value. This will be either the value of the log-likelihood function or the minimized function value. For single equation estimation, available temporary variables are *$SSE* and *$SIG2*, and when the **RSTAT** option is used additional temporary variables are *$R2OP* and *$RHO*.

NONLINEAR LEAST SQUARES

The nonlinear equation with additive errors has the general form:

$$Y_t = f(X_t, \beta) + \varepsilon_t \qquad \text{for } t = 1, \ldots, N$$

The residual sum of squares is: $S(\beta) = \sum_{t=1}^{N} [Y_t - f(X_t, \beta)]^2$

With $\varepsilon \sim N(0, \sigma^2 I_N)$ the maximum likelihood estimator for σ^2 is $\tilde{\sigma}^2 = S(\beta)/N$ and the maximum likelihood estimator is the value of β that maximizes the concentrated log-likelihood function (see Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Equation 12.2.85]):

$$L(\beta) = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln\left(\frac{S(\beta)}{N}\right) - \frac{N}{2}$$

When the errors are normally distributed the maximum likelihood estimator is identical to the nonlinear least squares estimator which globally minimizes S(β).

The estimates have an interpretation as estimates from a linearized model that is constructed from a Taylor series approximation. Define the matrix of first derivatives evaluated at the converged estimates $\tilde{\beta}$ as:

$$Z(\tilde{\beta}) = \left.\frac{\partial f(X,\beta)}{\partial \beta}\right|_{\tilde{\beta}}$$

(The Z matrix can be saved with the **ZMATRIX=** option on the **NL** command). The linear pseudomodel (see Judge et al. [1988, Equation 12.2.14]) is:

$$\overline{Y}(\tilde{\beta}) = Z(\tilde{\beta})\beta + \varepsilon \qquad \text{where} \qquad \overline{Y}(\tilde{\beta}) = Y - f(X,\tilde{\beta}) + Z(\tilde{\beta})\tilde{\beta}$$

An OLS regression of $\overline{Y}(\tilde{\beta})$ on $Z(\tilde{\beta})$ will reproduce the parameter estimate $\tilde{\beta}$.

The example below uses the **NL** command to estimate a demand equation with the Theil textile data. The model has one equation and three coefficients.

```
|_NL 1 / NCOEF=3
...NOTE..SAMPLE RANGE SET TO:     1,    17
|_EQ CONSUME=A+B*INCOME+C*PRICE
|_COEF  B 1  C  -1  A 50
   3 VARIABLES IN  1 EQUATIONS WITH   3 COEFFICIENTS
        17 OBSERVATIONS

COEFFICIENT STARTING VALUES
A          50.000      B          1.0000      C         -1.0000
      100 MAXIMUM ITERATIONS, CONVERGENCE =  .000010

INITIAL STATISTICS :
TIME =       .090 SEC.   ITER. NO.     0   FUNCT. EVALUATIONS     1
LOG-LIKELIHOOD FUNCTION=   -93.26261
COEFFICIENTS
   50.00000       1.000000      -1.000000
GRADIENT
   .2884071       29.68023       21.50215

INTERMEDIATE STATISTICS :
TIME =       .150 SEC.   ITER. NO.    15   FUNCT. EVALUATIONS    26
LOG-LIKELIHOOD FUNCTION=   -54.73018
COEFFICIENTS
   70.36305       1.643421      -1.360359
GRADIENT
  -.5982383      -68.90536      -52.52647

FINAL STATISTICS :
TIME =       .180 SEC.   ITER. NO.    26   FUNCT. EVALUATIONS    37
LOG-LIKELIHOOD FUNCTION=   -51.64706
COEFFICIENTS
   130.7066       1.061709      -1.382986
GRADIENT
   .6334375E-07   .6693914E-05   .5618457E-05

MAXIMUM LIKELIHOOD ESTIMATE OF SIGMA-SQUARED =   25.489
GTRANSPOSE*INVERSE(H)*G  STATISTIC  -  =   .10715E-13

      COEFFICIENT    ST. ERROR   T-RATIO
```

```
A          130.71       24.827       5.2647
B          1.0617       .24390       4.3531
C         -1.3830       .76344E-01 -18.115
|_END
```

Note that the **EQ** command supplies names for the coefficients to be estimated. In the above example, the coefficients are A, B and C, the variables are *CONSUME, INCOME* and *PRICE*. The **COEF** command immediately follows the **EQ** command and specifies starting values for the coefficients. An **END** command should follow the **COEF** command. Notice that the estimation results for this example are identical to that illustrated in the chapter *ORDINARY LEAST SQUARES*. In the case of a linear equation, the **NL** command gives the same estimated coefficients as the **OLS** command, but the computational time required to run **NL** regressions is much higher.

A sufficient number of starting values must be included for all coefficients or SHAZAM will not run the estimation. If the **COEF** command is omitted and no starting values are assigned then SHAZAM uses a starting value of 1.0 for all coefficients.

In the above example the starting values are specified with the command:

```
coef  b 1  c  -1  a 50
```

If the coefficient names are not specified on the **COEF** command, SHAZAM assumes that the starting values appear in the same order as they appear on the **EQ** command. In the above example the coefficients on the **EQ** command appear in the order A, B and C. So an alternative way of entering starting values is with:

```
coef
 50  1  -1
```

Testing for Autocorrelation

In a nonlinear model it is often desirable to use the Durbin-Watson statistic to test for autocorrelation. Following the method in White [1992] it is easy to approximate the exact distribution of the Durbin-Watson statistic using SHAZAM. In that article a reference was made to SHAZAM code to perform this test. Since the *Review of Economics and Statistics* does not like to print computer code, the method is shown here in the context of estimation of a CES (constant elasticity of substitution) production function. The form of the production function is:

$$Q = \alpha[\delta L^{-\rho} + (1-\delta)K^{-\rho}]^{-\eta/\rho} \qquad (\alpha > 0;\ 0 < \delta < 1;\ \rho > -1;\ \rho \neq 0,\ \text{and}\ \eta > 0)$$

where Q is output, and L and K represent two factors of production. The statistical model can be expressed as:

$$\log(Q) = \gamma - \frac{\eta}{\rho}\log[\delta L^{-\rho} + (1-\delta)K^{-\rho}] + \varepsilon$$

where $\gamma = \log(\alpha)$ and ε is a random error term. The CES function is discussed and a data set is provided in Griffiths, Hill and Judge [1993, Chapter 22]. The next list of SHAZAM commands sets up the nonlinear estimation and then obtains a p-value for the Durbin-Watson test statistic.

```
sample 1 30
read(table22.4) l k q
genr logq=log(q)

* Estimate the CES production function
nl 1 / ncoef=4 pcov zmatrix=z coef=beta predict=yhat
eq logq=gamma-(eta/rho)*log(delta*l**(-rho)+(1-delta)*k**(-rho))
coef  rho 1  delta .5  gamma 1  eta 1
end

* Estimate the elasticity of substitution
test 1/(1+rho)

* Generate the linear pseudomodel and compute the DURBIN-WATSON p-
value
matrix ybar=logq-yhat+z*beta
ols ybar z / noconstant dwpvalue
```

The option **NCOEF=4** on the **NL** command specifies that there are 4 coefficients to estimate. The coefficients are GAMMA, ETA, RHO and DELTA. The **COEF** command immediately follows the **EQ** command and gives the starting values for the iterative estimation. The Durbin-Watson p-value printed by the final **OLS** command above can be used with the Durbin-Watson statistic so it is not necessary to try to apply Durbin-Watson tables to a nonlinear problem.

MAXIMIZING A FUNCTION

This example is discussed in Greene [2003, p. 943]. The problem is to maximize a function of a single variable: $f(\theta) = \ln\theta - 0.1\cdot\theta^2$. This problem can be solved by using the

MAXFUNC option on the **NL** command. The SHAZAM commands that follow specify a starting value of $\theta = 5$ for the iterative procedure.

```
sample 1 1
nl 1 / ncoef=1 maxfunc
eq log(theta)-0.1*theta**2
coef theta 5
end
```

NONLINEAR SEEMINGLY UNRELATED REGRESSION

A set of M nonlinear equations can be written as:

$$Y_i = f_i(X, \beta) + \varepsilon_i \qquad \text{for } i = 1, \ldots, M$$

Note that the inclusion of the matrix X and the coefficient vector β in all equations allows for common explanatory variables and coefficients across equations. It is assumed that there is contemporaneous correlation between errors in different equations. Let S be the M x M matrix with $(i, j)^{th}$ element equal to:

$$\varepsilon_i' \varepsilon_j = [Y_i - f_i(X, \beta)]'[Y_j - f_j(X, \beta)]$$

With the assumption that the errors have a multivariate normal distribution the maximum likelihood estimator for β is obtained by maximizing the concentrated log-likelihood function (see Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Section 12.4.2]):

$$L(\beta) = -\frac{NM}{2}\ln(2\pi) - \frac{N}{2}\ln(|S/N|) - \frac{NM}{2}$$

The maximum likelihood estimator is the value of β that minimizes $|S|$.

The next example shows how to set up a nonlinear estimation of the linear expenditure system as discussed in Judge et al. [1988, Section 12.4.3]. In this system it is assumed that consumer income *Y* is divided between 3 goods *Q1, Q2* and *Q3* whose prices are *P1, P2* and *P3* respectively. The coefficients to be estimated are the marginal budget shares (B1, B2, B3) along with the subsistence quantities (G1, G2, G3). Since this is a complete system of demand equations, it is well known that only 2 of the 3 equations need to be estimated (B3=1–B1–B2). Thus, there are 2 equations and 5 coefficients to estimate. The SHAZAM commands might look like:

```
sample 1 30
read (table11.3) p1 p2 p3 y q1 q2 q3
genr pq1=p1*q1
genr pq2=p2*q2
nl 2 / ncoef=5 pcov
eq pq1=p1*g1+b1*(y-p1*g1-p2*g2-p3*g3)
eq pq2=p2*g2+b2*(y-p1*g1-p2*g2-p3*g3)
coef g1 2.903 g2 1.36 g3 13.251 b1 .20267 b2 .13429
end

* Compute B3
test 1-b1-b2
```

It is important to remember to eliminate 1 equation from the model when estimating systems of demand equations so that the system is not overdetermined.

ESTIMATION WITH AUTOREGRESSIVE ERRORS

The estimation of models with autoregressive errors is implemented with the **AUTO** option on the **NL** command. The estimation algorithm uses numeric derivatives and so the **NUMERIC** option is automatically set.

Single Equation Estimation

The nonlinear equation with errors that follow an autoregressive process of order p (AR(p) errors) has the form:

$$Y_t = f(X_t,\beta) + \varepsilon_t \qquad \text{with} \qquad \varepsilon_t = \sum_{k=1}^{p} \rho_k \varepsilon_{t-k} + v_t$$

Parameter estimates are obtained by minimizing the objective function:

$$S(\beta,\rho) = \sum_{t=p+1}^{N} v_t^2 = \sum_{t=p+1}^{N} \left[(Y_t - f(X_t,\beta)) - \sum_{k=1}^{p} \rho_k (Y_{t-k} - f(X_{t-k},\beta)) \right]^2$$

Note that initial observations are dropped. The method is a variation of the method described in Pagan [1974] and implemented for linear models with the **PAGAN** option on the **AUTO** command (see the chapter *AUTOCORRELATION MODELS*). Note that the Pagan method sets the pre-sample residuals to zero.

The number of coefficients to estimate is K+p where K must be specified with the **NCOEF=** option and p is specified with the **ORDER=** option. Starting values can be requested with the **START=** option and values must be specified for the K coefficients in β followed by the p autocorrelation coefficients.

Nonlinear Seemingly Unrelated Regression Estimation

For the SUR model with M equations let $\varepsilon_{(t)}$ denote the vector of M disturbances for observation t and assume an AR(p) process represented as:

$$\varepsilon_{(t)} = \sum_{k=1}^{p} R_k \varepsilon_{(t-k)} + v_{(t)}$$

where the R_k are M x M matrices of autocorrelation coefficients. Further discussion is in Judge, Griffiths, Hill, Lütkepohl, and Lee [1985, Section 12.3].

The default estimation method is to assume that $R_k = \rho_k I_M$. If the **DRHO** option is used then the assumption is that R_k is a diagonal matrix. The **ACROSS** option implements the general case of a vector autoregressive model with no restrictions on the R_k matrix. When the **ACROSS** option is specified the autocorrelation coefficients are printed in column order (VEC(R_k)).

Let S be the M x M matrix with $(i,j)^{th}$ element equal to: $\sum_{t=p+1}^{N} v_{it} v_{jt}$

Parameter estimates are obtained by minimizing $|S|$.

NONLINEAR TWO-STAGE LEAST SQUARES (N2SLS)

Consider a model of the form: $Y = f(Z,\beta) + \varepsilon$

where Y is an N x 1 vector of observations on the dependent variable, Z is the matrix of right-hand side variables in the equation, β is a p x 1 vector of unknown parameters and ε is a random error vector. With X as a matrix of instrumental variables (usually all the exogenous variables in the system) the parameter estimates are obtained by minimizing the objective function:

$\varepsilon' X (X'X)^{-1} X'\varepsilon$ where $\varepsilon = Y - f(Z,\beta)$

With estimated coefficients $\hat{\beta}$ the estimated N2SLS residuals are obtained as:

$$e = Y - f(Z, \hat{\beta})$$

The estimated covariance matrix of $\hat{\beta}$ is:

$$\hat{\sigma}^2 (g'X (X'X)^{-1}X'g)^{-1} \qquad \text{where} \qquad \hat{\sigma}^2 = e'e / N$$

and g is the N x p matrix of derivatives $\partial f(Z,\beta)/\partial\beta$ evaluated at $\hat{\beta}$.

An example of N2SLS using the first equation of the Klein Model described in the chapter *TWO STAGE LEAST SQUARES AND SYSTEMS OF EQUATIONS* follows. To implement N2SLS with the **NL** command the user must specify a set of instrumental variables to be used. In the example the list of instrumental variables is given as *W2, T, G, TIME1, PLAG, KLAG* and *XLAG.* In addition, SHAZAM automatically includes a constant in the list of instrumental variables.

```
|_* NONLINEAR TWO STAGE LEAST SQUARES
|_NL 1 W2 T G TIME1 PLAG KLAG XLAG / NCOEF=4
...NOTE..SAMPLE RANGE SET TO:      1,     21
|_EQ C=A1*PLAG+A2*P+A3*W1W2+A0
|_END
  11 VARIABLES IN  1 EQUATIONS WITH   4 COEFFICIENTS
NONLINEAR TWO-STAGE LEAST SQUARES: USING   8 INSTRUMENTAL EXOGENOUS VARIABLES
        21 OBSERVATIONS

COEFFICIENT STARTING VALUES
A1         1.0000      A2         1.0000      A3         1.0000
A0         1.0000
      100 MAXIMUM ITERATIONS, CONVERGENCE =  .000010

INITIAL STATISTICS :
TIME =       .130 SEC.   ITER. NO.     0   FUNCT. EVALUATIONS     1
FUNCTION VALUE=   11111.95     FUNCTION VALUE/N =   529.1406
COEFFICIENTS
   1.000000       1.000000       1.000000       1.000000
GRADIENT
   16117.80       16557.11       39581.22       913.6000

FINAL STATISTICS :
TIME =       .250 SEC.   ITER. NO.     9   FUNCT. EVALUATIONS    13
FUNCTION VALUE=   9.157975     FUNCTION VALUE/N =   .4360940
COEFFICIENTS
   .2162340       .1730222E-01   .8101827       16.55476
GRADIENT
   .1799183E-04   .1988876E-04   .4711613E-04   .1456845E-05

MAXIMUM LIKELIHOOD ESTIMATE OF SIGMA-SQUARED =   1.0441
GTRANSPOSE*INVERSE(H)*G  STATISTIC  -  =   .29445E-12
```

```
       COEFFICIENT     ST. ERROR    T-RATIO
A1         .21623       .10727       2.0158
A2         .17302E-01   .11805        .14657
A3         .81018       .40250E-01   20.129
A0        16.555        1.3208       12.534
|_END
```

N2SLS Estimation with Autoregressive Errors

If the **AUTO** option is used for estimation with autoregressive errors then an appropriate set of instrumental variables must be specified in the *exogs* list on the **NL** command (see, for example, the discussion in Greene [2000, pp. 688-9]). For the model with AR(1) errors a choice of instrumental variables may be: Y_{t-1}, X_t and X_{t-1}.

NONLINEAR THREE STAGE LEAST SQUARES (N3SLS)

Now consider a system of M equations such that equation i has the general form:

$$f_i(Y, X, \beta) = \varepsilon_i \qquad \text{for} \quad i = 1, \ldots, M$$

The vector β has p parameters. The error covariances are given by $E(\varepsilon_i \varepsilon_j') = \sigma_{ij} I_N$. Denote Σ as the M x M matrix with individual elements σ_{ij} and stack the ε_i vectors to obtain an MN x 1 vector ε. The N3SLS estimator is obtained by minimizing the objective function:

$$\varepsilon' \, [\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X'] \, \varepsilon$$

where $\hat{\Sigma}$ is constructed from the N2SLS residuals. With the estimated N2SLS residuals for equation i in the N x 1 vector e_i the individual elements of $\hat{\Sigma}$ are obtained as $\hat{\sigma}_{ij} = e_i' e_j / N$. The covariance matrix of coefficients is estimated as:

$$[G'(\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X') \, G]^{-1} \qquad \text{where} \qquad G = \begin{bmatrix} g_1 \\ | \\ . \\ | \\ g_M \end{bmatrix}$$

The g_i matrix is N x p and it contains the partial derivatives of $f_i(Y, X, \beta)$ with respect to β evaluated at the parameter estimates.

SHAZAM output for N3SLS estimation of the Klein Model is:

```
|_* NONLINEAR THREE STAGE LEAST SQUARES
|_NL 3 W2 T G TIME1 PLAG KLAG XLAG / NCOEF=12 PITER=50
...NOTE..SAMPLE RANGE SET TO:      1,     21
```

```
|_EQ C=A1*PLAG+A2*P+A3*W1W2+A0
|_EQ I=B1*PLAG+B2*KLAG+B3*P+B0
|_EQ W1=C1*TIME1+C2*XLAG+C3*X+C0
|_END
  14 VARIABLES IN  3 EQUATIONS WITH  12 COEFFICIENTS
NONLINEAR TWO-STAGE LEAST SQUARES: USING   8 INSTRUMENTAL EXOGENOUS VARIABLES
        21 OBSERVATIONS

COEFFICIENT STARTING VALUES
A1         1.0000      A2         1.0000      A3         1.0000
A0         1.0000      B1         1.0000      B2         1.0000
B3         1.0000      B0         1.0000      C1         1.0000
C2         1.0000      C3         1.0000      C0         1.0000
      100 MAXIMUM ITERATIONS, CONVERGENCE =  .000010

INITIAL STATISTICS :
TIME =       .090 SEC.   ITER. NO.     0   FUNCT. EVALUATIONS     1
FUNCTION VALUE=   1348508.     FUNCTION VALUE/N =   64214.68
COEFFICIENTS
   1.000000       1.000000       1.000000       1.000000       1.000000
   1.000000       1.000000       1.000000       1.000000       1.000000
   1.000000       1.000000
GRADIENT
   16117.80       16557.11       39581.22       913.6000       161577.1
   1970580.       165951.4       9806.800       46201.60       233550.0
   242535.3       3934.600

FINAL STATISTICS :
TIME =      2.130 SEC.   ITER. NO.    19   FUNCT. EVALUATIONS    31
FUNCTION VALUE=   17.62146     FUNCTION VALUE/N =   .8391173
COEFFICIENTS
   .2162340       .1730221E-01   .8101827       16.55476       .6159436
  -.1577876       .1502218       20.27821       .1303957       .1466738
   .4388591       .6594422E-01
GRADIENT
  -.9135559E-07   .1209752E-06  -.1557105E-07  -.2684200E-08  -.1069811E-06
  -.1600394E-05  -.1663395E-06  -.6411252E-08  -.1361892E-07  -.8107014E-07
   .1330693E-07  -.9246286E-07

SIGMA MATRIX
   1.0441
   .43785       1.3832
  -.38523       .19261       .47643

GTRANSPOSE*INVERSE(H)*G  STATISTIC  -  =   .99128E-14

      COEFFICIENT    ST. ERROR   T-RATIO
A1        .21623      .10727      2.0158
A2        .17302E-01  .11805      .14657
A3        .81018      .40250E-01  20.129
A0       16.555      1.3208      12.534
B1        .61594      .16279      3.7838
B2       -.15779      .36126E-01 -4.3677
B3        .15022      .17323      .86718
B0       20.278      7.5427      2.6885
C1        .13040      .29141E-01  4.4746
C2        .14667      .38836E-01  3.7767
C3        .43886      .35632E-01  12.316
C0        .65944E-01  1.0377      .63550E-01

*** NONLINEAR THREE STAGE LEAST SQUARES ***
```

```
INITIAL STATISTICS :
TIME =      2.340 SEC.   ITER. NO.     0   FUNCT. EVALUATIONS     1
FUNCTION VALUE=   28.61320     FUNCTION VALUE/N =   1.362534
COEFFICIENTS
   .2162340       .1730221E-01   .8101827       16.55476       .6159436
  -.1577876       .1502218       20.27821       .1303957       .1466738
   .4388591       .6594422E-01
GRADIENT
  -3.710973      -19.98431      -40.19389      -.1978416E-06   2.223335
   72.48382       11.97309       .1136923E-06  -79.17372      -57.11702
  -8.732483      -.4000084E-06

FINAL STATISTICS :
TIME =      6.560 SEC.   ITER. NO.    23   FUNCT. EVALUATIONS    29
FUNCTION VALUE=   24.29102     FUNCTION VALUE/N =   1.156715
COEFFICIENTS
   .1631441       .1248905       .7900809       16.44079       .7557240
  -.1948482      -.1307918E-01   28.17785       .1496741       .1812910
   .4004919       .1508024
GRADIENT
  -.1155376E-06  -.1648650E-06  -.4578952E-06  -.8789430E-07  -.1000770E-06
  -.4155699E-05  -.3814232E-06  -.2193214E-07  -.1618987E-05  -.2499567E-05
  -.1481715E-05  -.1650535E-06

SIGMA MATRIX
   .89176
   .41132       2.0930
  -.39361       .40305       .52003

GTRANSPOSE*INVERSE(H)*G  STATISTIC  -  =   .19942E-13

      COEFFICIENT    ST. ERROR   T-RATIO
A1        .16314      .10044      1.6243
A2        .12489      .10813      1.1550
A3        .79008      .37938E-01  20.826
A0       16.441      1.3045      12.603
B1        .75572      .15293      4.9415
B2       -.19485      .32531E-01 -5.9897
B3       -.13079E-01  .16190     -.80787E-01
B0        28.178      6.7938      4.1476
C1        .14967      .27935E-01  5.3579
C2        .18129      .34159E-01  5.3073
C3        .40049      .31813E-01  12.589
C0        .15080      1.0150      .14858
|_END
```

GENERALIZED METHOD OF MOMENTS ESTIMATION

Generalized method of moments (GMM) estimation is described in Andrews [1991], Davidson and MacKinnon [1993, Chapter 17], Gallant [1987], Greene [2003, Chapter 18], Hansen and Singleton [1982] and Newey and West [1987 and 1991].

Single Equation Estimation

In a single equation the model follows the notation of the Nonlinear Two-Stage Least Squares model which is a special case of GMM. The general form of the equation is:

$$Y = f(Z,\beta) + \varepsilon$$

where Y is an $N \times 1$ vector of observations on the dependent variable, Z is the matrix of right-hand side variables in the equation, β is a $P \times 1$ vector of unknown parameters and ε is a random error vector. The model assumptions are $E(\varepsilon)=0$ and $E(\varepsilon\varepsilon')=\Omega$ where Ω is unrestricted. With X as an $N \times K$ matrix of instrumental variables (usually all the predetermined variables in the system) the parameter estimates are obtained by minimizing the objective function:

$$\varepsilon' X (X'\Omega X)^{-1} X'\varepsilon \qquad \text{where} \qquad \varepsilon = Y - f(Z,\beta)$$

The matrix $(X'\Omega X)$ is known as the weighting matrix and the user must specify this matrix with the **GMM=** option either as the name of a matrix variable which contains the desired values or as one of the pre-set SHAZAM options described below. The estimated covariance matrix of the GMM estimates $\tilde{\beta}$ is:

$$(g'X (X'\Omega X)^{-1} X'g)^{-1}$$

where g is the $N \times P$ matrix of derivatives $\partial f(Z,\beta)/\partial\beta$ evaluated at $\tilde{\beta}$.

There are a few preset options to allow SHAZAM to automatically compute the weighting matrix. If **GMM=HETCOV** is specified the White [1980] estimate of the matrix is used. The weighting matrix $(X'\Omega X)$ is estimated using the residuals e_t estimated from N2SLS as:

$$W_0 = \frac{1}{N}\sum_{t=1}^{N} e_t^2 X_t X_t'$$

where X_t is a K x 1 vector of instrumental variables for observation t. If the disturbances are autocorrelated then the estimator proposed by Newey and West [1987] is:

$$W = W_0 + \frac{1}{N}\sum_{j=1}^{L}\sum_{t=j+1}^{N} w_j e_t e_{t-j}\left(X_t X_{t-j}' + X_{t-j} X_t'\right)$$

The weights w_j and the maximum lag length L must be chosen in advance. A number of alternative schemes are implemented in SHAZAM with the **GMM=** option using the

keywords **BARTLETT**, **TRUNC**, **PARZEN**, **QS** or **TUKEY**. The lag length L can be specified with the **AUTCOV=** option and if this option is not specified a default setting of L will be used. The weighting scheme and the default value of L is set for the alternative methods as follows.

If **GMM=BARTLETT** is specified then: $$w_j = 1 - \frac{j}{L+1}$$

If the **AUTCOV=** option is not specified then the automatic bandwidth formula in Newey and West [1991] is used:

$$L = 4(N/100)^{(2/9)} \qquad \text{(rounded down to the nearest integer)}$$

If **GMM=TRUNC** is specified then: $w_j = 1$ for all j, and

$$L = 4(N/100)^{(1/4)}$$

If **GMM=PARZEN** is specified then:

$$w_j = 1 - 6\left(\frac{j}{L+1}\right)^2 + 6\left(\frac{j}{L+1}\right)^3 \quad \text{for} \quad \left(\frac{j}{L+1}\right) \le 0.5$$

$$w_j = 2\left(1 - \frac{j}{L+1}\right)^3 \quad \text{for} \quad \left(\frac{j}{L+1}\right) > 0.5$$

$$L = 4(N/100)^{(4/25)}$$

If **GMM=QS** is specified then the **Q**uadratic **S**pectral estimator is used as suggested in Andrews [1991] with:

$$w_j = \left(\frac{25}{12\pi^2 (j/(L+1))^2}\right)\left(\frac{\sin(6\pi j/(5(L+1)))}{6\pi j/(5(L+1))} - \cos(6\pi j/(5(L+1)))\right)$$

$$L = 4(N/100)^{(2/25)}$$

If **GMM=TUKEY** is specified then the Tukey-Hanning estimator is used with:

$$w_j = \left(1 + \cos\left(\frac{\pi j}{L+1}\right)\right) \Big/ 2$$

$$L = 4\,(N/100)^{(1/4)}$$

Estimation of a System of Equations

When there is more than one equation the notation follows that of the N3SLS model. Given a system of M equations such that equation i has the general form:

$$f_i(Y, X, \beta) = \varepsilon_i \qquad \text{for} \quad i = 1, \ldots, M$$

Stack the ε_i vectors to obtain an MN x 1 vector ε. The GMM estimator minimizes:

$$\varepsilon'(X \otimes I)\,[(X \otimes I)'\Omega(X \otimes I)]^{-1}\,(X \otimes I)'\varepsilon$$

where I is an MxM identity matrix and Ω is now a (MNxMN) matrix. The matrix $(X \otimes I)$ is the block diagonal matrix:

$$\begin{bmatrix} X & 0 & . & 0 \\ 0 & X & . & 0 \\ . & . & . & . \\ 0 & 0 & . & X \end{bmatrix}$$

When **GMM=HETCOV** is used the weighting matrix $((X \otimes I)'\Omega(X \otimes I))$ is estimated as:

$$\frac{1}{N}\sum_{t=1}^{N}(e_t \otimes X_t)(e_t \otimes X_t)'$$

where e_t is an Mx1 vector of estimated N3SLS residuals and X_t is a $K \times 1$ vector of instrumental variables for observation t.

A similar multivariate procedure analagous to the single equation case is used for the other **GMM=** options. Note that the dimensions of the weighting matrix is now $MK \times MK$. When the weighting matrix is shown on SHAZAM output it has been standardized by dividing by N. A `TEST OF THE OVERIDENTIFYNG RESTRICTIONS` is obtained by multiplying the minimized function value by N. This resulting test statistic (usually called J) is

distributed χ^2 with MK–P degrees of freedom under the null hypothesis where P is the number of parameters in the system.

The minimized function value is available in the temporary variable *$LLF*. Note, however, that the function is not a log-likelihood function but the *$LLF* variable is used in all SHAZAM nonlinear estimation to hold the function value.

Users should be aware that the weighting matrix can easily be singular especially for small sample sizes. In this case it is not possible to compute the inverse of the matrix and estimation will not be successful. If alternative forms of the matrix are used with possibly different values for the **AUTCOV=** option then it may become possible to create a non-singular weighting matrix.

SHAZAM commands for the estimation of the first two equations of the Klein Model are:

```
* Generalized Method of Moments estimation
nl 2 w2 t g time1 plag klag xlag / ncoef=8 gmm=hetcov nopsigma
eq c=a1*plag+a2*p+a3*w1w2+a0
eq i=b1*plag+b2*klag+b3*p+b0
end
```

SIMULATED ANNEALING

Simulated Annealing can be used to find solutions to problems that traditional gradient approaches are unable to solve. One advantage that Simulated Annealing has over more traditional gradient methods is that the fitting parameters can be bounded to an area of interest and initial conditions do not necessarily have to be specified. However, Simulated Annealing may also struggle to find a solution, if care is not taken with the option parameters.

The following example is taken from the NIST Nonlinear Least Squares Regression sample problems. This particular problem presents many challenges for the traditional gradient approaches, and without a reasonably good set of starting conditions, i.e. within 1% of the certified answers, the correct solution is normally not found.

Simulated annealing alone also struggles to find the certified solution. However, it does find a rough and ready solution, which can then be used as a good set of starting conditions. This is the essence of the **HYBRID** approach - Simulated Annealing finds a reasonable solution and then using the highly efficient Gradient method of your choice to find the final solution. An example of this follows:

```
* Set sample size and read data
sample 1 6

read y / cols=1 rows=6
109
149
149
191
213
224

read x / cols=1 rows=6
 1
 2
 3
 5
 7
10

* Gradient method
nl 1 / ncoef=2 opgcov nodn
eq y=b1*(1-exp(-b2*x))
coef b1 1 b2 1
gen1 k=$k
format(2f30.15)

* Simulated Annealing Estimate
nl 1/ ncoef=2  saiter=1e8 opgcov saupfac=1000 salowfac=0 sans=100
     method=sa nodn
eq y=b1*(1-exp(-b2*x))
coef b1 1 b2 1
gen1 k=$k
format(2f30.15)

* Hybrid Approach
nl 1/ ncoef=2  saiter=1e8 opgcov  saupfac=1000 salowfac=0 sans=100
     hybrid nodn
eq y=b1*(1-exp(-b2*x))
coef b1 1 b2 1
gen1 k=$k
format(2f30.15)
```

The example demonstrates the limitations of the traditional gradient approach. The algorithm fails to find a sensible solution. The use of Simulated Annealing achieves a better solution, though the best solution is obtained by using a **HYBRID** option whereby

the traditional gradient method seeks out the certified solution, BUT starting from the estimate obtained by the initial Simulated Annealing algorithm.

23. NONPARAMETRIC METHODS

"What is sought is found."

Sophocles, *Oedipus Tyrannus*

The **NONPAR** command provides features for nonparametric density estimation and regression smoothing techniques.

DENSITY ESTIMATION

The Univariate Kernel Method

The kernel method is a nonparametric approach to density estimation and a good exposition of this method is in Silverman [1986]. For observations X_t for t = 1, . . . , N kernel estimates of the probability density function are obtained as:

$$\hat{f}_N(x) = \frac{1}{N \cdot h} \sum_{t=1}^{N} \mathbf{K}\{(x - X_t)/h)\}$$

where h is a bandwidth or smoothing parameter and **K** is a kernel function with the property:

$$\int_{-\infty}^{\infty} \mathbf{K}(u)du = 1$$

The **DENSITY** option on the **NONPAR** command is used to obtain kernel density estimates. The **METHOD=** option allows the use of the kernel functions:

Kernel	**METHOD=** *option*	**K**(u)
Gauss	**NORMAL**	$\frac{1}{\sqrt{2\pi}}\exp(-u^2/2)$
Epanechnikov	**EPAN**	$I(\lvert u\rvert \le 1)\cdot\frac{3}{4}(1-u^2)$
Quartic	**QUARTIC**	$I(\lvert u\rvert \le 1)\cdot\frac{15}{16}(1-u^2)^2$
Triangular	**TRIANG**	$I(\lvert u\rvert \le 1)\cdot(1-\lvert u\rvert)$
Uniform	**UNIFORM**	$I(\lvert u\rvert \le 1)\cdot 0.5$

The indicator function I = 1 if $|u| \le 1$ and 0 otherwise. The default setting for the bandwidth parameter is $h = \lambda \cdot \hat{\sigma}_x$ where

$$\lambda = \{4/(3 \cdot N)\}^{1/5} \qquad \text{and} \qquad \hat{\sigma}_x^2 = \frac{1}{N}\sum_{t=1}^{N}(X_t - \overline{X})^2$$

This approximately optimal bandwidth for a normal kernel is described in Silverman [1986, p.45]. Alternative values for λ can be specified with the **SMOOTH=** option. The density estimates can be saved with the **PREDICT=** option.

The Multivariate Kernel Method

For the multivariate case where $X_t = (X_{1t}, \ldots, X_{Kt})$ a multivariate kernel function can be stated as a product of univariate kernel functions. A kernel function for K-dimensional u is:

$$\mathbf{K}(u) = \mathbf{K}(u_1, \ldots, u_K) = \prod_{j=1}^{K}\mathbf{K}(u_j)$$

The multivariate kernel density estimate can then be expressed as:

$$\hat{f}_N(x) = \frac{1}{N \cdot h}\sum_{t=1}^{N}\mathbf{K}_h(x - X_t) = \frac{1}{N \cdot h}\sum_{t=1}^{N}\prod_{j=1}^{K}\mathbf{K}\{(x_j - X_{jt})/h_j\} \qquad \text{where} \qquad h = \prod_{j=1}^{K} h_j$$

The bandwidth parameters are $h_j = \lambda \cdot \hat{\sigma}_{x_j}$ and default settings are:

$$\lambda = \{4/(N(2 \cdot K + 1))\}^{1/(K+4)} \qquad \text{and} \qquad \hat{\sigma}_{x_j}^2 = \frac{1}{N}\sum_{t=1}^{N}(X_{jt} - \overline{X}_j)^2 \qquad \text{for } j = 1, \ldots, K$$

Alternative values for λ can be specified with the **SMOOTH=** option and other estimates of the variance $\sigma_{x_j}^2$ can be requested with the **INCOVAR=** option.

To recognize covariance in the sample Rust [1988] uses a multivariate normal kernel function and this is implemented with the **METHOD=MULTI** option. Standardized variables are created as:

$$Z_t = A(X_t - \overline{X})/\lambda \qquad \text{where} \qquad A\hat{\Sigma}_X A' = I_K$$

The smoothing parameter λ can be set with the **SMOOTH=** option and the default setting is:

$$\lambda = \{4/(N(2 \cdot K + 1))\}^{1/(K+4)}$$

The $\hat{\Sigma}_X$ matrix can be input with the **INCOVAR=** option and the default is to use the sample variance-covariance matrix with a divisor of N. The multivariate normal kernel is then obtained as a product of univariate standard normal kernel functions:

$$\hat{f}_N(x) = \frac{1}{N \cdot h} \sum_{t=1}^{N} \mathbf{K}_h(x - X_t) = \frac{1}{N \cdot h} \sum_{t=1}^{N} \prod_{j=1}^{K} \mathbf{K}(z_j - Z_{jt}) \qquad \text{where} \qquad h = \lambda^K \cdot \left|\hat{\Sigma}_x\right|^{1/2}$$

NONPARAMETRIC REGRESSION

The **NONPAR** command provides nonparametric smoothing approaches to estimating the regression relationship:

$$Y_t = m(X_t) + \varepsilon_t \qquad \text{for } t = 1, \ldots, N$$

where m = E(Y|X=x) is the unknown regression function. An informative presentation of the subject is contained in the monograph by Härdle [1990].

Kernel Estimators

The Nadaraya-Watson estimator is:

$$\hat{m}_N(x) = \frac{1}{N \cdot h} \sum_{t=1}^{N} W_{ht}(x) \cdot Y_t \qquad \text{where} \qquad W_{ht}(x) = \mathbf{K}_h(x - X_t) \Big/ \hat{f}_N(x)$$

The kernel function **K** can be specified with the **METHOD=** option. The **SMOOTH=** and **INCOVAR=** options can also be used as described above.

In matrix notation the predicted values can be stated as $\hat{Y} = S'Y$. The N x N smoother matrix S can be saved with the **SMATRIX=** option. A feature of S is that it does not depend on Y and so the nonparametric estimate is linear in Y. This property facilitates model evaluation that is described later in this chapter.

The selection of an optimal value for the smoothing parameter λ is the subject of much discussion in the literature. Smaller values of λ give rougher estimators (with more wiggles) and larger values of λ give smoother estimators. One selection approach is to choose λ to minimize the CV or GCV (described below).

The estimated residuals are $e_t = Y_t - \hat{Y}_t$. A variance estimate (see Härdle [1990, p.100]) conditional on X = x is:

$$\hat{\sigma}^2(x) = \frac{1}{N \cdot h} \sum_{t=1}^{N} W_{ht}(x) \cdot e_t^2$$

The conditional standard deviations evaluated at $X_1, \ldots, X_N$ can be saved with the **SIGMA=** option on the **NONPAR** command.

The asymptotic variance (from Algorithm 4.2.1 in Härdle [1990]) is:

$$V_N(x) = \frac{c_K \hat{\sigma}^2(x)}{N \cdot h \cdot \hat{f}_N(x)} \qquad \text{where} \qquad c_K = \int \mathbf{K}^2(u)du$$

For the Gaussian kernel $c_K = 1/(2\sqrt{\pi})$, for the Epanechnikov kernel $c_K = 3/5$, for the quartic kernel $c_K = 5/7$, for the triangular kernel $c_K = 2/3$ and for the uniform kernel $c_K = 0.5$. An approximate 95 % confidence interval can then be constructed as:

$$\hat{m}_N(x) \pm 1.96\sqrt{V_N(x)}$$

The prediction standard errors can be saved with the **FCSE=** option.

By taking derivatives of the regression function with respect to x_j slope estimates can be obtained at point x (see Härdle [1990, pp. 33-34]). The slope estimates are computed as:

$$b_j(x) = \frac{-1}{N \cdot h} \left[\sum_{t=1}^{N} G_{jt} \cdot \mathbf{K}_h(x - X_t) \cdot Y_t - \hat{m}_N(x) \sum_{t=1}^{N} G_{jt} \cdot \mathbf{K}_h(x - X_t) \right] \Big/ \hat{f}_N(x)$$

where, for the multivariate normal kernel,

$$G_{jt} = \frac{1}{\lambda^2} \sum_{k=1}^{K} \hat{\sigma}_x^{jk}(x_k - X_{kt}) \qquad \text{and} \qquad \hat{\sigma}_x^{jk} \text{ is the (j,k) element of } \hat{\Sigma}_x^{-1}$$

When the Gaussian kernel is selected, the slope estimates are evaluated at each point and printed with the **PCOEF** option.

Kernel smoothing methods are less accurate near the boundary of the observation interval. Boundary modifications have been proposed by Rice [1984] (also described in Härdle [1990, pp. 130-132]) and others (see Hall and Wehrly [1991]). A multivariate extension of the Rice method is available as a **NONPAR** command option. A Euclidean distance measure is used to select points near the boundary. The idea is that if x is more than the arbitrary rule of 2 1/2 bandwidths from the center then a boundary modified estimate is used. If

$$\sqrt{\frac{1}{K}\sum_{j=1}^{K}\{(x_j - \overline{X}_j)/h_j\}^2} > 2\tfrac{1}{2}$$

then the modified estimate is: $\tilde{m}_N(x) = \hat{m}_N(x;\lambda) + \gamma[\hat{m}_N(x;\lambda) - \hat{m}_N(x;\alpha\lambda)]$

where $\gamma = R(\rho)/[\alpha R(\rho/\alpha) - R(\rho)]$, $R(v) = w_1(v)/w_0(v)$ and

$w_0(v) = \int_{-1}^{v} \mathbf{K}(u)du$ and $w_1(v) = \int_{-1}^{v} u\,\mathbf{K}(u)du$

The value for ρ (in the interval [0,1]) can be set with the **BRHO=** option. With $\rho \geq 1$, $\gamma = 0$. A value for α must also be set and SHAZAM uses $\alpha = 2 - \rho$ as recommended in Rice [1984]. Note that this boundary modified estimate is still linear in Y.

SHAZAM makes direct use of the formula for the kernel regression estimator to compute estimates at each X_t for t = 1, . . . , N. The computation time increases rapidly with N and the use of efficient algorithms has been advocated. Algorithms that use fast Fourier transforms are available for calculating kernel estimators for bivariate data (see, for example, Härdle [1987]). The use of binning methods is described in Fan and Marron [1994]. These methods are not implemented here.

Locally Weighted Regression

The **METHOD=LOWESS** option on the **NONPAR** command implements the locally weighted regression method described in Cleveland [1979], Cleveland and Devlin [1988] and Cleveland, Devlin and Grosse [1988]. This method applies to the two-variable simple regression model. Suppose $X_{a(t)}, \ldots, X_{b(t)}$ are the ordered r nearest neighbors of X_t. For a value of f ($0 < f \leq 1$) let r be $f \cdot N$ rounded down to the nearest integer. The smoothing value f can be set with the **SMOOTH=** option.

For each t the locally weighted regression method finds estimates of β to minimize:

$$\sum_{\tau=a(t)}^{b(t)} w_\tau(X_t)\{Y_\tau - \beta(X_\tau - \overline{X}_t^w)\}^2$$

where $\overline{X}_t^w$ is the weighted average $\overline{X}_t^w = \sum_{\tau=a(t)}^{b(t)} w_\tau(X_t) X_\tau$

The weights for the weighted least squares regression are computed as:

$$w_\tau(X_t) = W\left((X_\tau - X_t)/h_t\right) \qquad \text{where} \qquad h_t = \max(X_t - X_{a(t)},\ X_{b(t)} - X_t)$$

and W is the "tricube" weight function $W(x) = I(|x| \le 1)\cdot(1 - |x|^3)^3$

The fitted values can be expressed as $\hat{Y}_t = \sum_{\tau=a(t)}^{b(t)} s_\tau(X_t) Y_\tau$

In matrix notation this can be stated as $\hat{Y} = S'Y$. The N x N matrix S is a linear smoother and can be saved with the **SMATRIX=** option. The estimated residuals are $e_t = Y_t - \hat{Y}_t$.

The prediction standard errors that can be saved with the **FCSE=** option are:

$$\hat{\sigma}_t = \sqrt{\hat{\sigma}^2 \sum_{\tau=a(t)}^{b(t)} s_\tau^2(X_t)}$$

The $\hat{\sigma}^2$ estimate is described in the section on model evaluation later in this chapter. An approximate $100\cdot(1-\alpha)\%$ confidence interval (see Cleveland and Devlin [1988, p. 599]) can then be obtained as:

$$\hat{Y}_t \pm t_{(\rho,\alpha/2)}\hat{\sigma}_t$$

where $t_{(\rho,\alpha/2)}$ is the critical value from a t-distribution with ρ degrees of freedom and

$$\rho = tr\{(I-S)(I-S)'\}^2 / tr\{[(I-S)(I-S)']^2\}$$

The value for ρ is reported on the SHAZAM output as the LOOKUP DEGREES OF FREEDOM and is saved in the temporary variable *$DF1*.

The weighted least squares procedure can be iterated with recalculated weights to get robust locally weighted regression estimates. Let m be the median of the $|e_t|$ and define robustness weights by:

$$\delta_t = \mathbf{K}(e_t / 6m) \qquad \text{where } \mathbf{K} \text{ is the quartic kernel} \quad \mathbf{K}(u) = I(|u| \le 1) \cdot (1 - u^2)^2 \ .$$

The robustness weights δ_t can be saved with the **RWEIGHT=** option. The weighted least squares regression is repeated with the weights:

$$\delta_t \, w_\tau(X_t) \qquad \text{for} \qquad \tau = a(t), \ldots, b(t)$$

Robust locally-weighted regression is recommended for data sets with outliers or long-tailed error distributions (see Cleveland, Devlin and Grosse [1988, p.111]). However, the smoother matrix S now depends on the ε_t and so the model diagnostics described below are not valid.

Computations can be reduced by noting that if $X_{t+1} = X_t$ then $\hat{Y}_{t+1} = \hat{Y}_t$. Computations can be speeded by obtaining an interpolation for $\hat{Y}_{t+1}$ when $X_{t+1} \le X_t + \Delta$. A value for Δ can be specified with the **DELTA=** option.

A general implementation of the LOWESS method considers polynomial regressions and this is not available with the **NONPAR** command.

Model Evaluation

The R^2 measure that is reported on the SHAZAM output is calculated as:

$$R^2 = 1 - \frac{e'e}{Y'Y - N\bar{Y}^2}$$

The error variance reported as SIGMA**2 on the SHAZAM output is computed as:

$$\hat{\sigma}^2 = \frac{1}{n_1} \sum_{t=1}^{N} e_t^2 \qquad \text{where} \qquad n_1 = \text{tr}\{(I - S)(I - S)'\}$$

The derivation of the degrees of freedom n_1 is discussed in Cleveland and Devlin [1988] and Hall and Marron [1990]. The "equivalent number of parameters" is obtained as $k_1 = N - n_1$. Note that k_1 is not necessarily an integer as is the case with OLS. Two other definitions for degrees of freedom are discussed in Buja, Hastie and Tibshirani [1989, pp. 469-470] and these compute the equivalent number of parameters as $k_2 = tr(SS')$ and alternatively $k_3 = tr(S)$. These measures are also reported on the SHAZAM output.

The value for n_1 is available in the temporary variable *$DF*, the value for k_1 is stored in the temporary variable *$K* and $\hat{\sigma}^2$ is saved in *$SIG2*.

The adjusted R^2 and model selection statistics including the Akaike information criterion (AIC), the generalized cross-validation (GCV) statistic and others are calculated using the formula given in the chapter *A CHILD'S GUIDE TO RUNNING REGRESSIONS* where K is replaced with k_1.

The SHAZAM output also reports the cross-validation mean square error computed as:

$$CV = \frac{1}{N}\sum_{t=1}^{N}\{e_t/(1 - s_{tt})\}^2$$

where s_{tt} is the t^{th} diagonal element of the smoother matrix S. The CV statistic is saved in the temporary variable *$CV*. A discussion of the CV, GCV and other criterion is available in Eubank [1988, Chapter 2].

NONPAR *COMMAND OPTIONS*

In general, the format of the **NONPAR** command for regression estimation is:

NONPAR *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of options.

The format of the **NONPAR** command for kernel density estimation is:

NONPAR *vars* / **DENSITY** *options*

where *vars* is a list of variables.

Options as defined for the **OLS** command that are available are:

LIST, **BEG=**, and **END=**

The following additional options are available on the **NONPAR** command:

DENSITY Calculates kernel density estimates. The kernel function is specified with the **METHOD=** option as **EPAN**, **MULTI, NORMAL**, **QUARTIC**, **TRIANG** or **UNIFORM**. The default is **METHOD=NORMAL** for the univariate case and **METHOD=MULTI** for the multivariate case. An example of the use of this option is in the section on *Bootstrapping Regression Coefficients* in the chapter *PROGRAMMING IN SHAZAM*.

GRAPH Prepares gnuplot plots of the residuals and the fitted values. With the **DENSITY** option a plot of the density function is given (available for univariate density estimation only). For more information on this option see the chapter *PLOTS AND GRAPHS*. With the **GRAPH** option the **APPEND**, **OUTPUT=**, **DEVICE=**, **PORT=** and **COMMFILE=** options are also available as described for the **GRAPH** command.

PCOEF For regression estimation with **METHOD=NORMAL**, **METHOD=MULTI** or **METHOD=LOWESS** this option prints the estimated slope coefficients evaluated at every point.

BRHO= Used with kernel smoothing methods to specify a value of ρ ($0 \leq \rho < 1$) to use in calculating the Rice [1984] boundary modified estimate. This is not available with the Epanechnikov kernel. With the Gaussian or triangular kernel the value for **BRHO=** is rounded down to the nearest tenth and numeric integration is used. Integrals are computed explicitly for other kernels. (When this option is used the **DENSITY** and **FCSE=** options are not available).

COEF= For regression estimation with **METHOD=NORMAL, METHOD=MULTI** or **METHOD=LOWESS** this option saves the estimated slope coefficients in an N x K matrix.

DELTA= Used with **METHOD=LOWESS** to group the observations as described above. The default is **DELTA=0**. When this option is used some model diagnostics are not provided and the **FCSE=** option is not available.

FCSE= Saves the prediction standard errors in the variable specified. This option is not available with robust locally weighted regression.

HATDIAG= Saves the diagonal elements of the smoother matrix S in the variable specified.

INCOVAR= Used with kernel density estimation methods. For **METHOD=MULTI** the covariance matrix $\hat{\Sigma}_X$ must be a symmetric matrix stored in lower-triangular form such as that produced by the **COV=** option on the **STAT** command or the **SYM** function on the **MATRIX** command. For the product kernel methods this must be a K x 1 vector of variance estimates $\hat{\sigma}^2_{x_j}$. If this option is not specified then default values will be set as described above.

ITER= Used with **METHOD=LOWESS** to specify the number of iterations for robust locally weighted regression. The default is **ITER=0**. With **ITER=0** the estimation is by nonrobust locally weighted regression.

METHOD= Specifies the kernel function or regression method. The available options are **EPAN**, **MULTI, NORMAL**, **QUARTIC**, **TRIANG**, **UNIFORM** and **LOWESS**. The default is **METHOD=NORMAL** for the univariate case and **METHOD=MULTI** for the multivariate case.

PREDICT= Saves the predictions from the regression estimation in the variable specified. When the **DENSITY** option is used the **PREDICT=** option saves the density estimates in the variable specified.

RESID= Saves the residuals from the regression estimation in the variable specified.

RWEIGHTS= For **METHOD=LOWESS** saves the robustness weights in the variable specified. This is not used when **ITER=0**.

SIGMA= For kernel smoothing methods saves the conditional standard errors $\hat{\sigma}_t$ in the variable specified.

SMATRIX= Saves the N x N smoother matrix in the variable specified.

SMOOTH= Specifies the value of the smoothing parameter. For kernel density estimation methods the default values are described above. For

METHOD=LOWESS the default for the smoothing fraction f is **SMOOTH=0.5**.

Temporary variables that are available following regression estimation are:

$ADR2, $CV, $DF, $DF1, ERR, $K, $N, $R2, $SIG2 and *$SSE*.

The model selection test statistics are available in the temporary variables:

$AIC, $FPE, $GCV, $HQ, $LAIC, $LSC, $RICE, $SC and *$SHIB*

EXAMPLES

This example is from Rust [1988] and is designed to show the limitations of OLS when the relationship is nonlinear. With N=50, values for X_t are generated from a uniform distribution on (0,1) and values for the errors ε_t are generated from a N(0, 0.1) distribution. The Y_t are generated as:

$$Y_t = 1 - 4(X_t - 0.5)^2 + \varepsilon_t$$

The SHAZAM command file is:

```
sample 1 50
set ranfix
genr x=uni(1)
genr e=nor(.1)
genr y = 1 - 4*(x-.5)**2 + e
ols y x / anova
nonpar y x
stop
```

SHAZAM results comparing OLS regression and nonparametric regression with a normal kernel function are:

```
|_SAMPLE 1 50
|_SET RANFIX
|_GENR X=UNI(1)
|_GENR E=NOR(.1)
|_GENR Y = 1 - 4*(X-.5)**2 + E
|_OLS Y X / ANOVA
 OLS ESTIMATION
      50 OBSERVATIONS     DEPENDENT VARIABLE = Y
...NOTE..SAMPLE RANGE SET TO:    1,    50
```

```
 R-SQUARE =    .0001     R-SQUARE ADJUSTED =   -.0208
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .98797E-01
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .31432
SUM OF SQUARED ERRORS-SSE=   4.7423
MEAN OF DEPENDENT VARIABLE =   .70496
LOG OF THE LIKELIHOOD FUNCTION = -12.0592

MODEL SELECTION TESTS - SEE JUDGE ET AL. (1985,P.242)
 AKAIKE (1969) FINAL PREDICTION ERROR - FPE =      .10275
    (FPE IS ALSO KNOWN AS AMEMIYA PREDICTION CRITERION - PC)
 AKAIKE (1973) INFORMATION CRITERION - LOG AIC =  -2.2755
 SCHWARZ (1978) CRITERION - LOG SC =              -2.1990
MODEL SELECTION TESTS - SEE RAMANATHAN (1992,P.167)
 CRAVEN-WAHBA (1979)
    GENERALIZED CROSS VALIDATION - GCV =           .10291
 HANNAN AND QUINN (1979) CRITERION =               .10578
 RICE (1984) CRITERION =                           .10309
 SHIBATA (1981) CRITERION =                        .10243
 SCHWARZ (1978) CRITERION - SC =                   .11091
 AKAIKE (1974) INFORMATION CRITERION - AIC =       .10274

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      48 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
X         -.82941E-02  .1595      -.5200E-01  .959 -.008     -.0075     -.0054
CONSTANT   .70876      .8555E-01   8.285      .000  .767      .0000     1.0054

|_NONPAR Y X
       50 OBSERVATIONS     DEPENDENT VARIABLE = Y
...NOTE..SAMPLE RANGE SET TO:    1,   50

THE BANDWIDTH/SMOOTHING PARAMETER IS SET BY DEFAULT
NONPARAMETRIC REGRESSION USING KERNEL=NORMAL
NUMBER OF VARIABLES=   1     NUMBER OF OBSERVATIONS=   50

         BANDWIDTH PARAMETER =    .48439

VARIABLE     MEAN         VARIANCE
X            .45826         .77671E-01

 R-SQUARE =    .7607     R-SQUARE ADJUSTED =    .7442
ERROR VARIANCE         SIGMA**2 =        .24756E-01
STANDARD ERROR         SIGMA =           .15734
SUM OF SQUARED ERRORS  SSE =            1.1347
EQUIVALENT NUMBER OF PARAMETERS - K1 =   4.1670
                                - K2 =   2.4716
                                - K3 =   3.3193

CROSS-VALIDATION MEAN SQUARE ERROR =     .27498E-01

MODEL SELECTION TESTS - SEE JUDGE ET AL. (1985,P.242)
 AKAIKE (1969) FINAL PREDICTION ERROR - FPE =      .26820E-01
    (FPE IS ALSO KNOWN AS AMEMIYA PREDICTION CRITERION - PC)
 AKAIKE (1973) INFORMATION CRITERION - LOG AIC =  -3.6190
 SCHWARZ (1978) CRITERION - LOG SC =              -3.4597
MODEL SELECTION TESTS - SEE RAMANATHAN (1992,P.167)
 CRAVEN-WAHBA (1979)
    GENERALIZED CROSS VALIDATION - GCV =           .27007E-01
 HANNAN AND QUINN (1979) CRITERION =               .28486E-01
 RICE (1984) CRITERION =                           .27232E-01
 SHIBATA (1981) CRITERION =                        .26476E-01
```

```
 SCHWARZ (1978) CRITERION - SC =                    .31440E-01
 AKAIKE (1974) INFORMATION CRITERION - AIC =        .26809E-01
|_STOP
```

24. POOLED CROSS-SECTION TIME-SERIES

"Branch banking...will mean, I suggest in all humility, the beginning of the end of the capitalist system."

John T. Flynn
Business writer, 1933

Pooling methods can be used to combine cross-section and time series data. Consider N cross-sectional units (a cross-sectional unit is, for example, a household, an industry or a region) with T_i observations for cross-section i, i = 1, . . . , N. The regression equation can be written as:

$$Y_{it} = X'_{it}\beta + \varepsilon_{it} \qquad \text{for } t = 1, \ldots, T_i \; ; \quad i = 1, \ldots, N$$

where β is a K x 1 vector of unknown parameters and ε_{it} is a random error.

Cross-Section Heteroskedasticity and Time-wise Autoregression

The Parks [1967] method (described in Kmenta [1986, Section 12.2, pp.616-625] and Greene [2003, Section 13.9]) employs a set of assumptions on the disturbance covariance matrix that gives a cross-sectionally heteroskedastic and timewise autoregressive model. The **POOL** command in SHAZAM provides features for estimating this model and some variations of it. The assumptions of the model are:

$E(\varepsilon_{it}^2) = \sigma_i^2$	heteroskedasticity
$E(\varepsilon_{it}\varepsilon_{jt}) = 0$	for $i \neq j$, cross-section independence
$\varepsilon_{it} = \rho_i \varepsilon_{i,t-1} + v_{it}$	autoregression

and $E(v_{it}) = 0$, $E(v_{it}^2) = \phi_{ii}$, $E(v_{it}v_{js}) = 0$ for $i \neq j$ or $t \neq s$, and $E(\varepsilon_{i,t-1}v_{jt}) = 0$.

A balanced panel data set contains cross-sections observed over the same time period so that each cross-section has T observations. The total number of observations is then N·T. With balanced panels, the assumptions can be extended to allow for contemporaneous cross-section correlation so that $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_{ij}$ and $E(v_{it}v_{jt}) = \phi_{ij}$.

An estimate for β is obtained by a generalized least squares (GLS) procedure. The estimation proceeds with the following steps.

STEP 1: Estimate β by OLS and obtain residuals e_{it}.

STEP 2: Use the residuals to compute $\hat{\rho}_i$ as estimates of the ρ_i. The **POOL** command allows for different estimation methods. The least squares estimation method (the default method) is:

$$\hat{\rho}_i = \sum_{t=2}^{T_i} e_{it} e_{i,t-1} \Big/ \sum_{t=2}^{T_i} e_{i,t-1}^2 \qquad \text{for } i = 1, \ldots, N$$

On the SHAZAM output the $\hat{\rho}_i$ are listed as `RHO VECTOR`. When the **SAME** option is specified the same autoregressive parameter is used for all cross-sections as follows:

$$\hat{\rho}_1 = \ldots = \hat{\rho}_N = \sum_{i=1}^{N} \sum_{t=2}^{T_i} e_{it} e_{i,t-1} \Big/ \sum_{i=1}^{N} \sum_{t=2}^{T_i} e_{i,t-1}^2$$

The **CORCOEF** option ensures values in the interval [–1,+1] by estimating the autoregressive parameter as the sample correlation coefficient between e_{it} and $e_{i,t-1}$.

STEP 3: Use the $\hat{\rho}_i$'s to transform the observations, including the first observation (see Kmenta [1986, Equation 12.27, p.619]) and apply OLS to the transformed model. In matrix notation, the transformed model can be expressed as:

$$\hat{P}Y = \hat{P}X\beta + v$$

where $\hat{P}$ is the block diagonal matrix:

$$\hat{P} = \begin{bmatrix} \hat{P}_1 & 0 & . & 0 \\ 0 & \hat{P}_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \hat{P}_N \end{bmatrix}$$

and $\hat{P}_i$ is the T_i x T_i matrix:

$$\hat{P}_i = \begin{bmatrix} \sqrt{1-\hat{\rho}_i^2} & 0 & 0 & . & 0 \\ -\hat{\rho}_i & 1 & 0 & . & 0 \\ 0 & -\hat{\rho}_i & 1 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & -\hat{\rho}_i & 1 \end{bmatrix} \qquad \text{for } i = 1, \ldots, N$$

With residuals $\hat{v}_{it}$ the estimated cross-section error variances (reported as the DIAGONAL OF PHI MATRIX on the SHAZAM output) are:

$$\hat{\phi}_{ii} = \frac{1}{T_i - K} \sum_{t=1}^{T_i} \hat{v}_{it}^2 \qquad \text{for } i = 1, \ldots, N$$

If the **DN** option is used the divisor is T_i instead of $T_i - K$. The following matrix is constructed:

$$\hat{V} = \begin{bmatrix} \hat{\phi}_{11} I_{T_1} & 0 & . & 0 \\ 0 & \hat{\phi}_{22} I_{T_2} & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \hat{\phi}_{NN} I_{T_N} \end{bmatrix}$$

With balanced panels, if the **FULL** option is specified then contemporaneous error correlation is recognized and the matrix $\hat{\Phi}$ is calculated with elements:

$$\hat{\phi}_{ij} = \frac{1}{T - K} \sum_{t=1}^{T} \hat{v}_{it} \hat{v}_{jt}$$

The $\hat{V}$ matrix is then generalized to: $\hat{V} = \hat{\Phi} \otimes I_T$

On the SHAZAM output the estimated error covariance matrix $\hat{\Phi}$ is reported as the PHI MATRIX. When T < N the matrix $\hat{\Phi}$ is singular and the **FULL** option cannot be used.

STEP 4: GLS estimates are obtained as:

$$\tilde{\beta} = \left(X'\hat{\Omega}^{-1}X\right)^{-1} X'\hat{\Omega}^{-1}Y \qquad \text{where} \qquad \hat{\Omega}^{-1} = \hat{P}'\hat{V}^{-1}\hat{P}$$

The procedure may be iterated to convergence with the **ITER=** option. However this may not lead to efficiency gains in small samples. Let $\tilde{\beta}^{(i)}$ be the parameter estimates at iteration i. The iterative estimation stops when the following convergence criteria is met:

$$\left|\tilde{\beta}_k^{(i)} - \tilde{\beta}_k^{(i-1)}\right| \Big/ \left|\tilde{\beta}_k^{(i)}\right| < \delta \qquad \text{for } k = 1, \ldots, K$$

where δ can be set with the **CONV=** option and the default is $\delta = 0.001$.

Consider the lower triangular matrix $\hat{P}^*$ such that $\hat{\Omega}^{-1} = \hat{P}^{*\prime}\hat{P}^*$. The residuals are calculated as:

$$\tilde{v} = \hat{P}^* Y - \hat{P}^* X\tilde{\beta}$$

The variance of the residuals (reported as `SIGMA**2` on the SHAZAM output) is computed as:

$$\hat{\sigma}^2 = \frac{1}{\sum_{i=1}^{N} T_i - K} \sum_{i=1}^{N} \sum_{t=1}^{T_i} \tilde{v}_{it}^2$$

When the **DN** option is used the divisor is $\sum_{i=1}^{N} T_i$.

As a goodness-of-fit measure the SHAZAM output reports the Buse R^2 (see Buse [1973]) as described in the chapter *GENERALIZED LEAST SQUARES.* This is computed using the *estimated* `PHI MATRIX`. The result is that, unlike the usual R^2, the Buse R^2 is not guaranteed to be a nondecreasing function of the number of explanatory variables.

The Durbin-Watson statistic (reported with the **RSTAT** option) is calculated as:

$$\sum_{i=1}^{N} \sum_{t=2}^{T_i} (\tilde{v}_{it} - \tilde{v}_{i,t-1})^2 \Big/ \sum_{i=1}^{N} \sum_{t=1}^{T_i} \tilde{v}_{it}^2$$

Note that the Durbin-Watson statistic is appropriate as a test for autocorrelation only when the option **RHO=0** is used and when there are no lagged dependent variables as explanatory variables.

When the **UT** option is specified the residuals that are saved with the **RESID=** option are calculated as the untransformed residuals:

$$\tilde{e}_{it} = Y_{it} - X'_{it}\tilde{\beta}$$

When the **BLUP** option is specified the predicted values that are saved with the **PREDICT=** option are computed as:

$$\left.\begin{aligned} \hat{Y}_{i1} &= X'_{i1}\tilde{\beta} \qquad \text{and} \\ \hat{Y}_{it} &= X'_{it}\tilde{\beta} + \hat{\rho}_i(\hat{Y}_{i,t-1} - X'_{i,t-1}\tilde{\beta}) \quad \text{for} \quad t = 2, \ldots, T_i \end{aligned}\right\} \quad \text{for } i = 1, \ldots, N$$

Lagrange Multiplier Tests

With balanced panels and serially uncorrelated errors, the SHAZAM output reports test statistics computed from the pooled OLS residuals e_{it}. A Lagrange multiplier statistic (see Greene [2003, p. 328]) for testing for cross-section heteroskedasticity is calculated as:

$$\frac{T}{2}\sum_{i=1}^{N}\left(\frac{\hat{\sigma}_{ii}}{\hat{\sigma}^2} - 1\right)^2 \qquad \text{where} \qquad \hat{\sigma}_{ij} = \frac{1}{T}\sum_{t=1}^{T} e_{it}e_{jt} \qquad \text{and} \qquad \hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}\hat{\sigma}_{ii}$$

With the assumption of normality, there is evidence to reject the hypothesis of homoskedasticity if the statistic exceeds the critical value from a chi-square distribution with N–1 degrees of freedom.

The Breusch-Pagan [1980] Lagrange multiplier test gives a test for a diagonal covariance matrix (that is, no cross-section correlation). The statistic is:

$$T\sum_{i=2}^{N}\sum_{j=1}^{i-1} r_{ij}^2 \qquad \text{where} \qquad r_{ij}^2 = \frac{\hat{\sigma}_{ij}^2}{\hat{\sigma}_{ii}\hat{\sigma}_{jj}}$$

Under the null hypothesis of a diagonal covariance structure the statistic has an asymptotic chi-square distribution with N(N–1)/2 degrees of freedom.

Panel-Corrected Standard Errors

Some applied researchers (see Beck and Katz [1995] and Greene [2003, p. 323]) recommend the use of pooled OLS estimation with standard errors that are adjusted for cross-section heteroskedasticity and cross-section correlation ("panel corrected standard errors"). The **HETCOV** option on the **POOL** command implements OLS estimation with the calculation of a heteroskedastic-consistent covariance matrix for pooled regression models.

With balanced panels, when the **HETCOV** option is specified, the covariance matrix for OLS is estimated as:

$$(X'X)^{-1}\left(X'(\hat{\Phi} \otimes I_T)X\right)(X'X)^{-1}$$

where $\hat{\Phi}$ is an NxN matrix with the (i,j)th element $\sum_{t=1}^{T} e_{it}e_{jt} / T$.

When the **CSINDEX=** and **HETCOV** options are specified on the **POOL** command a diagonal Φ matrix is assumed. The covariance matrix for OLS is estimated as:

$$(X'X)^{-1}\left(\sum_{i=1}^{N} \hat{\sigma}_{ii} X_i' X_i\right)(X'X)^{-1}$$

When the **AR1** and **HETCOV** options are specified, parameter estimates are obtained by applying OLS to data transformed to correct for serial correlation. Panel corrected standard errors are then calculated and reported. A common cross-section autocorrelation coefficient, as recommended by Beck and Katz [1995], can be forced with the **SAME** option.

POOL *COMMAND OPTIONS*

The format of the **POOL** command is:

POOL *depvar indeps* / **NCROSS=** *options*

where *depvar* and *indeps* are the names of the dependent and independent variables. The **NCROSS=** option **must** specify the number of cross-sectional units in the data. SHAZAM will then figure out the number of time periods from the total number of observations. With balanced panels, the number of time periods can be specified with the **NTIME=** option. In this case SHAZAM will figure out the number of cross-sectional units. For unbalanced panels and data sets with missing values, the **CSINDEX=** option must be specified.

The data should be arranged so that all observations of a particular cross-sectional unit are together. Therefore, SHAZAM will require a complete time-series for the first group followed by a time-series for the second group, etc. If the data is not set up in this fashion it must be sorted before estimation of the model. In some cases the SHAZAM **SORT** command will help to rearrange the data.

Estimation with AR(1) errors assumes that the time series observations for each cross-section are equally spaced. This may not be appropriate with missing values.

Options as defined for the **OLS** command that are available are:

ANOVA, **DLAG**, **DUMP**, **GF**, **LININV**, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **MAX**, **NOCONSTANT**, **PCOR**, **PCOV**, **RESTRICT**, **RSTAT**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=** and **TRATIO=**

Options as defined for the **GLS** command that are available are:

BLUP and **UT**

Additional options available on the **POOL** command are:

AR1 Transform the observations to correct for AR(1) errors, but do not allow for cross-section heteroskedasticity. When this option is specified the **SAME**, **CORCOEF** and **HETCOV** options are also available.

CORCOEF Estimates the autoregressive parameters ρ_i using the correlation coefficient form, the alternative method described by Kmenta [1986, Equation 12.26]. This method confines the estimate of ρ_i to the interval [–1,+1].

DN Uses a divisor of T_i instead of $T_i - K$, i = 1, . . . , N when calculating the PHI matrix. That is, no degrees of freedom correction is considered in the estimation of error variances.

FIXED Estimates a fixed effects model. This model assumes that the intercept varies across cross-sections. Discussion is available in Greene [2003, Chapter 13] and Wooldridge [2006, Chapter 14].

FULL Estimates the **FULL** cross-sectionally correlated and time-wise autoregressive model (see Kmenta [1986, pp. 622-625]). This option is not available with the **CSINDEX=** option. If the **FULL** option is not specified then the model assumptions are cross-sectional heteroskedasticity with cross-sectional independence. The **UT** or **BLUP** options are recommended as the transformed residuals in the **FULL** model are sensitive to order of the cross-sections and have no useful interpretation. The **FULL** option is not available when N > T (when there are a large

number of cross-sections but the number of time series is few). When N > T the estimated error covariance matrix PHI is singular and SHAZAM will terminate with the message `MATRIX IS NOT POSITIVE DEFINITE`.

HETCOV For pooled OLS estimation, calculate a panel-corrected covariance matrix of the coefficient estimates and report the panel-corrected standard errors. If the **AR1** option is also specified then the estimation considers AR(1) errors for each cross-section.

MULSIGSQ The estimated covariance matrix of coefficients is calculated using Equation 12.39 in Kmenta [1986, p. 623]. However, some econometricians believe that this matrix should be multiplied by the overall estimate of $\hat{\sigma}^2$. The **MULSIGSQ** option does this multiplication. This option is the default, but could be turned off with the **NOMULSIGSQ** option.

OLS Estimation by pooled OLS.

PCOV Prints the covariance matrix of coefficients. In addition it prints the PHI matrix if **NCROSS=** is greater than 8. If **NCROSS=** is less than 8 the PHI matrix is always printed.

SAME Forces the ρ_i to be the same for each of the cross-sectional units.

COEF= Saves the estimated coefficients, the RHO vector, and the square root of the diagonal elements of the PHI matrix in the variable specified.

CONV= Specifies a convergence criterion to stop the iterative procedure when the **ITER=** option is used. The default is 0.001.

CSINDEX= Specifies a variable that contains the cross-section identifier for each observation. The cross-section identifier must be a positive number. The **NCROSS=** option must also be specified. If the **CSINDEX=** option is not specified then the estimation assumes balanced panels.

ITER= Specifies the maximum number of iterations if an iterative procedure is desired. If this option is not specified then one iteration is done.

NCROSS= Specifies the number of cross-sectional units in the data. This option is required. With balanced panels, the **NTIME=** option can be used as a substitute.

NTIME= With balanced panels, specifies the number of time periods in the data. This option is required if the **NCROSS=** option is not used. If both **NCROSS=** and **NTIME=** are specified, **NCROSS=** is ignored.

RHO= Specifies a fixed value of ρ to use. If this is not specified the autoregressive parameters are estimated. When this option is used the **SAME** option is automatically in effect. This option is commonly used with **RHO=0** to suppress the autocorrelation correction so that only the heteroskedastic correction is performed.

The available temporary variables on the **POOL** command are:

$ANF, *$DF*, *$DW*, *$ERR*, *$K*, *$LLF*, *$N*, *$R2*, *$R2OP*, *$RAW*, *$RHO*, *$SIG2*, *$SSE*, *$SSR*, *$SST*, *$ZANF*, *$ZDF*, *$ZSSR* and *$ZSST*.

For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

EXAMPLES

This example uses a data set from Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Table 11.5, p. 477]. The data set contains 10 years of cost and production data for four industries. The SHAZAM commands below set the data up in the form that is accepted by the **POOL** command. That is, the cost data is stacked in the variable *C* and the production data is stacked in the variable *Q*.

```
*Set Sample and read data inline
sample 1 10
read c1 c2 c3 c4 q1 q2 q3 q4
 43.72  51.03  43.90  64.29  38.46  32.52  32.86  41.86
 45.86  27.75  23.77  42.16  35.32  18.71  18.52  28.33
  4.74  35.72  28.60  61.99   3.78  27.01  22.93  34.21
 40.58  35.85  27.71  34.26  35.34  18.66  25.02  15.69
 25.86  43.28  40.38  47.67  20.83  25.58  35.13  29.70
 36.05  48.52  36.43  45.14  36.72  39.19  27.29  23.03
 50.94  64.18  19.31  35.31  41.67  47.70  16.99  14.80
 42.48  38.34  16.55  35.43  30.71  27.01  12.56  21.53
 25.60  45.39  30.97  54.33  23.70  33.57  26.76  32.86
 49.81  43.69  46.60  59.23  39.53  27.32  41.42  42.25

* Stack the columns into a long vector
matrix c=(c1'|c2'|c3'|c4')'
```

```
matrix q=(q1'|q2'|q3'|q4')'
sample 1 40

* Pooling by OLS with panel-corrected standard errors
pool c q / ncross=4 ols hetcov rstat

* Pooling with cross-section heteroskedasticity,
* contemporaneous cross-section correlation and AR1 errors
pool c q / ncross=4 full dn nomulsigsq

* Create cross-section dummy variables.
*  Set the number of cross-sections
gen1 nc=4
matrix csdum=seas(40,-nc)

*  Set the number of time periods
gen1 nt=10

*  Generate an index for each cross-section
genr csindex=sum(seas(nt))

*  Generate a repeating time index for the 10 observations
genr tindex=time(0)-nt*(csindex-1)

* Estimation with cross-section dummy variables
pool c q csdum / ncross=4 noconstant full dn nomulsigsq
```

The above commands show a number of alternative estimation methods available with the **POOL** command. The **DN** and **NOMULSIGSQ** options are used to follow the presentation in Greene [2003, Section 13.9]. The example shows how to generate cross-section dummy variables as well as how to construct a time index for each cross-section. The SHAZAM output is:

```
|_SAMPLE 1 10
|_READ C1 C2 C3 C4 Q1 Q2 Q3 Q4
   8 VARIABLES AND       10 OBSERVATIONS STARTING AT OBS       1

|_* Stack the columns into a long vector
|_MATRIX C=(C1'|C2'|C3'|C4')'
|_MATRIX Q=(Q1'|Q2'|Q3'|Q4')'

|_SAMPLE 1 40
|_* Pooling by OLS with panel-corrected standard errors
|_POOL C Q / NCROSS=4 OLS HETCOV RSTAT
POOLED CROSS-SECTION TIME-SERIES ESTIMATION
    40 TOTAL OBSERVATIONS
     4 CROSS-SECTIONS
    10 TIME-PERIODS

DEPENDENT VARIABLE = C
```

```
POOLING BY OLS
USING PANEL-CORRECTED COVARIANCE MATRIX

LM TEST FOR CROSS-SECTION HETEROSKEDASTICITY   5.8261
CHI-SQUARE WITH    3 D.F.     P-VALUE= 0.12039

BREUSCH-PAGAN LM TEST FOR DIAGONAL COVARIANCE MATRIX   25.240
CHI-SQUARE WITH    6 D.F.     P-VALUE= 0.00031

R-SQUARE = 0.7147
VARIANCE OF THE ESTIMATE-SIGMA**2 =   48.394
STANDARD ERROR OF THE ESTIMATE-SIGMA =   6.9566
SUM OF SQUARED ERRORS-SSE=   1839.0
MEAN OF DEPENDENT VARIABLE =   39.836
LOG OF THE LIKELIHOOD FUNCTION = -133.319

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      38 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
Q          1.1316     0.1042       10.86     0.000 0.870     0.8454     0.8146
CONSTANT   7.3852      2.998        2.464     0.018 0.371     0.0000     0.1854

DURBIN-WATSON = 0.6458    VON NEUMANN RATIO = 0.6623    RHO =  0.63342
RESIDUAL SUM =  0.24869E-13  RESIDUAL VARIANCE =   48.394
SUM OF ABSOLUTE ERRORS=   228.79
R-SQUARE BETWEEN OBSERVED AND PREDICTED = 0.7147
RUNS TEST:    7 RUNS,   18 POS,    0 ZERO,   22 NEG  NORMAL STATISTIC = -4.4668

|_* Pooling with cross-section heteroskedasticity,
|_* contemporaneous cross-section correlation and AR1 errors
|_POOL C Q / NCROSS=4 FULL DN NOMULSIGSQ
POOLED CROSS-SECTION TIME-SERIES ESTIMATION
    40 TOTAL OBSERVATIONS
     4 CROSS-SECTIONS
    10 TIME-PERIODS

DEPENDENT VARIABLE = C
    THE DN OPTION IS IN EFFECT

MODEL ASSUMPTIONS:
    DIFFERENT ESTIMATED RHO FOR EACH CROSS-SECTION
    FULL PHI MATRIX - CROSS-SECTION CORRELATION

OLS COEFFICIENTS
   1.1316       7.3852

RHO VECTOR
  0.53953      0.45388E-02  0.94290      0.70678

SAME ESTIMATED RHO FOR ALL CROSS-SECTIONS =  0.63342

VARIANCES (DIAGONAL OF PHI MATRIX)
   33.206       16.460       9.0686       36.046
PHI MATRIX
   33.206
   1.1912       16.460
  -2.2770      -5.1726       9.0686
  -25.487      -7.6641      0.68614       36.046

BUSE [1973] R-SQUARE = 0.9260      BUSE RAW-MOMENT R-SQUARE = 0.9920
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.99598
```

```
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.99799
SUM OF SQUARED ERRORS-SSE=   39.839
MEAN OF DEPENDENT VARIABLE =   39.836
LOG OF THE LIKELIHOOD FUNCTION = -112.331

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
Q          1.1658     0.5221E-01   22.33     0.000 0.964     0.8710     0.8392
CONSTANT   7.4442      1.742       4.273     0.000 0.570     0.0000     0.1869
|_* Create cross-section dummy variables.
|_*  Set the number of cross-sections
|_GEN1 NC=4
|_MATRIX CSDUM=SEAS(40,-NC)
|_*  Set the number of time periods
|_GEN1 NT=10
|_*  Generate an index for each cross-section
|_GENR CSINDEX=SUM(SEAS(NT))
|_*  Generate a repeating time index for the 10 observations
|_GENR TINDEX=TIME(0)-NT*(CSINDEX-1)

|_* Estimation with cross-section dummy variables
|_POOL C Q CSDUM / NCROSS=4 NOCONSTANT FULL DN NOMULSIGSQ
POOLED CROSS-SECTION TIME-SERIES ESTIMATION
    40 TOTAL OBSERVATIONS
     4 CROSS-SECTIONS
    10 TIME-PERIODS

DEPENDENT VARIABLE = C
    THE DN OPTION IS IN EFFECT

MODEL ASSUMPTIONS:
    DIFFERENT ESTIMATED RHO FOR EACH CROSS-SECTION
    FULL PHI MATRIX - CROSS-SECTION CORRELATION

OLS COEFFICIENTS
   1.1190       2.3150       10.110       2.3854       16.171

RHO VECTOR
 -0.36486     -0.23682     -0.76528E-01 -0.50981

SAME ESTIMATED RHO FOR ALL CROSS-SECTIONS = -0.33758

VARIANCES (DIAGONAL OF PHI MATRIX)
   12.536       13.099       5.5972       11.077
PHI MATRIX
   12.536
  0.80966       13.099
  -2.8281      -1.1276       5.5972
  -8.1934     -0.80424      0.60875       11.077

BUSE [1973] R-SQUARE = 0.9608      BUSE RAW-MOMENT R-SQUARE = 0.9988
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.99771
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.99885
SUM OF SQUARED ERRORS-SSE=   39.908
MEAN OF DEPENDENT VARIABLE =   39.836
LOG OF THE LIKELIHOOD FUNCTION = -98.8112

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
```

```
Q          1.0687     0.3995E-01   26.75     0.000 0.976     0.7984     0.7693
CSDUM      3.7587      1.473       2.551     0.011 0.396     0.1282     0.0236
CSDUM      11.444      1.516       7.547     0.000 0.787     0.3904     0.0718
CSDUM      3.6919      1.251       2.952     0.003 0.446     0.1259     0.0232
CSDUM      17.644      1.306       13.51     0.000 0.916     0.6018     0.1107
| STOP
```

The final estimation includes a set of cross-section dummy variables in the matrix variable *CSDUM*. This permits different cross-section intercepts. The **NOCONSTANT** option is specified to avoid the dummy variable trap. The estimation output shows that the individual intercepts are 3.76, 11.44, 3.69 and 17.64 for industries 1, 2, 3 and 4 respectively.

25. PROBIT AND LOGIT REGRESSION

"The deliverance of the saints must take place some time before 1914."
Charles Taze Russell
American religious leader, 1910

"The deliverance of the saints must take place some time after 1914."
Charles Taze Russell
American religious leader, 1923

The probit and logit models can be used for the analysis of binary choice models where the dependent variable Y_t is a 0–1 dummy variable. Some references are Greene [2003, Chapter 21]; Wooldridge [2006, Chapter 17]; Chow [1983, Chapter 8]; Griffiths, Hill and Judge [1993, Chapter 23]; Hanushek and Jackson [1977, Chapter 7]; Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 18]; Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 19]; Maddala [1977, Chapter 9-7]; Maddala [1983, Chapter 2]; and Pindyck and Rubinfeld [1998, Chapter 11]. Examples of probit and logit estimation using SHAZAM can be found in, respectively, White [1975] and Cameron and White [1986].

The estimation algorithms implemented with the **PROBIT** and **LOGIT** commands use fast iterative methods which usually converge in 4 or 5 iterations. The maximum likelihood estimation routines are based on computer programs originally written by John Cragg. SHAZAM **TEST** commands can be used following estimation. **RESTRICT** commands are not permitted.

A "utility index" I is defined for individual t as $I_t = X_t'\beta$.

The choice probabilities must lie between zero and one. However, the index I is in the range $(-\infty, +\infty)$. This can be translated to a 0–1 range by the use of a cumulative distribution function so that $\text{Prob}(Y_t = 1) = P_t = F(I_t) = F(X_t'\beta)$. Two alternative choices for F are:

The probit model:
$$F(X_t'\beta) = \int_{-\infty}^{I_t} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt$$

The logit model:
$$F(X_t'\beta) = \frac{1}{1 + \exp(-X_t'\beta)}$$

Maximum Likelihood Estimation

With the assumption of independent observations the log-likelihood function for a sample of N observations is:

$$L(\beta) = \sum_{t=1}^{N} \{Y_t \ln[F(X_t'\beta)] + (1 - Y_t) \ln[1 - F(X_t'\beta)]\}$$

The estimate of the asymptotic covariance matrix for the maximum likelihood estimates $\hat{\beta}$ is computed as the inverse of the negative of the matrix of second derivatives of the log-likelihood function.

A test of the null hypothesis that all the slope coefficients are zero can be carried out using a likelihood ratio test. If S is the number of successes (Y_t=1) observed in N observations, then for both the probit and logit models, the maximum value of the log likelihood function under the null hypothesis is:

$$L(0) = S \ln\left(\frac{S}{N}\right) + (N - S) \ln\left(\frac{N - S}{N}\right)$$

The above statistic is printed on the SHAZAM output as `LOG-LIKELIHOOD FUNCTION WITH CONSTANT TERM ONLY` or as `LOG-LIKELIHOOD (0)`. The `BINOMIAL ESTIMATE` is S/N. If all coefficients except the intercept are zero the `LIKELIHOOD RATIO TEST` statistic $2[L(\hat{\beta}) - L(0)]$ has an asymptotic $\chi^2_{(k-1)}$ distribution.

Interpreting the Results

Predicted probabilities are computed as $\hat{Y}_t = \hat{P}_t = F(X_t'\hat{\beta})$.

The coefficients tell the effect of a change in the independent variable on the utility index. The impact of a unit increase in an explanatory variable on the choice probability is obtained by estimating the marginal effects as:

$$\frac{\partial \hat{P}_t}{\partial X_{kt}} = \text{scale} \cdot \hat{\beta}_k \qquad \text{where}$$

for the probit model: $\text{scale} = f(X_t'\hat{\beta})$ f() is the normal density function

and for the logit model: $\text{scale} = \dfrac{\exp(-X_t'\hat{\beta})}{[1+\exp(-X_t'\hat{\beta})]^2}$

For reporting the marginal effects, the scale factor is evaluated at the sample means of the variables.

Suppose the k^{th} explanatory variable is a 0–1 dummy variable. The change in the probability of a success (Y=1) that results from changing X_k from zero to one, holding all other variables at some fixed values, denoted by X_*, is given by the difference:

$$\text{Prob}(Y=1|X_k=1, X_*) - \text{Prob}(Y=1|X_k=0, X_*)$$

Values must be set for X_*. An approach is to set values to represent a "typical case". A "typical case" can be defined by setting all dummy variables to their modal values and all other variables to their mean values.

The SHAZAM output reports marginal effects for all dummy explanatory variables in the section labelled `PROBABILITIES FOR A TYPICAL CASE`. For a dummy explanatory variable X_k, the column labelled `X=0` reports the probability:

$$\text{Prob}(Y=1|X_k=0, X_*)$$

and the column labelled `X=1` gives the probability:

$$\text{Prob}(Y=1|X_k=1, X_*)$$

The final column labelled `MARGINAL EFFECT` reports the difference between the two probabilities. The values used for X_* are reported in the column labelled `CASE VALUES`. These are set as the modal values for dummy variables and sample averages for other variables.

The elasticity gives the percentage change in the choice probability in response to a percentage change in the explanatory variable. For the k^{th} coefficient this is estimated as:

$$E_{kt} = \left(\frac{\partial \hat{P}_t}{\partial X_{kt}}\right)\frac{X_{kt}}{F(X_t'\hat{\beta})}$$

Since the elasticity is different for every observation it is often reported at the mean values of X. On the SHAZAM output the ELASTICITY AT MEANS is computed as:

$$\bar{E}_k = \left(\frac{\partial \hat{P}}{\partial \bar{X}_k} \right) \frac{\bar{X}_k}{F(\bar{X}'\hat{\beta})}$$

Alternatively, Hensher and Johnson [1981, Eq. 3.44] propose the WEIGHTED AGGREGATE ELASTICITY calculated as:

$$\bar{E}_k^W = \sum_{t=1}^{N} \hat{P}_t E_{kt} \Big/ \sum_{t=1}^{N} \hat{P}_t$$

Measuring Goodness of Fit

Various researchers have proposed different ways of computing R^2 measures for the probit and logit models and the practitioner must decide which statistic is the most appealing. The SHAZAM output prints a variety of these statistics to choose from.

	Goodness-of-fit measure	*Reference*
ESTRELLA R-SQUARE	$1 - \left(L(\hat{\beta}) / L(0)\right)^{(-2/N)\cdot L(0)}$	Estrella [1998]
MADDALA R-SQUARE	$1 - \exp\{\ 2[L(0) - L(\hat{\beta})] / N\}$	Maddala [1983, Eq.2.44]
CRAGG-UHLER R-SQUARE	$\dfrac{1 - \exp\{\ 2[L(0) - L(\hat{\beta})] / N\}}{1 - \exp\{\ 2L(0) / N\}}$	Cragg &Uhler [1970, p. 400]
MCFADDEN R-SQUARE	$1 - L(\hat{\beta}) / L(0)$	McFadden [1974]
CHOW R-SQUARE	$1 - \dfrac{\sum_{t=1}^{N} (Y_t - \hat{Y}_t)^2}{\sum_{t=1}^{N} (Y_t - \bar{Y})^2}$	Chow [1983, p. 262, Eq. 47]

Denote R^2 as the McFadden goodness-of-fit measure. Hensher and Johnson [1981, p. 52] recommend adjusting this for degrees of freedom and on the SHAZAM output the statistic ADJUSTED FOR DEGREES OF FREEDOM is computed as:

$$1 - \frac{L(\hat{\beta})/(N-K)}{L(0)/(N-1)}$$

An alternative to the likelihood ratio test is proposed in Chow [1983, p. 263]. Under the null hypothesis that all the slope coefficients are zero the statistic:

$$\frac{R^2/(K-1)}{(1-R^2)/K}$$ is APPROXIMATELY F DISTRIBUTED with (K–1, K) degrees of freedom.

A summary of predictive ability is the 2 x 2 PREDICTION SUCCESS TABLE reported as:

		ACTUAL	
		0	1
	0	N_{11}	N_{12}
PREDICTED	1	N_{21}	N_{22}

The decision rule is to predict $Y_t = 0$ when $\hat{P}_t < 0.5$ (equivalent to $\hat{I}_t < 0$) and predict $Y_t = 1$ when $\hat{P}_t \geq 0.5$. The table tells the numeric counts of hits and misses using this decision rule.

A naive model predicts 0 if most observations are 0 and predicts 1 if most observations are 1. The NAIVE MODEL PERCENTAGE OF RIGHT PREDICTIONS is then S/N if S > N/2 and (N–S)/N otherwise.

Other calculated statistics are:

EXPECTED OBSERVATIONS AT 0 $\sum_{t=1}^{N}(1-\hat{P}_t)$

EXPECTED OBSERVATIONS AT 1 $\sum_{t=1}^{N}\hat{P}_t$

SUM OF SQUARED RESIDUALS $\sum_{t=1}^{N}(Y_t-\hat{Y}_t)^2$

WEIGHTED SUM OF SQUARED RESIDUALS $\sum_{t=1}^{N}\frac{(Y_t-\hat{Y}_t)^2}{\hat{Y}_t(1-\hat{Y}_t)}$

A discussion on the interpretation of these statistics in the context of binary choice models is given in Amemiya [1981, pp. 1502-7].

Hensher and Johnson [1981, p.54] provide a more enhanced version of the prediction success table where the N_{ij} elements are defined somewhat differently as:

$$N_{11} = \sum_{t=1}^{N} (1 - Y_t)(1 - \hat{P}_t) \; ; \quad N_{12} = \sum_{t=1}^{N} (1 - Y_t)\hat{P}_t \; ; \quad N_{21} = \sum_{t=1}^{N} Y_t(1 - \hat{P}_t) \quad \text{and} \quad N_{22} = \sum_{t=1}^{N} Y_t\hat{P}_t$$

The HENSHER-JOHNSON PREDICTION SUCCESS TABLE is then obtained as:

ACTUAL	PREDICTED CHOICE 0	1	OBSERVED COUNT	OBSERVED SHARE
0	N_{11}	N_{12}	$N_{1\cdot}$	$N_{1\cdot}/N_{\cdot\cdot}$
1	N_{21}	N_{22}	$N_{2\cdot}$	$N_{2\cdot}/N_{\cdot\cdot}$
PREDICTED COUNT	$N_{\cdot 1}$	$N_{\cdot 2}$	$N_{\cdot\cdot}$	1
PREDICTED SHARE	$N_{\cdot 1}/N_{\cdot\cdot}$	$N_{\cdot 2}/N_{\cdot\cdot}$	1	
PROP. SUCCESSFUL	$N_{11}/N_{\cdot 1}$	$N_{22}/N_{\cdot 2}$	$(N_{11}+N_{22})/N_{\cdot\cdot}$	
SUCCESS INDEX	$\frac{N_{11}}{N_{\cdot 1}} - \frac{N_{\cdot 1}}{N_{\cdot\cdot}}$	$\frac{N_{22}}{N_{\cdot 2}} - \frac{N_{\cdot 2}}{N_{\cdot\cdot}}$	$\sum_{i=1}^{2} \frac{N_{ii}}{N_{\cdot\cdot}} - \left(\frac{N_{\cdot i}}{N_{\cdot\cdot}}\right)^2$	
PROPORTIONAL ERROR	$(N_{\cdot 1}-N_{1\cdot})/N_{\cdot\cdot}$	$(N_{\cdot 2}-N_{2\cdot})/N_{\cdot\cdot}$		

The NORMALIZED SUCCESS INDEX is the sum of the SUCCESS INDEXES weighted by the PROPORTIONAL ERROR.

PROBIT *AND* LOGIT *COMMAND OPTIONS*

In general, the formats of the **PROBIT** and **LOGIT** commands are:

PROBIT *depvar indeps / options*

LOGIT *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of desired options. Options as defined for the **OLS** command that are available are:

MAX, **NOCONSTANT**, **NONORM**, **PCOR**, **BEG=**, **END=** **COEF=**, **COV=**, **STDERR=**, **TRATIO=** and **WEIGHT=**

Additional options available on the **PROBIT** and **LOGIT** commands are:

DUMP **DUMP**s the matrix of second derivatives and possibly some other output that the user normally does not want to see.

LIST **LIST**s, for each observation, the value of the index I, the predicted and observed values of the dependent variable, and a plot of the predicted values. It will also print the residual statistics obtained with **RSTAT**. Note that the predicted values, rather than the residuals, are plotted with the **LIST** option in the probit and logit models since a plot of the residuals is less useful.

LOG Computes elasticities at the mean and weighted aggregate elasticities assuming that the explanatory variables are **LOG**-transformed variables.

PCOV **P**rints the estimated asymptotic **COV**ariance matrix of the coefficients. The matrix is the inverse of the negative of the matrix of second derivatives of the log-likelihood function.

RSTAT Prints **R**esidual **STAT**istics, but no listing of the observations. While a variety of statistics are printed (for example, Durbin-Watson), they should be used with care since they may not be valid for the probit and logit models.

CONV= Sets the **CONV**ergence criterion for the log-likelihood function. Let $L^{(i)}$ be the value of the log-likelihood function at iteration i. The estimation converges when $(L^{(i)} - L^{(i-1)})/|L^{(i)}| < \delta$. The value for δ is set with **CONV=** and the default is 0.001.

IMR= Saves the computed inverse Mill's ratio (or hazard rate) in the variable specified (only for use with **PROBIT**). This is estimated as $f(X_t'\hat{\beta})/F(X_t'\hat{\beta})$ if Y_t=1 where f( ) is the standard normal density function and F( ) is the standard normal cumulative distribution function. When Y_t=0 SHAZAM computes the negative of the hazard rate as $f(X_t'\hat{\beta})/[F(X_t'\hat{\beta}) - 1]$. The ratio is discussed in Heckman [1979, p.156] and

applications can be found in Maddala [1983, Chapter 8-9] and Berndt [1991, Chapter 11]. An example of the use of the **IMR=** option is given for a two-stage estimation procedure at the end of this chapter. In the first stage a probit model is specified and estimated. In the second stage the inverse Mill's ratio is included as a regressor in an OLS estimation with a selected sample. This is intended as a correction for sample selection bias.

INDEX= Saves the computed **INDEX** in the variable specified.

ITER= Sets the maximum number of **ITER**ations allowed. The default is 25.

PITER= Specifies the frequency with which the **ITER**ations will be **P**rinted. The default is **PITER=1**. To prevent any information from being printed **PITER=0** must be specified.

PREDICT= Saves the **PREDICT**ed probabilities in the variable specified.

If the **WEIGHT=** option is used, the method used follows that described for the **REPLICATE** option in **OLS**.

The temporary variables available after the **PROBIT** or **LOGIT** commands are:

$ERR	Error code
$K	Number of coefficients
$LLF	Log of the Likelihood Function
$N	Number of observations
$VAL1	Scale factor used to calculate marginal effects

EXAMPLES

Logit Model Estimation

This example shows the estimation results from the logit procedure using the voting data set from Pindyck and Rubinfeld [1998, Table 11.8, p. 332]. The variable *YESVM* is equal to 1 if the individual voted yes and 0 if the individual voted no.

```
|_LOGIT YESVM PUB12 PUB34 PUB5 PRIV YEARS SCHOOL LOGINC PTCON / PITER=0
 LOGIT ANALYSIS     DEPENDENT VARIABLE =YESVM    CHOICES =  2
      95. TOTAL OBSERVATIONS
      59. OBSERVATIONS AT ONE
      36. OBSERVATIONS AT ZERO
  25 MAXIMUM ITERATIONS
CONVERGENCE TOLERANCE =0.00100

LOG OF LIKELIHOOD WITH CONSTANT TERM ONLY =    -63.037
BINOMIAL  ESTIMATE = 0.6211

ITERATION  5      LOG OF LIKELIHOOD FUNCTION =   -53.303
ITERATION  5 ESTIMATES
 0.58364      1.1261     0.52606    -0.34142    -0.26127E-01  2.6250
  2.1872     -2.3945     -5.2014

                                ASYMPTOTIC                          WEIGHTED
VARIABLE    ESTIMATED       STANDARD     T-RATIO    ELASTICITY      AGGREGATE
  NAME     COEFFICIENT       ERROR                   AT MEANS      ELASTICITY
PUB12         0.58364      0.68778      0.84858      0.93986E-01  0.91051E-01
PUB34          1.1261      0.76820       1.4659      0.11827      0.96460E-01
PUB5          0.52606       1.2693      0.41445      0.73664E-02  0.69375E-02
PRIV         -0.34142      0.78299     -0.43605     -0.11952E-01 -0.12037E-01
YEARS        -0.26127E-01  0.26934E-01 -0.97006     -0.73996E-01 -0.68592E-01
SCHOOL         2.6250       1.4101       1.8616      0.10108      0.28999E-01
LOGINC         2.1872      0.78781       2.7763       7.2529       6.7561
PTCON         -2.3945       1.0813      -2.2145      -5.5262      -5.1745
CONSTANT      -5.2014       7.5503     -0.68890      -1.7298      -1.6137

SCALE FACTOR =   0.22197

VARIABLE      MARGINAL      ----- PROBABILITIES FOR A TYPICAL CASE -----
  NAME         EFFECT         CASE          X=0          X=1         MARGINAL
                             VALUES                                  EFFECT
PUB12         0.12955       0.0000      0.44231      0.58706      0.14476
PUB34         0.24996       0.0000      0.44231      0.70978      0.26747
PUB5          0.11677       0.0000      0.44231      0.57304      0.13073
PRIV         -0.75785E-01   0.0000      0.44231      0.36049     -0.81814E-01
YEARS        -0.57995E-02   8.5158
SCHOOL        0.58267       0.0000      0.44231      0.91631      0.47400
LOGINC        0.48548       9.9711
PTCON        -0.53150       6.9395

LOG-LIKELIHOOD FUNCTION =  -53.303
LOG-LIKELIHOOD(0)  =   -63.037
LIKELIHOOD RATIO TEST  =    19.4681    WITH     8  D.F.   P-VALUE= 0.01255

ESTRELLA R-SQUARE           0.19956
MADDALA R-SQUARE            0.18529
CRAGG-UHLER R-SQUARE        0.25218
MCFADDEN R-SQUARE           0.15442
     ADJUSTED FOR DEGREES OF FREEDOM        0.75759E-01
     APPROXIMATELY F-DISTRIBUTED    0.20544      WITH      8  AND   9  D.F.
CHOW R-SQUARE               0.17197

         PREDICTION SUCCESS TABLE
                     ACTUAL
                0            1
          0     18.           7.
PREDICTED 1     18.          52.
```

```
NUMBER OF RIGHT PREDICTIONS =         70.0
PERCENTAGE OF RIGHT PREDICTIONS =    0.73684
NAIVE MODEL PERCENTAGE OF RIGHT PREDICTIONS =    0.62105

EXPECTED OBSERVATIONS AT 0  =         36.0   OBSERVED =     36.0
EXPECTED OBSERVATIONS AT 1  =         59.0   OBSERVED =     59.0
SUM OF SQUARED "RESIDUALS" =           18.513
WEIGHTED SUM OF SQUARED "RESIDUALS" =     86.839

HENSHER-JOHNSON PREDICTION SUCCESS TABLE
                                                 OBSERVED    OBSERVED
                    PREDICTED  CHOICE              COUNT       SHARE
        ACTUAL            0          1
           0           17.591     18.409     36.000      0.379
           1           18.409     40.591     59.000      0.621

PREDICTED COUNT        36.000     59.000     95.000      1.000
PREDICTED SHARE         0.379      0.621      1.000
PROP. SUCCESSFUL        0.489      0.688      0.612
SUCCESS INDEX           0.110      0.067      0.083
PROPORTIONAL ERROR      0.000      0.000
NORMALIZED SUCCESS INDEX                       0.177
```

For the school budget voting study, the variable *SCHOOL* is a dummy variable equal to one if the individual is employed as a school teacher and zero otherwise. A question to consider is: Are school teachers more likely to vote yes in the school budget referendum, holding all other variables fixed (that is, relative to individuals that are not school teachers but otherwise have similar characteristics) ? The positive estimated coefficient on the school dummy variable indicates a higher probability of a yes vote for a school teacher.

To get an estimate of the magnitude of the effect, set the explanatory variables to values that represent a "typical voter" in the sample. The definitions of the variables in the data set can be reviewed. The dummy variables *PUB12*, *PUB34*, *PUB5* and *PRIV* all have a mode of zero. This describes a voter with no children in public or private school. The "typical case" values for the other explanatory variables (*YEARS*, *LOGINC* and *PTCON*) are the sample means.

An individual that is not a school teacher, with "typical" characteristics on all variables in the model, has a probability of a yes vote of 0.44231. If the individual is a school teacher the probability increases to 0.91631. The marginal effect is the difference 0.474.

Note that the marginal effect obtained using the less precise method of taking the partial derivative with respect to the school dummy variable was calculated to be 0.58267.

Probit Model Estimation

The SHAZAM output given below uses data from T. Mroz included in Berndt [1991, Chapter 11]. The labor force participation for married women is given by the 0-1 dummy variable *LFP*. The explanatory variables measure various attributes of the woman and her family. The wage rate *WW* is observed only for the working women in the sample. To obtain a wage rate estimate for the entire sample a wage determination equation is first estimated and used for extrapolation purposes. The **PROBIT** command is then used to estimate a model of labor force participation that predicts the probability that an individual will join the labor force.

```
|_SAMPLE 1 753
|_READ (MROZ) LFP WHRS KL6 K618 WA WE WW RPWG HHRS HA HE HW FAMINC &
|             MTR WMED WFED UN CIT AX  / SKIPLINES=1
UNIT 88 IS NOW ASSIGNED TO: MROZ
  19 VARIABLES AND      753 OBSERVATIONS STARTING AT OBS       1
|_* Analyze wife's property income
|_GENR PRIN=(FAMINC-WW*WHRS)/1000
|_DIM LWW 753
|_GENR WA2=WA*WA
|_* Restrict the sample to those who work
|_SAMPLE 1 428
|_GENR LWW=LOG(WW)
|_* Estimate a wage determination equation
|_?OLS LWW WA WA2 WE CIT AX
|_* Estimate a predicted wage for non-workers
|_?FC / PREDICT=LWW BEG=429 END=753
|_* Probit Estimation
|_SAMPLE 1 753
|_PROBIT LFP LWW PRIN KL6 K618 WA WE UN CIT / PITER=0
PROBIT ANALYSIS     DEPENDENT VARIABLE =LFP      CHOICES =  2
     753. TOTAL OBSERVATIONS
     428. OBSERVATIONS AT ONE
     325. OBSERVATIONS AT ZERO
  25 MAXIMUM ITERATIONS
CONVERGENCE TOLERANCE = .00100

LOG OF LIKELIHOOD WITH CONSTANT TERM ONLY =    -514.87
BINOMIAL  ESTIMATE =  .5684

ITERATION  3      LOG OF LIKELIHOOD FUNCTION =   -450.72
ITERATION  3 ESTIMATES
  .23998     -.21238E-01 -.87938     -.32061E-01 -.34542E-01  .13204
 -.10666E-01  .11466E-01  .53839

                                ASYMPTOTIC                        WEIGHTED
VARIABLE    ESTIMATED       STANDARD     T-RATIO    ELASTICITY     AGGREGATE
  NAME     COEFFICIENT       ERROR                  AT MEANS      ELASTICITY
LWW            .23998       .93507E-01   2.5664       .18017       .15423
PRIN         -.21238E-01    .46991E-02  -4.5195      -.29063      -.25371
KL6          -.87938        .11450      -7.6799      -.14212      -.11515
K618         -.32061E-01    .40672E-01  -.78829      -.29497E-01  -.25672E-01
WA           -.34542E-01    .76642E-02  -4.5069      -.99895      -.88764
WE            .13204        .25962E-01   5.0861       1.1030       .95967
UN           -.10666E-01    .15955E-01  -.66851      -.62531E-01  -.54989E-01
```

```
CIT             .11466E-01   .10749       .10667       .50106E-02   .44439E-02
CONSTANT        .53839       .48129       1.1186       .36602       .32165

SCALE FACTOR =   0.39166

VARIABLE      MARGINAL      ----- PROBABILITIES FOR A TYPICAL CASE -----
  NAME         EFFECT        CASE          X=0          X=1         MARGINAL
                            VALUES                                  EFFECT
LWW           0.93991E-01   1.1043
PRIN         -0.83180E-02   20.129
KL6          -0.34442      0.23772
K618         -0.12557E-01   1.3533
WA           -0.13529E-01   42.538
WE            0.51716E-01   12.287
UN           -0.41774E-02   8.6235
CIT           0.44909E-02   1.0000      0.57321      0.57770      0.44923E-02

LOG-LIKELIHOOD FUNCTION =  -450.72
LOG-LIKELIHOOD(0)  =   -514.87
LIKELIHOOD RATIO TEST  =    128.306    WITH     8  D.F.   P-VALUE= 0.00000

ESTRELLA R-SQUARE           0.16638
MADDALA R-SQUARE            0.15667
CRAGG-UHLER R-SQUARE        0.21022
MCFADDEN R-SQUARE           0.12460
     ADJUSTED FOR DEGREES OF FREEDOM        0.11519
     APPROXIMATELY F-DISTRIBUTED    0.16013      WITH      8  AND   9  D.F.
CHOW R-SQUARE               0.15809

         PREDICTION SUCCESS TABLE
                       ACTUAL
                0             1
          0    163.           79.
PREDICTED 1    162.          349.

NUMBER OF RIGHT PREDICTIONS =        512.
PERCENTAGE OF RIGHT PREDICTIONS =    0.67995
NAIVE MODEL PERCENTAGE OF RIGHT PREDICTIONS =    0.56839

EXPECTED OBSERVATIONS AT 0  =        323.5   OBSERVED =    325.0
EXPECTED OBSERVATIONS AT 1  =        429.5   OBSERVED =    428.0
SUM OF SQUARED "RESIDUALS" =           155.52
WEIGHTED SUM OF SQUARED "RESIDUALS" =     746.36

HENSHER-JOHNSON PREDICTION SUCCESS TABLE
                                              OBSERVED    OBSERVED
                    PREDICTED  CHOICE          COUNT       SHARE
        ACTUAL           0          1
           0          168.692    156.308    325.000       .432
           1          154.761    273.239    428.000       .568

PREDICTED COUNT       323.454    429.546    753.000      1.000
PREDICTED SHARE          .430       .570      1.000
PROP. SUCCESSFUL         .522       .636       .587
SUCCESS INDEX            .092       .066       .077
PROPORTIONAL ERROR      -.002       .002
NORMALIZED SUCCESS INDEX                        .157
```

26. ROBUST ESTIMATION

"39..This appears to be the first uninteresting number, which of course makes it an especially interesting number, because it is the smallest number to have the property of being uninteresting. It is therefore also the first number to be simultaneously interesting and uninteresting."

David Wells, 1986
The Penguin Dictionary of Curious and Interesting Numbers

SHAZAM can perform the robust estimation methods described in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 22]. As a prelude to this chapter, the **OLS** command can be used to obtain regression diagnostics such as tests for normal errors and tests for detecting influential observations (see the **OLS** command options **DFBETAS**, **GF**, **INFLUENCE** and **RSTAT**). The **ROBUST** command provides a number of alternative robust estimation methods. These methods give coefficient estimators that do not necessarily follow a t-distribution, so hypothesis testing must be done carefully. Often the distribution is unknown.

ESTIMATION UNDER MULTIVARIATE t ERRORS

The multivariate-t regression model specifies an error distribution with fatter tails than the normal distribution. Two cases to consider are one where the residuals are independent and one where the residuals are uncorrelated but not independent. For the uncorrelated but not independent error case (specified with the **UNCOR** option on the **ROBUST** command) the coefficient estimates are identical to OLS estimates but the covariance matrix is estimated as:

$$\frac{v\tilde{\sigma}^2}{v-2}(X'X)^{-1} \qquad \text{where} \qquad \tilde{\sigma}^2 = \frac{e'e}{N}$$

and v is the degrees of freedom specified with the **MULTIT=** option and is only defined for $v > 2$.

The independent error case is discussed in Kelejian and Prucha [1985] and Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 22.3].

ESTIMATION USING REGRESSION QUANTILES

The method of regression quantiles is described in Koenker and Bassett [1978] and in Koenker and D'Orey [1987]. For a given a value of θ such that $0 < \theta < 1$ the θ^{th} sample regression quantile is found by minimizing the function:

$$\sum_{(t|Y_t \geq X_t'\beta)} \theta \left|Y_t - X_t'\beta\right| + \sum_{(t|Y_t < X_t'\beta)} (1-\theta) \left|Y_t - X_t'\beta\right|$$

The solution to the minimization problem is the estimator $\hat{\beta}^*(\theta)$. The value of the minimized function is printed on the SHAZAM output as OBJECTIVE FUNCTION. The output also includes a statistic labelled EMPIRICAL QUANTILE FUNCTION AT MEANS which is computed as the predicted value at the mean values of the independent variables.

Least Absolute Error Estimation

When $\theta = 0.5$ the objective is to find the set of coefficients that minimizes the sum of absolute errors:

$$\sum_{t=1}^{N} \left|Y_t - X_t'\beta\right|$$

The estimator $\hat{\beta}^*(0.5)$ is the least absolute errors (LAE) estimator, also known as minimum absolute deviations (MAD), least absolute values (LAV) or the L1 estimator. The covariance matrix of the LAE estimator is estimated as:

$$[2\hat{f}(0)]^{-2}(X'X)^{-1}$$

where f(0) is the value of the density at the median and an estimator is:

$$\hat{f}(0) = \frac{2d}{N(e_{(m+d)} - e_{(m-d)})}$$

where $e_{(t)}$ are ordered residuals and $m \approx N/2$. The value of d can be specified with the **DIFF=** option on the **ROBUST** command.

Linear Functions of Regression Quantiles

An estimator that is a linear function of regression quantiles is given by:

$$\hat{\beta}(\pi) = \sum_{i=1}^{M} \pi_i \hat{\beta}^*(\theta_i)$$

where $\pi = (\pi_1, \ldots, \pi_M)'$ is a symmetric weighting scheme. Alternative schemes described in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 22.4.2] and implemented with the **ROBUST** command are:

Estimator	**ROBUST** *option*	θ	π
Five quantile	**FIVEQUAN**	(.1, .25, .5, .75, .9)	(.05, .25, .4, .25, .05)
Gastwirth	**GASTWIRT**	(.33, .5, .67)	(.3, .4, .3)
Tukey trimean	**TUKEY**	(.25, .5, .75)	(.25, .5, .25)

Another estimator is: $\hat{\beta} = \frac{1}{M} \sum_{i=1}^{M} \hat{\beta}^*(\theta_i)$

where the θ_i values are equally spaced. This rule is used by specifying the **THETAB=**, **THETAE=** and **THETAI=** options on the **ROBUST** command.

Trimmed Least Squares

The trimmed least squares method constructs the estimators $\hat{\beta}^*(\alpha)$ and $\hat{\beta}^*(1-\alpha)$ where α is the desired trimming proportion ($0 < \alpha < 0.5$). The value for α is specified with the **TRIM=** option on the **ROBUST** command. The observations where $Y_t - X_t'\hat{\beta}^*(\alpha) \le 0$ or $Y_t - X_t'\hat{\beta}^*(1-\alpha) \ge 0$ are discarded and OLS is then applied to the remaining observations to get the estimator $\tilde{\beta}_\alpha$. The covariance matrix is estimated as:

$$V(\tilde{\beta}_\alpha) = \hat{\sigma}^2(\alpha)(X'X)^{-1}$$

where $$\hat{\sigma}^2(\alpha) = \frac{1}{(1-2\alpha)^2}\left(\frac{e'e}{N-K} + \alpha(c_1^2 + c_2^2) - \alpha^2(c_1 + c_2)^2\right)$$

with $c_1 = \overline{X}'[\hat{\beta}^*(\alpha) - \tilde{\beta}_\alpha]$ and $c_2 = \overline{X}'[\hat{\beta}^*(1-\alpha) - \tilde{\beta}_\alpha]$

ROBUST *COMMAND OPTIONS*

In general, the format of the **ROBUST** command is:

ROBUST *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of options.

Options as defined for the **OLS** command that are available are:

GRAPH, **LININV**, **LINLOG**, **LIST**, **LOGINV**, **LOGLIN**, **LOGLOG**, **MAX**, **NOCONSTANT**, **PCOR**, **PCOV**, **RSTAT**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT=**, **RESID=**, **STDERR=** and **TRATIO=**

Additional options available on the **ROBUST** command are:

FIVEQUAN Specifies the "five quantile estimator" as described above.

GASTWIRT Specifies the Gastwirth weighting scheme for the regression quantile estimator as described above.

LAE Specifies that least absolute error (**LAE**) estimation is desired. This is the default method. Also see the **DIFF=** option.

TUKEY Specifies the Tukey trimean weighting scheme in computing regression quantiles as described above.

UNCOR Used with the **MULTIT=** option described below to specify that the *uncorrelated* error model is requested. If this option is not used the *independent* error model is assumed.

CONV= Used with the **MULTIT=** option described below to specify a convergence criterion δ to stop the iterative procedure used to obtain the estimates. For $S^{(i)}$ the sum of squared errors at iteration i the iterations will stop when:

$$\left| S^{(i)} - S^{(i-1)} \right| < \delta$$

The default is **CONV=0.01**.

DIFF= Specifies a value of d to use with the **LAE** and **THETA=** options and for methods that use linear functions of regression quantiles. The parameter d is the differential used when selecting ordered residuals to use in computing the covariance matrix (details are in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 22.4]). The default value for d is $(N-K-1)/6$ (rounded down to the nearest integer). However, the maximum permitted value for d is $N \cdot \theta / 2$ (rounded down to the nearest integer). Judge et al. [1988, p. 903-4] note that the covariance estimator is unlikely to be satisfactory for large K/N.

If you specify **DIFF=–1** the method of Bofinger [1975] and Siddiqui [1960] is used which gives:

$$d = N^{(-1/5)}\left[\frac{4.5\,\varphi(x)^4}{(2x^2+1)^2}\right]^{(1/5)} \qquad \text{where} \qquad x = \Phi^{-1}(\theta)$$

φ and Φ^{-1} represent the standard normal density function and inverse of the standard normal cumulative distribution function respectively. In the above equation θ is the quantile required so $\theta = 0.5$ in the case of the LAE model. If **DIFF=–2** SHAZAM will use the method suggested by Hall and Sheather [1988] and Siddiqui [1960]:

$$d = N^{(-1/3)} z_\alpha^{2/3}\left[\frac{1.5\,\varphi(x)^2}{(2x^2+1)}\right]^{(1/3)} \qquad \text{where} \qquad x = \Phi^{-1}(\theta)$$

and z_α is the $\alpha/2$ critical value from the normal distribution. SHAZAM uses $\alpha = .05$ and therefore $z_\alpha = 1.96$.

ITER= Used with the **MULTIT=** option described below to specify the maximum number of iterations allowed in the iterative procedure used to obtain the estimates. The default is **ITER=10**.

MULTIT= Specifies the degrees of freedom parameter to be used for the multivariate-t error distribution. See also the **UNCOR**, **CONV=**, and **ITER=** options which can be used with the **MULTIT=** option.

THETA= Specifies a single value of θ to use for the regression quantile method.

THETAB=, THETAE=, THETAI= Specifies beginning (**THETAB=**), ending (**THETAE=**), and increments (**THETAI=**) to be used for θ. A sequence of regression quantile estimates is generated and coefficient estimates are obtained as the average (equal weights). If these options are specified the **THETA=** option is ignored.

TRIM= Specifies the value of the **TRIM**ming proportion α to use for the trimmed least squares estimation method. The value specified for α should be between 0 and 0.5. A listing of the observation numbers for the deleted observations is printed unless the **SET NOWARN** command has been used.

Only one estimation method is permitted on any **ROBUST** command. The default is the **LAE** method. Note that calculations of R^2 are not well defined in these models. Users may prefer to use the R^2 measure that is reported with the **RSTAT** option.

EXAMPLES

Examples of the use of the **ROBUST** command are provided in Chapter 22 of the *Judge Handbook*. An example of the **ROBUST** command to obtain the LAE estimates for the Theil textile data set is:

```
|_ROBUST CONSUME INCOME PRICE / LAE RSTAT
LEAST ABSOLUTE ERRORS REGRESSION
OBJECTIVE FUNCTION  =    35.244
NUMBER OF SIMPLEX ITERATIONS =    5.0000
EMPIRICAL QUANTILE FUNCTION AT MEANS =    136.24
SUM OF ABSOLUTE ERRORS =    70.487
  USING DIFF=    2 FOR COVARIANCE CALCULATIONS

VARIANCE OF THE ESTIMATE-SIGMA**2 =    68.326
STANDARD ERROR OF THE ESTIMATE-SIGMA =    8.2659
SUM OF SQUARED ERRORS-SSE=    570.95
MEAN OF DEPENDENT VARIABLE =    134.51

VARIABLE    ESTIMATED  STANDARD    T-RATIO         PARTIAL STANDARDIZED ELASTICITY
  NAME     COEFFICIENT   ERROR       14 DF    P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME       .69851      .3962       1.763       .100  .426       .1570      .5348
PRICE      -1.4421       .1245      -11.58       .000 -.952     -1.0316      -.8182
CONSTANT    174.36      40.26        4.331       .001  .757       .0000      1.2963

DURBIN-WATSON = 1.5333    VON NEUMANN RATIO = 1.6291    RHO =    .19972
RESIDUAL SUM =  -29.563      RESIDUAL VARIANCE =    40.782
SUM OF ABSOLUTE ERRORS=    70.487
R-SQUARE BETWEEN OBSERVED AND PREDICTED =   .9435
RUNS TEST:     7 RUNS,    10 POSITIVE,     7 NEGATIVE, NORMAL STATISTIC = -1.1583
```

27. TIME-VARYING LINEAR REGRESSION

"Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $250 and/or six months in prison."

Section 889, New York State Code of Criminal Procedure

Consider a linear regression model with time-varying coefficients:

$$Y_t = X_t' \beta_t + \varepsilon_t \qquad \text{for } t = 1, \ldots, N$$

The flexible least squares (FLS) method developed by Kalaba and Tesfatsion [1989] finds time paths of the coefficients which minimize the "incompatibility cost" function:

$$C(\beta; \delta, N) = \frac{1}{1-\delta}\left[\delta \sum_{t=1}^{N-1} (\beta_{t+1} - \beta_t)'(\beta_{t+1} - \beta_t) + (1-\delta)\sum_{t=1}^{N} (Y_t - X_t'\beta_t)^2\right]$$

$$= \frac{1}{1-\delta}\left[\delta\, r_D^2(\beta; N) + (1-\delta)\, r_M^2(\beta; N)\right]$$

where $\beta = (\beta_1, \beta_2, \ldots, \beta_N)$ is the time-path of coefficient vectors, r_D^2 is the sum of squared residual dynamic errors, r_M^2 is the sum of squared residual measurement errors and $\delta \in (0, 1)$ is a smoothness weight. The OLS extreme point occurs at $\delta = 1$ and equal weights apply when $\delta = 0.5$. The FLS solution is conditional on the choice of δ. The implementation of FLS with the **FLS** command is based on a program provided by Tesfatsion and Kalaba.

A strength of FLS is its ability to capture turning points and other systematic time variation in the coefficients. This is highlighted in the applied work of Tesfatsion and Veitch [1990] and Lütkepohl [1993]. FLS can be compared with other tests such as the Chow test and the CUSUM of recursive residuals tests (available with the **DIAGNOS** command). The Chow test requires the specification of a break-point and assumes coefficient constancy over a sub-period. Schneider [1991, p. 210] comments that the CUSUM test provides a global stability test and, in contrast to FLS, does not identify the sources of instability.

FLS offers an exploratory data analysis tool and requires no distributional assumptions on the error terms. However, Lütkepohl [1993, p. 733] discusses that, with some stochastic

assumptions, a likelihood function can be formed from a random walk model for the regression coefficients and FLS may be interpreted as a special Kalman filter.

FLS *COMMAND OPTIONS*

In general, the format of the **FLS** command is:

FLS *depvar indeps / options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables with coefficients which evolve over time and *options* is a list of desired options.

Options as defined for the **OLS** command that are available are:

BEG=, **END=**, **PREDICT=** and **RESID=**

Additional options available on the **FLS** command are:

GRAPH Prepares gnuplot plots of the time paths of the coefficients as well as the residuals and predicted values. For more information on this option see the chapter *PLOTS AND GRAPHS*. With the **GRAPH** option the **APPEND**, **OUTPUT=**, **DEVICE=**, **PORT=** and **COMMFILE=** options are also available as described for the **GRAPH** command.

MAX Reports the first-order necessary conditions for the minimization of the cost function as described in Kalaba and Tesfatsion [1989, Appendix A]. The columns on the output give the evaluations of the partial derivatives of the cost function with respect to the variables. The results should be close to zero and therefore may be machine dependent. The **MAX** option also prints the coefficient estimates.

NOCONSTANT Do not include an intercept term. Otherwise, a time-varying intercept is included in the model.

PCOEF Prints the coefficient estimates over the sample period.

COEF = Saves the FLS estimates in a N x K matrix.

DELTA= Specifies the smoothing weight δ in the range $0<\delta<1$. The default setting is **DELTA=0.5**.

The temporary variables available after the **FLS** command are:

$ERR	Error code
$K	Number of coefficients
$LLF	Value of the incompatibility cost function
$N	Number of observations
$SSE	Sum of squared residual measurement errors
$SSR	Sum of squared residual dynamic errors
$R2	R-square

Note that some of these temporary variables may have different definitions than that used by other SHAZAM commands.

EXAMPLES

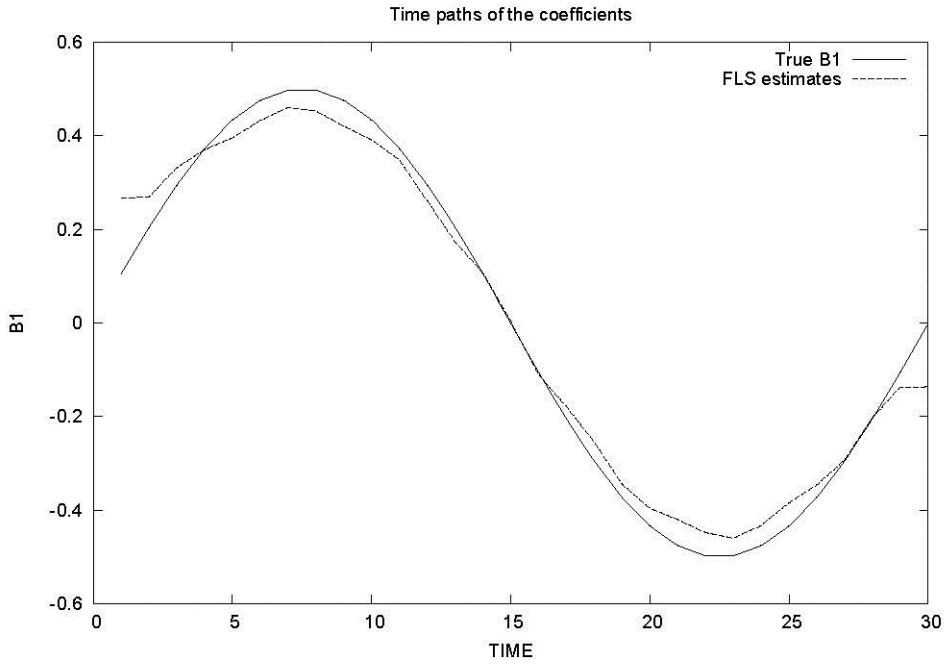
This example is inspired by simulation experiments reported in Kalaba and Tesfatsion [1989, Section 8]. The true coefficients $\beta_t = (\beta_{t1}, \beta_{t2})$ are generated so that they trace out an ellipse over time. The experiment illustrates the degree to which the FLS coefficient estimates are able to recover this time variation. The SHAZAM command file that generates the data and runs the FLS estimation is shown below.

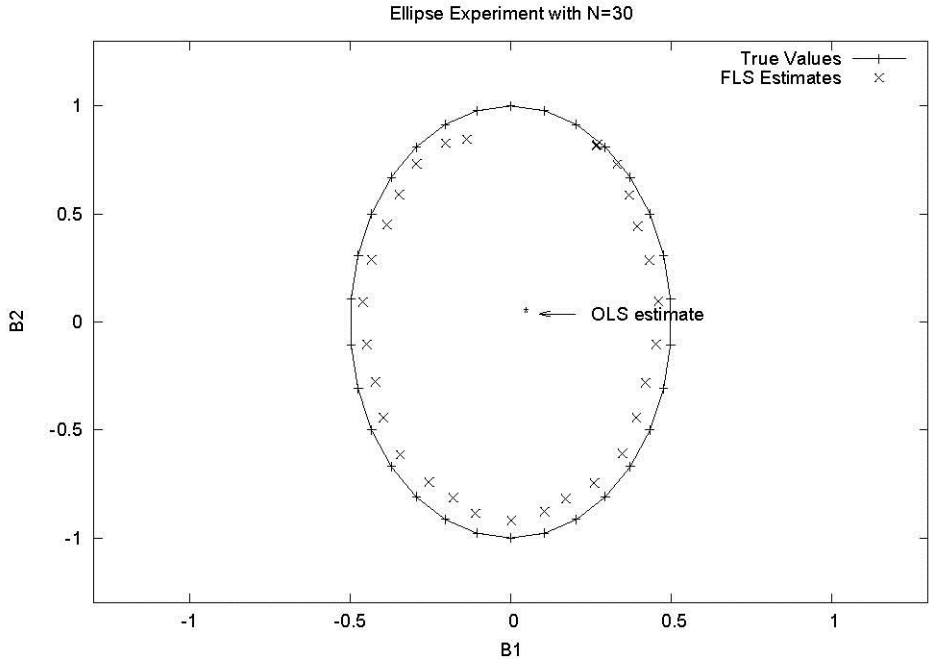
```
sample 1 30

* Generate the time-varying coefficients
genr ai=time(0)
genr b1=.5*sin(ai*2*$pi/30)
genr b2=sin( (ai*2*$pi/30) + $pi/2)

* Generate the data
genr x1=1
genr x2=1
sample 2 30
genr x1=sin(10+ai)+.01

* The next command uses the SIN function to generate COS(10 + AI)
genr x2=sin(10+ai + $pi/2)
sample 1 30
genr y=b1*x1+b2*x2

* Get the FLS estimates
fls y x1 x2 / noconstant pcoef
```

The FLS estimation output is:

```
|_FLS Y X1 X2 / NOCONSTANT PCOEF
      30 OBSERVATIONS     DEPENDENT VARIABLE = Y
...NOTE..SAMPLE RANGE SET TO:    1,   30

SMOOTHING WEIGHT : DELTA =  .500
 FLEXIBLE LEAST SQUARES ESTIMATES
             X1           X2
        1     .26646       .81866
        2     .26947       .82167
        3     .33164       .72990
        4     .36991       .58770
        5     .39536       .44375
        6     .43262       .28622
        7     .46050       .96371E-01
        8     .45298      -.10372
        9     .42074      -.28179
       10     .39140      -.44191
       11     .34850      -.60802
       12     .26072      -.74510
       13     .17287      -.81824
       14     .10615      -.87791
       15     .31137E-02  -.92037
       16    -.10953      -.88498
       17    -.17895      -.81339
       18    -.25383      -.74014
       19    -.34417      -.61391
       20    -.39581      -.44338
       21    -.42039      -.27711
       22    -.44792      -.10402
       23    -.45966       .92531E-01
       24    -.43288       .28858
       25    -.38436       .45040
       26    -.34608       .59009
       27    -.29316       .73170
       28    -.20245       .82765
       29    -.13678       .84552
       30    -.13669       .84543

SUM OF SQUARED RESIDUAL MEASUREMENT ERRORS =   .65723E-01
SUM OF SQUARED RESIDUAL DYNAMIC ERRORS =       .62918
THE INCOMPATIBILITY COST =                     .69491

R-SQUARE =   .99328

  SUMMARY STATISTICS FOR THE FLS ESTIMATES
VARIABLE   MEAN        STDEV         STDEV       COEFFICIENT
  NAME                               OF MEAN     OF VARIATION
X1         .46592E-02   .33747       .61614E-01   72.431
X2        -.72609E-02   .64204       .11722      -88.425

OLS ESTIMATES CALCULATED FROM A MATRIX AVERAGE OF FLS ESTIMATES
   .38463E-01   .37439E-01
```

The first graph below compares the true sequence of coefficients for the variable *X1* with the FLS estimates. The second graph shows the elliptical shape traced out by the FLS estimates.





28. TOBIT REGRESSION

"That the automobile has reached the limit of its development is suggested by the fact that during the past year no improvements of a radical nature have been introduced."

Scientific American
January 2, 1909

The **TOBIT** command is available for regressions with limited dependent variables. The model is described in Tobin [1958, pp. 24-36]. References for tobit analysis (the name is coined from "Tobin's probit") include Goldberger [1964, pp. 248-255]; Maddala [1977, Chapter 9.7]; Greene [2003, Chapter 22]; Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Chapter 19]; and Maddala [1983, Chapter 6]. An application of tobit analysis using SHAZAM can be found in Deegan and White [1976].

The tobit model estimation algorithm is based on a computer program originally written by John Cragg [1971]. SHAZAM **TEST** commands can be used following estimation. **RESTRICT** commands are not permitted.

The tobit model can be defined with the use of a latent variable as:

$$Y_t^* = X_t'\beta + \varepsilon_t$$

$$Y_t = 0 \qquad \text{if} \qquad Y_t^* \le 0$$

$$Y_t = Y_t^* \qquad \text{if} \qquad Y_t^* > 0$$

If a limit value other than 0 is required this can be specified with the **LIMIT=** option on the **TOBIT** command. For the latent variable,

$$E(Y_t^*) = \sigma I_t \qquad \text{where} \qquad I_t = X_t'\alpha = X_t'(\beta/\sigma)$$

The expectation of Y_t is: $E(Y_t \mid I_t) = \sigma I_t F(I_t) + \sigma f(I_t)$

where F() is the cumulative normal distribution function and f() is the normal density function. The expectation of Y_t conditional on $Y_t > 0$ is:

$$E(Y_t \mid I_t, Y_t > 0) = \sigma I_t + \frac{\sigma f(I_t)}{F(I_t)}$$

The parameters of the tobit model are the vector α and a normalizing parameter σ. Users should note that other computer programs may use a different parameterization.

Maximum Likelihood Estimation

The log-likelihood function for the tobit model can be expressed as:

$$L = \sum_{Y_t>0} -\frac{1}{2}\left[\ln(2\pi) + \ln(\sigma^2) + \left(\frac{1}{\sigma}Y_t - X_t'\alpha\right)^2\right] + \sum_{Y_t=0} \ln\left[1 - F(X_t'\alpha)\right]$$

The maximum likelihood estimates of α and σ are denoted by $\hat{\alpha}$ and $\hat{\sigma}$. An estimate of the index variable is $\hat{I}_t = X_t'\hat{\alpha}$. The regression coefficients are estimated as $\hat{\beta} = \hat{\sigma}\hat{\alpha}$. On the SHAZAM output the column `NORMALIZED COEFFICIENT` reports $\hat{\alpha}$ and the column `REGRESSION COEFFICIENT` reports $\hat{\beta}$. The coefficient listed for the dependent variable is actually equal to $(1/\hat{\sigma})$ while the statistic labelled `STANDARD ERROR OF ESTIMATE` is $\hat{\sigma}$ and `VARIANCE OF THE ESTIMATE` is $\hat{\sigma}^2$.

Interpreting the Results

Analysis of the tobit results is complicated by the fact that all computations are performed on the *normalized* α vector, and the estimated standard errors of the coefficients are those of the α vector and not the β. However, it is quite easy to perform hypotheses on the regression coefficients β by working on the α vector in the manner described in the Tobin article. SHAZAM **TEST** commands can be used to test hypotheses about the β vector as shown in the example in this chapter.

The predicted value of the dependent variable can be computed as:

$$\hat{Y}_t = \hat{\sigma}\hat{I}_t F(\hat{I}_t) + \hat{\sigma} f(\hat{I}_t)$$

The expectation of Y estimated at the mean values is given by:

$$Y^E = \hat{\sigma}(\overline{X}'\hat{\alpha}) F(\overline{X}'\hat{\alpha}) + \hat{\sigma} f(\overline{X}'\hat{\alpha})$$

Alternative approaches to the interpretation of results from tobit analysis can be considered and this may depend on the specific goals of the study (see, for example, the discussion in Greene [2003, pp. 764-6]).

The ELASTICITY OF INDEX for the k[th] variable (the percentage change in the index I for a percentage change in X_k at the sample means) is estimated as: $\hat{\beta}_k(\overline{X}_k/\overline{Y})$.

Another response to consider is that for Y, given the censoring, the marginal effect is:

$$\frac{\partial E(Y_t|I_t)}{\partial X_t} = \beta F(I_t)$$

This result is then used to compute the ELASTICITY OF E(Y) for the k[th] variable, evaluated at sample means, as:

$$\hat{\beta}_k F(\overline{X}'\hat{\alpha})\left(\overline{X}_k/Y^E\right)$$

An example of a study that reports these elasticities is Shishko and Rostker [1976].

With S the number of NON-LIMIT OBSERVATIONS the calculations for other statistics on the SHAZAM output are:

PREDICTED PROBABILITY OF Y > LIMIT GIVEN AVERAGE X(I) = $F(\overline{X}'\hat{\alpha})$

THE OBSERVED FREQUENCY OF Y > LIMIT IS = S/N

AT MEAN VALUES OF ALL X(I), E(Y) = Y^E

MEAN SQUARE ERROR $\frac{1}{N}\sum_{t=1}^{N}(Y_t - \hat{Y}_t)^2$

MEAN ABSOLUTE ERROR $\frac{1}{N}\sum_{t=1}^{N}\left|Y_t - \hat{Y}_t\right|$

When the **LIST** or **MAX** options are used output will include, for each observation:

INDEX	$\hat{I}_t$	OBSERVED	Y_t	
PROB(X)	$F(\hat{I}_t)$	EXPECTED	$\hat{Y}_t$	
DENSITY(X)	$f(\hat{I}_t)$	CONDITIONAL	$\hat{Y}_t / F(\hat{I}_t)$	for $Y_t > 0$

TOBIT *COMMAND OPTIONS*

In general, the format of the **TOBIT** command is:

TOBIT *depvar indeps /options*

where *depvar* is the dependent variable, *indeps* is a list of independent variables, and *options* is a list of desired options. Options as defined for the **OLS** command that are available are:

MAX, **NOCONSTANT**, **NONORM**, **BEG=**, **END=**, **COEF=**, **COV=**, **STDERR=**, **TRATIO=** and **WEIGHT=**

Additional options available on the **TOBIT** command are:

DUMP **DUMP**s the matrix of second derivatives, the moment matrices of limit and non-limit observations, and some other output that is probably not useful to the average user.

LIST **LIST**s, for each observation, the value of the index I, the density and cumulative probabilities corresponding to the index I, and the observed, expected and conditional values of the dependent variable. No plot is produced.

NEGATIVE The observed value of the dependent variable can be greater than or less than the limit value.

PCOR **P**rints the **COR**relation matrix of the *normalized* coefficients.

PCOV **P**rints the estimated asymptotic **COV**ariance matrix of the *normalized* coefficients. The matrix is the inverse of the negative of the matrix of second derivatives of the log-likelihood function. In a large sample the *normalized* coefficients will be normally distributed.

UPPER Used if the limit is an **UPPER** limit rather than a lower limit.

CONV= Sets the **CONV**ergence criterion for the *normalized* coefficients. The default is .00000001 (that is, 1×10^{-8}). This is the same as using **CONV=1E-8**.

INDEX= Saves the computed **INDEX** in the variable specified.

ITER= Sets the maximum number of **ITER**ations allowed. The default is 25.

LIMIT= Specifies the **LIMIT**ing value of the dependent variable. The default is **LIMIT=** 0.

PITER= Specifies the frequency with which **ITER**ations are to be **P**rinted. The default is **PITER=1**. If **PITER=0** is specified, no iterations are printed.

PREDICT= Saves the **PREDICT**ed expected values in the variable specified.

Note that the **PCOR**, **COEF=**, **COV=**, **STDERR=** and **TRATIO=** options all refer to the *normalized* coefficients. If the **WEIGHT=** option is used the method performed follows that described for the **REPLICATE** option in **OLS**.

The temporary variables available with the **TOBIT** command are:

$ERR, *$K*, *$LLF*, *$N*, *$SIG2*, and *$SSE*.

The variable *$K* is the number of estimated coefficients (the number of independent variables plus one for the estimate of σ).

EXAMPLES

The following is an example of tobit model estimation using data found in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Table 19.2, p. 800]. The SHAZAM commands include **TEST** commands to derive the standard error and associated asymptotic normal test statistics for the regression coefficients. This is easy to do since as explained earlier the coefficient on the dependent variable Y is simply 1/σ. The command file is:

```
sample 1 20
read (table19.2) y x
tobit y x / list piter=0
test x/y
test constant/y
```

The SHAZAM output is:

```
 |_SAMPLE 1 20
```

```
|_READ (TABLE19.2) Y X
   2 VARIABLES AND       20 OBSERVATIONS STARTING AT OBS       1

|_TOBIT Y X / LIST PITER=0
TOBIT ANALYSIS, LIMIT=     .00        25 MAX ITERATIONS
        6 LIMIT OBSERVATIONS
       14 NON-LIMIT OBSERVATIONS

 FIRST DERIVATIVES OF LOG OF LIKELIHOOD FUNCTION EVALUATED AT MAXIMUM
    .19539925E-13    .88817842E-15   -.23236187E-13

NUMBER OF ITERATIONS =  3

 DEPENDENT VARIABLE = Y
VARIANCE OF THE ESTIMATE =   13.184
STANDARD ERROR OF THE ESTIMATE =   3.6310

                              ASYMPTOTIC
VARIABLE  NORMALIZED     STANDARD   T-RATIO    REGRESSION ELASTICITY ELASTICITY
         COEFFICIENT      ERROR               COEFFICIENT  OF INDEX   OF E(Y)
X          .24820       .58959E-01   4.2096       .90120       1.9458   1.9970
CONSTANT  -1.5786       .60413      -2.6130      -5.7319
Y          .27541       .53292E-01   5.1679

THE PREDICTED PROBABILITY OF Y > LIMIT GIVEN AVERAGE X (I) =  .8479
 THE OBSERVED FREQUENCY OF Y > LIMIT IS =  .7000
 AT MEAN VALUES OF ALL X(I), E(Y) =       4.0177

                                                    DEPENDENT VARIABLE
 OB   INDEX      PROB(X)      DENSITY(X)  OBSERVED    EXPECTED   CONDITIONAL
   1  -1.3304      .91692E-01  .16465      .00000      .15491      ------
   2  -1.0822      .13958      .22212      .00000      .25805      ------
   3  -.83402      .20214      .28175      .00000      .41091      ------
   4  -.58582      .27900      .33604      .00000      .62670      ------
   5  -.33762      .36782      .37684      3.1348      .91739      2.4941
   6  -.89429E-01  .46437      .39735      3.5080      1.2920      2.7822
   7   .15877      .56307      .39395      .83120      1.7550      3.1169
   8   .40696      .65798      .36724      8.0064      2.3057      3.5042
   9   .65516      .74382      .32189      .00000      2.9382      ------
  10   .90335      .81683      .26528      .00000      3.6425      ------
  11   1.1515      .87525      .20557      2.9352      4.4061      5.0341
  12   1.3997      .91921      .14978      3.9048      5.2157      5.6741
  13   1.6479      .95032      .10261      6.5144      6.0590      6.3758
  14   1.8961      .97103      .66099E-01  5.9772      6.9254      7.1321
  15   2.1443      .98400      .40034E-01  3.7260      7.8069      7.9338
  16   2.3925      .99163      .22799E-01  10.412      8.6974      8.7708
  17   2.6407      .99586      .12208E-01  16.906      9.5932      9.6330
  18   2.8889      .99807      .61466E-02  9.2968      10.492      10.512
  19   3.1371      .99915      .29098E-02  7.8916      11.392      11.401
  20   3.3853      .99964      .12952E-02  14.216      12.292      12.297

LOG-LIKELIHOOD FUNCTION= -41.255240
MEAN-SQUARE ERROR=  8.0589568
MEAN ERROR= -.40295353E-02
MEAN ABSOLUTE ERROR=  2.1501402
SQUARED CORRELATION BETWEEN OBSERVED AND EXPECTED VALUES=   .66061
|_TEST X/Y
TEST VALUE =   .90120     STD. ERROR OF TEST VALUE   .17186
ASYMPTOTIC NORMAL STATISTIC =   5.2437210      P-VALUE=  .00000
WALD CHI-SQUARE STATISTIC =   27.496610     WITH    1 D.F.  P-VALUE=  .00000
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY =  .03637
```

```
|_TEST CONSTANT/Y
TEST VALUE =  -5.7319     STD. ERROR OF TEST VALUE   2.2383
ASYMPTOTIC NORMAL STATISTIC =  -2.5608774      P-VALUE=  .01044
WALD CHI-SQUARE STATISTIC =   6.5580928     WITH    1 D.F.  P-VALUE=  .01044
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY =  .15248
```

29. TWO-STAGE LEAST SQUARES AND SYSTEMS OF EQUATIONS

"You won't have Nixon to kick around anymore – because, gentlemen, this is my last press conference."

Richard M. Nixon
November 7, 1962
(after losing the California Governor Election)

A single linear equation can be estimated by Two Stage Least Squares (2SLS) with the **2SLS** command in SHAZAM. The **2SLS** command can be used for any instrumental variables estimation. The **INST** command can also be used, but the output is identical to **2SLS**. The **SYSTEM** command in SHAZAM provides features for joint estimation of a set of linear equations by either seemingly unrelated regression (SUR) estimation or by Three Stage Least Squares (3SLS). A reference for SUR is Greene [2003, Chapter 14] and a discussion of instrumental variable estimation and 3SLS is Greene [2003, Chapter 15].

Note that these estimation techniques are available for nonlinear models with the **NL** command described in the chapter *NONLINEAR REGRESSION*. The **SYSTEM** command does not estimate models with autoregressive errors. A set of seemingly unrelated regression equations with autoregressive errors can be estimated by using the **NL** command with the **AUTO** option.

2SLS OR INSTRUMENTAL VARIABLE ESTIMATION

Consider the model: $y = Z\delta + \varepsilon$

where y is a (N x 1) vector of observations on the dependent variable, Z is a (N x p) matrix of observations on the explanatory variables, ε is a (N x 1) vector of disturbances and the number of unknown parameters in the vector δ is p. Suppose that some of the regressors are correlated with the disturbance term. Introduce a (N x K) data matrix X composed of variables that are exogenous and independent of the disturbance. Premultiplying the above statistical model by X gives the transformed statistical model:

$$X'y = X'Z\delta + X'\varepsilon$$

The 2SLS estimator is: $\hat{\delta} = \left[Z'X(X'X)^{-1}X'Z\right]^{-1}Z'X(X'X)^{-1}X'y$

and the estimated covariance matrix of coefficients is given by:

$$\hat{\sigma}^2\left[Z'X(X'X)^{-1}X'Z\right]^{-1}$$

where SHAZAM uses: $\hat{\sigma}^2 = \frac{1}{N-p}(y - Z\hat{\delta})'(y - Z\hat{\delta})$

Some econometricians would argue that the denominator should just be N (see, for example, Greene [2003, pp. 77-8]). For a divisor of N, instead of N – p, the **DN** option should be specified on the **2SLS** command. The estimated coefficients are not affected in either case, but all the variances and t-ratios will be. Users should note that R^2 in **2SLS**, and in many other models, is not well defined and could easily be negative. In fact, the lower bound is minus infinity. Many econometricians would prefer to use the squared-correlation coefficient between the observed and predicted dependent variable instead for these models. This is obtained with the **RSTAT** option and is reported as `R-SQUARE BETWEEN OBSERVED AND PREDICTED`. It is also available in the temporary variable *$R2OP*.

2SLS *and* **INST** *Command Options*

In general, the format of the **2SLS** command is:

2SLS *depvar rhsvars (exogs) / options*

where *depvar* is the dependent variable, *rhsvars* is a list of all the right-hand side variables in the equation, *exogs* is a list of all the exogenous variables in the system, and *options* is a list of desired options.

Note that the list of *exogs* must be enclosed in parentheses, and that SHAZAM automatically includes a constant in the list of exogenous variables unless the **NOCONEXOG** option is used. If the number of exogenous variables is less than the number of right-hand side variables, the equation will be underidentified. Equation estimation requires $N \geq K \geq p$.

While most of the **OLS** options are also available as *options* on **2SLS**, the user should be aware that hypothesis testing in **2SLS** is complicated by the unknown distributions, so the normal t and F tests are invalid. At best these can be interpreted as being asymptotically normal and chi-square. The F test from an Analysis of Variance table is invalid, and the printed R^2, which is defined as 1 minus the unexplained proportion of the total variance,

may not coincide with that obtained by using other programs which do the calculation differently.

If the interest is instrumental variables estimation then the **INST** command can be used with the general format:

INST *depvar rhsvars (exogs) / options*

The output for the **INST** command is identical to **2SLS.**

Options as defined for **OLS** that are available on the **2SLS** and **INST** commands are:

DN, **DUMP**, **GF**, **LIST**, **MAX**, **NOCONSTANT**, **PCOR**, **PCOV**, **RESTRICT**, **RSTAT**, **BEG=**, **END=**, **COEF=**, **COV=**, **PREDICT= RESID=**, **STDERR=** and **TRATIO=**.

An additional option is:

NOCONEXOG Specifies that a constant is not to be included in the list of exogenous variables. If this option is not specified then SHAZAM includes a column of 1's in the X matrix.

The temporary variables available on the **2SLS** command are:

$ADR2, $DF, $DW, $ERR, $K, $N, $R2, $R2OP, $RAW, $RHO, $SIG2 and *$SSE*.

For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* and the chapter *ORDINARY LEAST SQUARES*.

Example

An example of the **2SLS** command using the first equation of the Klein model [Theil, 1971] is shown in the SHAZAM output:

```
|_2SLS C PLAG P WGWP (WG T G TIME PLAG KLAG XLAG) /  DN RSTAT
 TWO STAGE LEAST SQUARES - DEPENDENT VARIABLE = C
  7 EXOGENOUS VARIABLES
  3 POSSIBLE ENDOGENOUS VARIABLES
       21 OBSERVATIONS
DN OPTION IN EFFECT - DIVISOR IS N
```

```
 R-SQUARE =     .9767     R-SQUARE ADJUSTED =     .9726
VARIANCE OF THE ESTIMATE-SIGMA**2 =   1.0441
STANDARD ERROR OF THE ESTIMATE-SIGMA =   1.0218
SUM OF SQUARED ERRORS-SSE=   21.925
MEAN OF DEPENDENT VARIABLE =   53.995

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
PLAG        .21623      .1073       2.016      .044  .439      .1270      .0656
P           .17302E-01  .1180       .1466      .883  .036      .0106      .0054
WGWP        .81018      .4025E-01   20.13      .000  .980      .8922      .6224
CONSTANT   16.555      1.321        12.53      .000  .950      .0000      .3066

DURBIN-WATSON = 1.4851    VON NEUMANN RATIO = 1.5593    RHO =   .20423
RESIDUAL SUM =  -.15632E-12  RESIDUAL VARIANCE =   1.0441
SUM OF ABSOLUTE ERRORS=   17.866
R-SQUARE BETWEEN OBSERVED AND PREDICTED =  .9768
RUNS TEST:    9 RUNS,    9 POSITIVE,   12 NEGATIVE, NORMAL STATISTIC = -1.0460
```

SYSTEMS OF EQUATIONS

SHAZAM can estimate a system of linear equations. Linear restrictions can be imposed on the coefficients within or across equations. The seemingly unrelated regressions (SUR) case is also known as Zellner estimation, or multivariate regression. The simultaneous equation estimation technique is three-stage least squares (3SLS). As an option, SHAZAM will iterate until convergence.

A system of M equations may be written as:

$$y_1 = Z_1\delta_1 + \varepsilon_1$$
$$y_2 = Z_2\delta_2 + \varepsilon_2$$
$$\cdots$$
$$y_M = Z_M\delta_M + \varepsilon_M$$

or, more compactly, as: $y = Z\delta + \varepsilon$ where

$y = [\, y_1' \; y_2' \cdots y_M' \,]'$ is an $MN \times 1$ vector of observations on the left-hand side dependent variable.

$$Z = \begin{bmatrix} Z_1 & 0 & . & 0 \\ 0 & Z_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & Z_M \end{bmatrix}$$

is an $MN \times P$ matrix of jointly endogenous and exogenous variables in the system and $P = p_1 + p_2 + \ldots + p_M$ where p_i is the number of right hand side variables in equation i.

$\delta = [\,\delta_1' \;\delta_2' \;\ldots\; \delta_M'\,]'$ is a P x 1 vector of unknown parameters.

$\varepsilon = [\varepsilon_1' \;\varepsilon_2' \;\ldots\; \varepsilon_M'\,]'$ is an $M\,N \;x\; 1$ vector of disturbances.

All exogenous variables of the system are in the matrix X. In the case of SUR Z = X since there are no jointly endogenous variables in a SUR system.

The variance-covariance matrix of disturbances is given by:

$$E[\varepsilon\varepsilon'] = \begin{bmatrix} \sigma_{11}I & \sigma_{12}I & . & \sigma_{1M}I \\ \sigma_{21}I & \sigma_{22}I & . & \sigma_{2M}I \\ . & . & . & . \\ \sigma_{M1}I & \sigma_{M2}I & . & \sigma_{MM}I \end{bmatrix} = \Sigma \otimes I_N$$

Seemingly Unrelated Regression

The first step in the SUR estimation procedure is to estimate the σ_{ij} from OLS residuals as:

$$\hat{\sigma}_{ij} = \frac{1}{\tau}\left(y_i - Z_i\hat{\delta}_{OLS,i}\right)'\left(y_j - Z_j\hat{\delta}_{OLS,j}\right)$$

where $\hat{\delta}_{OLS,i} = (Z_i'Z_i)^{-1}Z_i'y_i$ and τ can be set using:

$\tau_1 = N$ or $\tau_2 = (MN - P)/M$

If the **DN** option is used SHAZAM will use $\tau = \tau_1$, otherwise $\tau = \tau_2$ is used.

With the matrix $\hat{\Sigma}$ containing individual elements $\hat{\sigma}_{ij}$ the SUR estimator is:

$$\hat{\delta}_{SUR} = [\,Z'(\hat{\Sigma}^{-1} \otimes I_N)Z]^{-1}Z'(\hat{\Sigma}^{-1} \otimes I_N)\,y$$

and the covariance matrix of the coefficients is estimated as: $[Z'(\hat{\Sigma}^{-1} \otimes I_N)Z]^{-1}$

With the assumption of normality the log-likelihood function can be expressed as:

$$L = -\frac{MN}{2}\ln(2\pi) - \frac{N}{2}\ln\left|\Sigma\right| - \frac{1}{2}(y - Z\delta)'(\Sigma^{-1} \otimes I_N)(y - Z\delta)$$

Restricted Seemingly Unrelated Regression

A set of q linear restrictions may be written in the form $R\delta = r$ where R and r are known matrices of dimensions (q x P) and (q x 1) respectively. When the **RESTRICT** and **ITER=0** options are specified on the **SYSTEM** command the restricted estimator is obtained from:

$$\hat{\delta}_R = \hat{\delta}_{OLS} + (Z'Z)^{-1}R'(R(Z'Z)^{-1}R')^{-1}(r - R\hat{\delta}_{OLS})$$

Estimates of the error variances and covariances are obtained from the residuals from the restricted estimation as:

$$\hat{\sigma}_{ij} = \frac{1}{\tau}\left(y_i - Z_i\hat{\delta}_{R,i}\right)'\left(y_j - Z_j\hat{\delta}_{R,j}\right)$$

With the **DN** option $\tau = \tau_1$ otherwise $\tau = \tau_2$ where

$$\tau_1 = N \qquad \text{or} \qquad \tau_2 = \left(MN - (P - q)\right)/M$$

When the **RESTRICT** and **ITER=1** options are specified on the **SYSTEM** command the restricted seemingly unrelated regression estimator is obtained from:

$$\hat{\delta}_{RSUR} = \hat{\hat{\delta}} + \hat{C}R'(R\hat{C}R')^{-1}(r - R\hat{\hat{\delta}})$$

where $\hat{C} = [Z'(\hat{\Sigma}^{-1} \otimes I_N)Z]^{-1}$ and $\hat{\hat{\delta}} = \hat{C}\,Z'(\hat{\Sigma}^{-1} \otimes I_N)\,y$

Three Stage Least Squares

The first step in the 3SLS estimation procedure is to estimate the σ_{ij} from 2SLS residuals as:

$$\hat{\sigma}_{ij} = \frac{1}{\tau}\left(y_i - Z_i\hat{\delta}_{2SLS,i}\right)'\left(y_j - Z_j\hat{\delta}_{2SLS,j}\right)$$

where τ is as described above. The 3SLS estimator is given by:

$$\hat{\delta} = \left\{Z'\left[\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X'\right]Z\right\}^{-1}Z'\left[\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X'\right]y$$

and the covariance matrix of the coefficients is estimated as:

$$\left[Z'\left(\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X'\right)Z\right]^{-1}$$

Iterative Estimation

When estimating a system of equations by SUR or 3SLS, the estimation may be iterated by using the **ITER=** option. In this case, the initial estimation is done to estimate $\hat{\delta}$. A new set of residuals is generated and used to estimate a new variance-covariance matrix. This matrix is then used to compute a new set of parameter estimates. The iterations proceed until the parameters converge or until the maximum number of iterations specified on the **ITER=** option is reached.

Model Diagnostics

A system R^2 is computed as: $\tilde{R}^2 = 1 - \left|E'E\right| / \left|Y_*'Y_*\right|$

where Y_* is an (N x M) matrix with column i containing the observations on the left hand side variable for equation i measured as deviations from the sample mean and E is an (N x M) matrix with the estimated residuals for equation i in column i. This statistic is frequently very high and should be interpreted with caution (see Berndt [1991, p. 468]). The system R^2 is not reported when the **NOCONSTANT** option is used.

The SHAZAM output from systems estimation includes a test of the null hypothesis that the slope coefficients are jointly zero. This is equivalent to the F-statistic used to determine whether all the slope coefficients in a multiple regression model are zero. The `TEST OF THE OVERALL SIGNIFICANCE` is calculated as:

$$-N \ln(1 - \tilde{R}^2)$$

The statistic can be compared with a chi-square distribution with degrees of freedom equal to the number of slope coefficients in the system.

With SUR estimation, the Breusch-Pagan [1980] Lagrange multiplier test gives a test for a diagonal covariance matrix. With the squared correlation coefficient of residuals given by:

$$r_{ij}^2 = \frac{\hat{\sigma}_{ij}^2}{\hat{\sigma}_{ii}\hat{\sigma}_{jj}}$$

the statistic reported on SHAZAM output as BREUSCH-PAGAN LM TEST FOR DIAGONAL COVARIANCE MATRIX is computed as:

$$N \sum_{i=2}^{M} \sum_{j=1}^{i-1} r_{ij}^2$$

Under the null hypothesis of a diagonal covariance structure the statistic has an asymptotic $\chi^2_{(M(M-1)/2)}$ distribution.

The SUR estimation results also report a LIKELIHOOD RATIO TEST OF DIAGONAL COVARIANCE MATRIX as an alternative to the Breusch-Pagan Lagrange multiplier test. This statistic is computed as:

$$N \left[\sum_{i=1}^{M} \ln(\hat{\sigma}_{ii}^2) - \ln \left| \hat{\Sigma} \right| \right] \qquad \text{with} \qquad \tau = N$$

The statistic has an asymptotic $\chi^2_{(M(M-1)/2)}$ distribution. For discussion see, for example, Greene [2003, p. 350].

SYSTEMS MODEL SPECIFICATION

As the setup for equation systems is quite general SHAZAM will handle many types of linear models. To set up a set of seemingly unrelated regression equations for estimation, the commands are:

SYSTEM *neq / options*
OLS *depvar indeps*
. . .
OLS *depvar indeps*

where *neq* is the number of equations, and *options* is a list of desired options. After the **SYSTEM** command there should be an **OLS** command for each equation in the system. Do NOT use *options* on the **OLS** commands. They must only be used with the **SYSTEM** command.

To set up a restricted SUR estimation, the commands are:

SYSTEM *neq* / **RESTRICT** *options*
OLS *depvar indeps*
. . .
OLS *depvar indeps*
RESTRICT *restriction*
. . .
RESTRICT *restriction*

END

The **RESTRICT** option on the **SYSTEM** command specifies that linear restrictions are to be imposed on the parameters in the system. The restrictions must be linear and are entered in a set of **RESTRICT** commands followed by **END**. Note that when **RESTRICT** commands are used the **RESTRICT** option on the **SYSTEM** command must be specified.

To set up a simultaneous equations system for estimation by 3SLS, the commands are:

SYSTEM *neq exogs / options*
OLS *depvar indeps*
. . .
OLS *depvar indeps*

where *neq* is the number of equations, *exogs* is a list of the exogenous variables and *options* is a list of desired options. **The** *exogs* **must ONLY be included for Three Stage Least Squares and not for Zellner estimation.** Linear restrictions on the parameters can be imposed by specifying the **RESTRICT** option on the **SYSTEM** command and including a set of **RESTRICT** commands terminated by an **END** command. Nonlinear restrictions cannot be imposed but may be tested using **TEST** commands.

Following model estimation linear or nonlinear hypotheses may be tested with **TEST** commands. For a discussion about linear and non-linear hypothesis testing, see the chapter *HYPOTHESIS TESTING*.

Estimation of systems of equations can be rather slow, especially if there are many equations or variables in the system. Options should be carefully specified so the system does not have to be re-estimated. Imposing restrictions on the coefficients will also slow the estimation. In large models, it is possible that SHAZAM will require additional memory. For information on how to proceed in this case, see the **PAR** command discussed in the

chapter *HOW TO RUN SHAZAM* and the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.

SYSTEM *Command Options*

The **OLS** options available are: **STDERR=** and **TRATIO=**. Additional options available on the **SYSTEM** command are:

DN This is a useful option and its use is strongly recommended. With this option the fact that the estimation procedure has good asymptotic properties is recognized. The covariance matrix is computed using **N** as the **D**ivisor, rather than a measure which provides a dubious degrees of freedom adjustment. The use of the **DN** option is consistent with Theil's development. Without the **DN** option, the degrees of freedom correction for the covariances is as described above.

DUMP **DUMP**s a lot of intermediate output after each iteration, including the system **X**'**X** matrix and its inverse, and the inverse of the residual covariance matrix (SIGMA). This option is primarily of interest to SHAZAM consultants.

FULL Lists **FULL** equation output. This output is similar to that of regular **OLS** regressions. **FULL** is automatically in effect except in TALK mode. It can be turned off with the **NOFULL** option.

LIST
RSTAT
MAX
GF Used as on **OLS** commands to control output. They should only be used when full output is required for each equation. Users should be aware when interpreting the equation-by-equation output that some statistics printed are not valid in system estimation. If systems analysis is properly understood the questionable statistics are easily identified. Since the analysis of variance F-test is invalid it is not printed; if it is desired, **TEST** commands should be used.

NOCONEXOG If a list of exogenous variables is included for either Two or Three Stage Least Squares SHAZAM will automatically add a *CONSTANT* to the list. If you do not want SHAZAM to automatically include a constant in the list of exogenous variables, specify the **NOCONEXOG** option.

NOCONSTANT Normally, SHAZAM will automatically put an intercept in each equation in the system. If there are some (or all) equations in which no

intercept is desired the **NOCONSTANT** option should be specified. Then, an intercept should be created by generating a variable of ones (1) with a **GENR** command and this variable should be included in each equation in which an intercept is desired. Without the **NOCONSTANT** option the intercept will only be printed in the **FULL** equation-by-equation output, and the variances of the intercepts will only be approximate unless the model converges.

PCOR
PCOV

Prints the **COR**relation and **COV**ariance matrices of all coefficients in the system after convergence. If these options are specified the matrices will also be printed for each equation in the equation-by-equation output.

PINVEV

Prints the **INV**erse of the **E**xogenous **V**ariables matrix $(X'X)^{-1}$ when 3SLS estimation is being used. This option is rarely needed.

PSIGMA

Prints the residual covariance matrix (**SIGMA**) after each iteration. If this option is not used the matrix will be printed for the first and final iterations. This option can add pages of output if there are many equations in the system and, thus, should not be specified unless this information is specifically needed.

RESTRICT

Forces linear restrictions as specified with **RESTRICT** commands. The **RESTRICT** commands follow the **OLS** commands. Only linear restrictions are permitted. Users should be aware that iterative estimates may not necessarily be maximum likelihood estimates if restrictions on the intercepts of the equation are imposed.

COEF=

Saves the values of the **COEF**ficients in the variable specified. The values for all equations will be in a single vector. No values for the equation intercepts will be saved so if these are required the directions above for the **NOCONSTANT** option should be followed.

COEFMAT=

Saves the values of the **COEF**ficients in the variable specified in **MAT**rix form with each column of the matrix representing one equation. Equation intercepts will also be saved. This is useful if the coefficients for a single equation need to be specified on the **FC** command.

CONV=

Specifies a **CONV**ergence criterion to stop the iterative procedure (subject to the maximum number of iterations specified with the **ITER=**

option). The criterion is the maximum desired percentage change in each of the coefficients. The default is 0.001.

COV= Saves the **COV**ariance matrix in the variable specified. No values for the variances and covariances of equation intercepts will be saved so if these are required the directions above for the **NOCONSTANT** option should be followed.

ITER= Specifies the maximum number of **ITER**ations performed if an iterative procedure is desired. If this option is not specified, one iteration is done. If **ITER=0** is specified, the system is estimated without the Generalized Least Squares procedure which uses the covariance matrix of residuals. This would be equivalent to running separate **OLS** regressions (or **2SLS**), but more expensive. **ITER=0** would be appropriate only if there were restrictions across equations or hypothesis testing across equations.

OUT=*unit*
IN= *unit*

These options are used to **OUT**put a dump of useful information on the unit specified at each iteration, so that the system can be restarted in another run at the same point by **IN**putting the dump. This can be very useful in expensive models to avoid re-estimation of already calculated data in the event that a time limit is reached. With the **OUT=** option, the information from the most recent iteration will be written on the specified unit. **OUT=** and **IN=** are usually assigned to the same unit so the latest information replaces existing information. Units 11-49 are available and may be assigned to a file with the SHAZAM **FILE** command or an operating system command.

NOTE: It is important to remember to create a system file to be attached to the appropriate unit before using the **OUT=** option. Without this file the information will be lost. The use of **OUT=** will add slightly to the cost, but will substantially lower the cost of restarting the model. It is also important to run the same system on subsequent runs with the **IN=** option. The **IN=** option should *never* be run before there is anything to input. See the example at the end of this chapter.

PITER= Specifies the frequency with which **ITER**ations are to be **P**rinted. The default is **PITER=1**. If **PITER=0** is specified, no iterations are printed.

PREDICT= Saves the **PREDICT**ed values of the dependent variable in a N x M matrix, where N is the number of observations and M is the number of equations.

RESID= Saves the values of the estimated **RESID**uals in a N x M matrix.

SIGMA= Saves the residual covariance matrix $\hat{\Sigma}$ in the variable specified.

The temporary variables available are:

$ERR, $N and *$SIG2*

When more than one equation is estimated the value of the temporary variable *$SIG2* is the value $\ln\left|\hat{\Sigma}\right|$. If there is only one equation, *$SIG2* will be the same as that obtained from OLS estimation.

With SUR estimation the temporary variable *$LLF* is also available.

EXAMPLES

The next list of SHAZAM commands shows the use of the **SYSTEM** command for 3SLS estimation of Klein's model I described in Theil [1971, Chapter 9.2].

```
system 3 wg t g time plag klag xlag / dn
ols c plag p wgwp
ols i plag klag p
ols wp time xlag x
test p:1=p:2
```

There are 3 equations in the system and the variables *WG, T, G, TIME, PLAG, KLAG* and *XLAG* are the exogenous variables. SHAZAM automatically includes a constant in the list of exogenous variables. (All identities have already been substituted.) Three **OLS** commands are included to describe the three equations in the system. The **TEST** command will test the hypothesis that the coefficient on variable *P* in equation 1 is equal to the coefficient on variable *P* in equation 2.

The SHAZAM output from the 3SLS estimation is:

```
|_SYSTEM 3 WG T G TIME PLAG KLAG XLAG / DN
|_OLS C PLAG P WGWP
|_OLS I PLAG KLAG P
```

```
|_OLS WP TIME XLAG X
THREE STAGE LEAST SQUARES--    3 EQUATIONS
  7 EXOGENOUS VARIABLES
  6 POSSIBLE ENDOGENOUS VARIABLES
   9 RIGHT-HAND SIDE VARIABLES IN SYSTEM
MAX ITERATIONS =       1         CONVERGENCE TOLERANCE =   0.10000E-02
       21 OBSERVATIONS
DN OPTION IN EFFECT - DIVISOR IS N

ITERATION     0 COEFFICIENTS
  0.21623      0.17302E-01  0.81018      0.61594     -0.15779      0.15022
  0.13040      0.14667      0.43886
ITERATION    0 SIGMA
   1.0441
  0.43785       1.3832
 -0.38523      0.19261      0.47643
LOG OF DETERMINANT OF SIGMA=  -1.2458

ITERATION    1 SIGMA INVERSE
   2.1615
 -0.98292       1.2131
   2.1451      -1.2852       4.3530
ITERATION     1 COEFFICIENTS
  0.16314      0.12489      0.79008      0.75572     -0.19485     -0.13079E-01
  0.14967      0.18129      0.40049
ITERATION    1 SIGMA
  0.89176
  0.41132       2.0930
 -0.39361      0.40305      0.52003
LOG OF DETERMINANT OF SIGMA=  -1.2623

SYSTEM R-SQUARE =  0.9995
TEST OF THE OVERALL SIGNIFICANCE =   159.41
CHI-SQUARE WITH    9 D.F.     P-VALUE= 0.00000

VARIABLE       COEFFICIENT    ST.ERROR       T-RATIO
PLAG         0.16314        0.10044         1.6243
P            0.12489        0.10813         1.1550
WGWP         0.79008        0.37938E-01     20.826
PLAG         0.75572        0.15293         4.9415
KLAG        -0.19485        0.32531E-01    -5.9897
P           -0.13079E-01     0.16190       -0.80787E-01
TIME         0.14967        0.27935E-01     5.3579
XLAG         0.18129        0.34159E-01     5.3073
X            0.40049        0.31813E-01     12.589

 EQUATION  1 OF  3 EQUATIONS
DEPENDENT VARIABLE = C                     21 OBSERVATIONS
 R-SQUARE =   0.9801
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.89176
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.94433
SUM OF SQUARED ERRORS-SSE=   18.727
MEAN OF DEPENDENT VARIABLE =   53.995

                            ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
PLAG      0.16314     0.1004       1.624     0.104 0.367     0.0958     0.0495
P         0.12489     0.1081       1.155     0.248 0.270     0.0768     0.0391
WGWP      0.79008     0.3794E-01   20.83     0.000 0.981     0.8701     0.6070
CONSTANT   16.441      1.302       12.63     0.000 0.951     0.0000     0.3045
```

```
 EQUATION  2 OF  3 EQUATIONS
DEPENDENT VARIABLE = I                    21 OBSERVATIONS
 R-SQUARE =   0.8258
VARIANCE OF THE ESTIMATE-SIGMA**2 =   2.0930
STANDARD ERROR OF THE ESTIMATE-SIGMA =   1.4467
SUM OF SQUARED ERRORS-SSE=   43.954
MEAN OF DEPENDENT VARIABLE =   1.2667

                            ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
PLAG      0.75572     0.1529       4.942     0.000 0.768     0.8570     9.7704
KLAG     -0.19485     0.3253E-01  -5.990     0.000-0.824    -0.5441   -30.8417
P        -0.13079E-01 0.1619     -0.8079E-01 0.936-0.020    -0.0155    -0.1744
CONSTANT   28.178      6.796       4.146     0.000 0.709     0.0000    22.2457

 EQUATION  3 OF  3 EQUATIONS
DEPENDENT VARIABLE = WP                   21 OBSERVATIONS
 R-SQUARE =   0.9863
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.52003
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.72113
SUM OF SQUARED ERRORS-SSE=   10.921
MEAN OF DEPENDENT VARIABLE =   36.362

                            ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
TIME      0.14967     0.2794E-01   5.358     0.000 0.793     0.1473     0.0453
XLAG      0.18129     0.3416E-01   5.307     0.000 0.790     0.2565     0.2891
X         0.40049     0.3181E-01   12.59     0.000 0.950     0.6745     0.6615
CONSTANT  0.15080      1.016      0.1484     0.882 0.036     0.0000     0.0041

|_TEST P:1=P:2
TEST VALUE =  0.13797     STD. ERROR OF TEST VALUE  0.16036
ASYMPTOTIC NORMAL STATISTIC =  0.86037777      P-VALUE= 0.38958
WALD CHI-SQUARE STATISTIC =  0.74024991     WITH    1 D.F.  P-VALUE= 0.38958
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 1.00000
```

Restrictions in a Systems Estimation

In this example a set of Seemingly Unrelated Regression equations is estimated. The iteration option is not requested. The model is estimated subject to the restriction that the coefficient on variable *B* in equation 1 plus the coefficient on variable *E* in equation 2 sum to zero.

```
system 2 / restrict
ols a b c
ols d e f
restrict b:1+e:2=0
end
```

Note that SHAZAM always puts a constant term in each equation unless the **NOCONSTANT** option is specified on the **SYSTEM** command. If restrictions are required on the constant terms, you should generate your own constant by creating a variable which is always equal to 1 and then suppress the usual SHAZAM constant term with the **NOCONSTANT** option. The next example illustrates a model where two constant terms must sum to one.

```
genr const=1
system 2 / restrict noconstant
ols a b c const
ols d e f const
restrict b:1+e:2=0
restrict const:1+const:2=1
end
```

Example using a restart file

If you use the **OUT=** and **IN=** options in a **SYSTEM** (or **NL**) problem, the first run would assign a file to a unit between from 11-49. Note that a decimal point is included after the unit number on the **FILE** command because a **BINARY** file is required. The general form is as follows:

```
file 17.  restartfile
sample beg end
.  .  .
system 3 / dn  iter=10  out=17
ols depvar indeps
ols depvar indeps
ols depvar indeps
.  .  .

stop
```

In the next run, to restart where you left off, you should add the **IN=** option. The **SYSTEM** command might read as follows:

```
system 3 / dn  iter=20  out=17  in=17
```

In this case, the model will continue at the point it left off in the first run and update the file assigned to unit 17 until iteration 20 is reached. Do not attempt to use the **IN=** option before you have anything to **IN**put.

30. DATA SMOOTHING, MOVING AVERAGES AND SEASONAL ADJUSTMENT

"Everything should be made as simple as possible, but not simpler."
Albert Einstein

The **SMOOTH** command provides options for smoothing time series by the methods of moving averages, exponential smoothing and seasonal adjustment. An introductory reference is Newbold [1995, Chapter 17].

A time series Y_t for t = 1, 2, . . . , N can be viewed as containing a trend component (T_t), a seasonal component (S_t), a cyclical component (C_t) and an irregular component (I_t). An additive model considers:

$$Y_t = T_t + S_t + C_t + I_t$$

Alternatively, a multiplicative model assumes:

$$Y_t = T_t \cdot S_t \cdot C_t \cdot I_t$$

Moving Averages

A smoothed series can be obtained by the p-period moving average: $Y_t^* = \frac{1}{p}\sum_{j=0}^{p-1} Y_{t-j}$

The value for p is set with the **NMA=** option. For an odd number p, a centered p-period moving average is obtained by considering m=(p–1)/2 neighbors on either side of the observation:

$$Y_t^* = \frac{1}{p}\sum_{j=-m}^{m} Y_{t+j}$$

For an even number p, consider m=p/2. A centered p-period moving average is calculated as:

$$Y_t^* = \frac{1}{p}\left(\sum_{j=-m+1}^{m-1} Y_{t+j} + \frac{1}{2}\left(Y_{t-m} + Y_{t+m}\right) \right)$$

Seasonal Adjustment

The centered moving averages represent the combined T and C components of the time series. For the multiplicative model, the seasonal irregular effect (labelled as `SEAS&IRREG` on the SHAZAM output) in the time series is obtained as:

$$R_t = Y_t \big/ Y_t^*$$

For the additive model: $R_t = Y_t - Y_t^*$

Suppose the time series is observed for s periods per year. Seasonal factors are calculated by averaging the seasonal irregular effects over all observations in the sample that occur in the same time period of each year. With p=s the averages include n=N/s–1 observations. For the multiplicative model, geometric averages are computed as:

$$SF_i = \left(\prod_{t \in \text{period } i} R_t \right)^{1/n} \quad \text{or} \quad \log(SF_i) = \frac{1}{n} \left(\sum_{t \in \text{period } i} \log(R_t) \right) \quad \text{for } i = 1, 2, \ldots, s$$

The seasonal factors are normalized to have a geometric mean of unity. For the additive model, arithmetic averages are computed as:

$$SF_i = \frac{1}{n} \left(\sum_{t \in \text{period } i} R_t \right)$$

In this case, the seasonal factors are normalized to have an arithmetic mean of zero.

Suppose the normalized seasonal factors are stacked in a seasonal index variable SI. The seasonally adjusted time series is obtained as Y_t / SI_t for the multiplicative model, or $Y_t - SI_t$ for the additive model.

Exponential Smoothing

The exponential moving average is calculated as:

$$\overline{Y}_1 = Y_1 \quad \text{and} \quad \overline{Y}_t = w\, Y_t + (1 - w)\, \overline{Y}_{t-1} \quad \text{for } t = 2, \ldots, N$$

The weight w must be in the range 0 < w < 1 and the default value is 2/(1+p). The weight can be specified with the **WEIGHT=** option.

Hodrick-Prescott Filter

The Hodrick-Prescott [1997] filter finds the T_t path to minimize the sum of squares:

$$\sum_{t=1}^{N}(Y_t - T_t)^2 + \lambda \sum_{t=2}^{N-1}\left[(T_{t+1} - T_t) - (T_t - T_{t-1})\right]^2$$

λ is set as the smoothing parameter.

SMOOTH *COMMAND OPTIONS*

In general, the format of the **SMOOTH** command is:

SMOOTH *var* / *options*

where *var* is a time series variable and *options* is a list of desired options. Options as defined for most SHAZAM commands that are available are **BEG=** and **END=**.

Additional options available on the **SMOOTH** command are:

ADDITIVE	Uses an additive model instead of the multiplicative model. If this option is used the **ARITH** option is automatically set.
ARITH	Used to compute an arithmetic average rather than a geometric average for the calculation of seasonal factors.
CENTRAL/ NOCENTRAL	By default, centered moving averages are calculated. Use **NOCENTRAL** for fully lagged moving averages.
HPFILTER	Implements the Hodrick-Prescott [1997] filter. The smoothing parameter is specified with the **LAMBDA=** option.
EMAVE=	Stores the exponential moving average in the variable specified.
LAMBDA=	Specifies the smoothing parameter for the Hodrick-Prescott filter.

MAVE= Stores the moving average in the variable specified.

NMA= Specifies the number of periods (p) to use in calculating moving averages. For seasonal adjustment **NMA=** is usually the same number as **NSPAN=**.

NSPAN= Specifies the number of seasonal periods s. Use 4 for quarterly data and 12 for monthly data.

SAMAVE= Stores the seasonally adjusted time series in the variable specified.

SFAC= Stores the seasonal adjustment factors in the variable specified.

WEIGHT= Specifies the weight w in the range 0 < w < 1 to be used to compute exponential moving averages. The default is 2/(1+p) where p is the value specified with the **NMA=** option.

A temporary variable available after the **SMOOTH** command is the error code *$ERR*.

EXAMPLES

This example is presented in Newbold [1995, Table 17.14, p. 702]. The data set is quarterly data on earnings per share of a corporation observed over a period of 8 years. Newbold uses a different method for calculating seasonal factors. Therefore the seasonally adjusted time series reported in the column `SA(EARNINGS)` is different from the adjusted series reported in Newbold.

```
|_SAMPLE 1 32
|_READ EARNINGS / BYVAR
   1 VARIABLES AND       32 OBSERVATIONS STARTING AT OBS       1

|_SMOOTH EARNINGS / NMA=4 NSPAN=4

CENTRAL MOVING AVERAGES - PERIODS=  4 NSPAN=  4 WEIGHT= 0.400

OBSERVATION  EARNINGS MOVING-AVE   SEAS&IRREG  SA(EARNINGS) EXP-MOV-AVE
          1  0.30000      -------      -------      0.45014      0.30000
          2  0.46000      -------      -------      0.44541      0.36400
          3  0.34500      0.50750      0.67980      0.42855      0.35640
          4  0.91000      0.52187       1.7437      0.50424      0.57784
          5  0.33000      0.54438      0.60620      0.49515      0.47870
          6  0.54500      0.57250      0.95197      0.52772      0.50522
          7  0.44000      0.60938      0.72205      0.54655      0.47913
          8   1.0400      0.64687       1.6077      0.57627      0.70348
          9  0.49500      0.67688      0.73130      0.74273      0.62009
         10  0.68000      0.72063      0.94363      0.65844      0.64405
```

11	0.54500	0.75812	0.71888	0.67698	0.60443
12	1.2850	0.78875	1.6292	0.71203	0.87666
13	0.55000	0.82688	0.66515	0.82525	0.74600
14	0.87000	0.87813	0.99075	0.84241	0.79560
15	0.66000	0.92000	0.71739	0.81983	0.74136
16	1.5800	0.94000	1.6809	0.87549	1.0768
17	0.59000	0.97625	0.60435	0.88527	0.88209
18	0.99000	1.0163	0.97417	0.95861	0.92525
19	0.83000	1.0375	0.80000	1.0310	0.88715
20	1.7300	1.0475	1.6516	0.95860	1.2243
21	0.61000	1.0662	0.57210	0.91528	0.97857
22	1.0500	1.1162	0.94065	1.0167	1.0071
23	0.92000	1.1662	0.78885	1.1428	0.97229
24	2.0400	1.2000	1.7000	1.1304	1.3994
25	0.70000	1.2400	0.56452	1.0503	1.1196
26	1.2300	1.2925	0.95164	1.1910	1.1638
27	1.0600	1.3425	0.78957	1.3167	1.1223
28	2.3200	1.3800	1.6812	1.2855	1.6014
29	0.82000	1.4263	0.57493	1.2304	1.2888
30	1.4100	1.5012	0.93922	1.3653	1.3373
31	1.2500	-------	-------	1.5527	1.3024
32	2.7300	-------	-------	1.5127	1.8734

4	SEASONAL	FACTORS
1	0.66646	
2	1.0327	
3	0.80505	
4	1.8047	

31. FINANCIAL TIME SERIES

"$100 million dollars is way too much to pay for Microsoft."
IBM, 1982

Four commands are available for the study of financial time series such as stock or asset prices:

STOCKGRAPH	Used to calculate and chart technical indicators of stock market prices.
PORTFOLIO	Used to compute risk return portfolios using Markowitz models.
CALL	Used for price valuation models for call options.
PUT	Used for price valuation models for put options.

Algorithms for computing technical indicators for stock market data can be found in various monthly issues of the magazine: *Technical Analysis of Stocks and Commodities*. Computational formula for Markowitz risk return frontiers are described in Campbell, Lo and MacKinlay [1997, Chapter 5]. An excellent reference for algorithms used to compute stock option prices for the **CALL** and **PUT** commands is Clewlow and Strickland [1998]. Most of the calculations follow the suggestions in this book.

For financial time series, a presentation tool is the display of graphs. The commands in this chapter prepare graphs with the GNUPLOT software as described in the *PLOTS AND GRAPHS* chapter. GNUPLOT Version 3.7 or later is required (this may not be available on all operating systems).

THE **STOCKGRAPH** *COMMAND*

Consider a financial time series P_t for t = 1, . . . , N. A smoothed series is obtained by a moving average. For example, a basic indicator is a moving average calculated from daily closing stock prices. A number of alternative moving average formula are available. A simple moving average is calculated as:

$$P_t^* = \begin{cases} \frac{1}{t}\sum_{j=0}^{t-1} P_{t-j} & \text{for} \quad t = 1, \ldots, n \\ \frac{1}{n}\sum_{j=0}^{n-1} P_{t-j} & \text{for} \quad t = n+1, \ldots, N \end{cases}$$

where the value for n is set with the **MASHORT=** option. The default value is n = 12. The **SOMA** option calculates a second-order moving average as:

$$P_t^{**} = \begin{cases} P_t^* + \frac{6}{(t+1)t}\sum_{j=0}^{t-1} P_{t-j}[t-(2\cdot j+1)]/2 & \text{for} \quad t = 1, \ldots, n \\ P_t^* + \frac{6}{(n+1)n}\sum_{j=0}^{n-1} P_{t-j}[n-(2\cdot j+1)]/2 & \text{for} \quad t = n+1, \ldots, N \end{cases}$$

An exponential moving average is computed with the **EMA** option. A weight w is set as 2/(1+n) and the smoothed series is calculated as:

$$\overline{P}_1 = P_1 \qquad \text{and} \qquad \overline{P}_t = w \cdot P_t + (1-w)\cdot \overline{P}_{t-1} \qquad \text{for } t = 2, \ldots, N$$

Bollinger Bands

An envelope or trading band is defined as an upper and lower boundary for a given moving average. For a simple moving average, Bollinger bands are calculated as:

$$\left[B_t^L,\ B_t^U\right] = \left[P_t^* - d\cdot\sqrt{V\left(P_t^*\right)},\ P_t^* + d\cdot\sqrt{V\left(P_t^*\right)}\right]$$

where the value for d is specified with the **BOLLINGER=** option. A typical choice is d = 2. The variance is:

$$V\left(P_t^*\right) = \begin{cases} \frac{1}{t}\left[\sum_{j=0}^{t-1} P_{t-j}^2 - t\left(P_t^*\right)^2\right] & \text{for} \quad t = 1, \ldots, n \\ \frac{1}{n}\left[\sum_{j=0}^{n-1} P_{t-j}^2 - n\left(P_t^*\right)^2\right] & \text{for} \quad t = n+1, \ldots, N \end{cases}$$

A feature of Bollinger bands is that during periods of relatively high volatility the bands are wider. The difference between the closing price P_t and the lower bound relative to the range of the Bollinger band is:

$$\left(P_t - B_t^L\right) / \left(B_t^U - B_t^L\right)$$

When the **LIST** option is used this calculation is reported in the %B column on the SHAZAM output. Another measure reported in the WIDTH column is:

$$\left(B_t^U - B_t^L\right) / P_t^*$$

Moving Average Convergence-Divergence Indicator

The moving average convergence-divergence (MACD) line is the difference between an m-period and n-period exponential moving average. The values for m and n are specified with the **MALONG=** and **MASHORT=** options respectively. The "signal" or "trigger" line is calculated as a p-period exponential moving average of this difference. The value for p is specified with the **MAMACD=** option. For daily data, standard choices are m = 26, n = 12 and p = 9.

STOCKGRAPH ***Command Options***

In general, the format of the **STOCKGRAPH** command is:

STOCKGRAPH *vars* / *options*

where *vars* is a list of variable names and *options* is a list of desired options. The variable list contains the stock price information to be used for technical analysis. This can include the open, high and low prices (*open*, *hi*, *low*), the closing price (*close*), the volume measured by the number of shares traded (*volume*) and the date (*date*). The command format must conform to one of the following.

STOCKGRAPH *close* / *options*

STOCKGRAPH *close date* / *options*

STOCKGRAPH *close volume* / **GRAPHVOL** *options*

STOCKGRAPH *close volume date* / **GRAPHVOL** *options*

STOCKGRAPH *open hi low close* / *options*

STOCKGRAPH *open hi low close date* / *options*

STOCKGRAPH *open hi low close volume* / **GRAPHVOL** *options*

STOCKGRAPH *open hi low close volume date* / **GRAPHVOL** *options*

The number of variables listed determines which of the above formats is effective. Variables must be listed in the order shown above. If data on the volume of shares traded is included, the **GRAPHVOL** option must be specified or SHAZAM might think that the volume variable is a date.

If a date variable is included it must be a numerical value. Before the **STOCKGRAPH** command, the **TIMEFMT** and **AXISFMT** commands should be used to specify the format of the date variable and the date format of the x-axis labels as described in the *PLOTS AND GRAPHS* chapter. In addition, the **AXISFMT** option should also be specified on the **STOCKGRAPH** command.

Options as defined for most SHAZAM commands that are available are **BEG=** and **END=**. Options as defined for the **GRAPH** command that are available are **AXISFMT** and **WIDE**. Additional options available on the **STOCKGRAPH** command are:

EMA Calculates an exponential moving average instead of a simple moving average. The number of time periods in the moving average is specified with the **MASHORT=** option.

GRAPHDATA Graphs the financial time series data. If the variable list includes *open, hi, low* and *close* then a bar chart is displayed. A vertical line marks the low and high values. The closing price is represented by a tick mark to the right of the bar and the opening value is given by a tick mark to the left of the bar.

GRAPHMA Graphs the moving average line. This option automatically turns on the **GRAPHDATA** option.

GRAPHMACD Graphs the moving average convergence-divergence (MACD) lines. This option automatically turns on the **GRAPHDATA** option. The chart with

the MACD line and the signal line is displayed at the bottom of the graph. The chart also shows a bar chart that represents the difference between the MACD and signal lines.

GRAPHVOL Graphs the volume of shares traded. If this option is used, the variable list must include the *volume* variable. This option automatically turns on the **GRAPHDATA** option. A volume bar chart is displayed at the bottom of the graph. The bottom of each volume bar is the value zero. The chart highlights the relative volume levels. The y-axis labels for the volume are not shown.

LIST Lists results on the SHAZAM output.

SOMA Calculates a second-order moving average instead of a simple moving average. The number of time periods in the moving average is specified with the **MASHORT=** option. This may result in a less smooth result compared to a simple moving average that assigns equal weight to each price in the calculation.

BOLLINGER= Specifies the number of standard deviations to use to calculate the lower and upper Bollinger bands. A common number is 2 standard deviations.

MALONG= Specifies the number of time periods to use for a long-term moving average calculation. The default is 26 periods.

MAMACD= Specifies the number of time periods to use for the moving average convergence-divergence trigger calculation. The default is 9 periods.

MASHORT= Specifies the number of time periods to use for a short-term moving average calculation. The default is 12 periods.

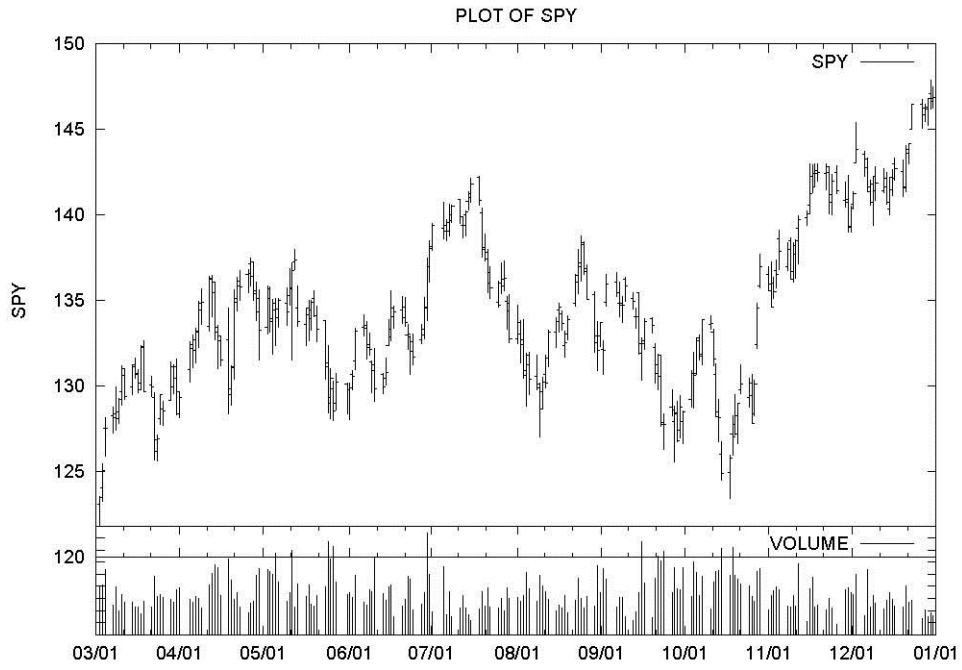
A temporary variable available after the **STOCKGRAPH** command is the error code *$ERR*.

EXAMPLES

A data set is available with the price history of S&P Depository Receipts identified by the trading symbol SPY. The data set contains data on the open, high, low and close daily prices as well as the volume (number of shares traded) for the period March 3, 1999 to March 1, 2000. A chart is prepared with the SHAZAM commands below.

```
sample 1 253

* The variable SPY is the daily closing price for the symbol SPY.
read (spy.dat) date open hi low spy volume
timefmt %Y%m%d
axisfmt %m/%d

* Set sample period from 1999/03/03 to 1999/12/31.
sample 1 212
stock open hi low spy volume date / graphvol axisfmt
stop
```

A daily bar chart is shown in the figure. The volume bar chart is displayed at the bottom of the figure.



THE PORTFOLIO *COMMAND*

Denote P_{it} for t = 1, . . . , N as prices for asset i. The percentage returns are:

$$R_{it} = 100 \cdot (P_{it} / P_{i,t-1} - 1) \qquad \text{for} \quad t = 2, \ldots, N$$

The means $\overline{R}_i$, variances $V(R_i)$ and covariances $Cov(R_i, R_j)$ are calculated as described for the **STAT** command in the chapter *DESCRIPTIVE STATISTICS.* Consider r as the return

on the risk-free asset. A value for r is specified with the **RISKFREE=** option. For asset i, the Sharpe ratio is defined as:

$$\left(\overline{R}_i - r\right) \Big/ \sqrt{V(R_i)}$$

The **INDEX=** option can be used to specify a variable that contains a price index for the market. Denote R_{mt} as the return on the market portfolio. The capital asset pricing model can be estimated and evaluated using the regression equation:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

where the ε_{it} are random errors. OLS estimates of the β_i and α_i are reported on the SHAZAM output in the columns labelled `BETA` and `ALPHA` respectively.

The problem is to find portfolio weights w_i. For an expected portfolio return of R_p, the minimum-variance portfolio is obtained as the solution to the problem:

$$\min_{w} \sum_i \sum_j w_i w_j \, Cov\,(R_i, R_j) \qquad \text{subject to} \qquad \sum_i w_i \overline{R}_i = R_p \quad \text{and} \quad \sum_i w_i = 1$$

With optimal weights w_i^p the mean return of the portfolio is: $\sum_i w_i^p \overline{R}_i$

and the variance is: $\sum_i \sum_j w_i^p w_j^p \, Cov\,(R_i, R_j)$

The standard deviation of the portfolio gives a measure of risk. The **PFRONTIER** option gives a listing of the means, standard deviations and Sharpe ratios for the minimum-variance portfolios. The **GRAPHFRONT** option displays a graph of the minimum-variance risk-return frontier. The problem now is to select a single optimum portfolio. An example is given later in this chapter.

The mean return and standard deviation of a user-specified portfolio can be calculated. An equal-weighted portfolio return is reported with the **EQUALWEIGHT** option. User-specified weights are recognized with the **WEIGHTS** option. Alternatively, with the **SHARES** option, the number of shares can be provided. Denote Q_{it} as the number of shares for asset i. Portfolio weights are calculated as:

$$w_i^a = \frac{1}{N-1}\sum_{t=2}^{N}\left(P_{it}Q_{it}\Big/\sum_i P_{it}Q_{it}\right)$$

The mean return of the portfolio is then obtained as: $\sum_i w_i^a \overline{R}_i$

PORTFOLIO ***Command Options***

In general, the formats of the **PORTFOLIO** command are:

PORTFOLIO *vars* / *options*

PORTFOLIO *vars weights* / **WEIGHTS** *options*

PORTFOLIO *vars shares* / **SHARES** *options*

where *vars* is a list of variables containing stock prices or returns for each of the assets. For example, if there are 12 stocks in the portfolio there would be 12 variables. If the user knows how many shares are owned of each stock then there would also be 12 *shares* variables indicating these numbers and the **SHARES** option would be specified. Alternatively, portfolio weights of each stock can be entered in 12 *weight* variables and the **WEIGHTS** option would be specified. Finally *options* is a list of desired options.

Options as defined for most SHAZAM commands that are available are **BEG=** and **END=**. An option as defined for the **GRAPH** command that is available is **WIDE**. Additional options available on the **PORTFOLIO** command are:

EQUALWEIGHT Specifies that no share or weight information is provided as the portfolio contains equal weights for each asset.

GRAPHDATA Displays a scatterplot of the risks and returns for each of the stocks in the portfolio.

GRAPHFRONT Plots the minimum-variance risk-return frontier for the stocks in the portfolio.

GRAPHLINE Adds a line on the frontier plot from the risk-free rate of interest to the tangency point on the risk-return frontier. This will help define risk-

return options if it is possible to borrow and lend at the risk-free rate of interest. This option automatically turns on the **GRAPHFRONT** option.

INRATES Specifies that the stock price data is in rates of return rather than the original stock price. The rate of return is defined as $100 \cdot (P_t / P_{t-1} - 1)$.

LIST Lists the rates of return on the SHAZAM output.

PFRONTIER Prints a listing of the minimum-variance risk-return frontier.

SHARES Specifies that the command line includes names of variables containing the corresponding number of shares for each stock price variable.

WEIGHTS Specifies that the command line includes names of variables containing the corresponding weights for each stock price variable. The weights are normalized to sum to unity.

INDEX= Specifies the name of a variable containing the price of a stock market index to be used in capital asset pricing model calculations.

RETURNS= Specifies a variable for saving the portfolio returns for the risk-return frontier (200 values are saved), the mean returns for each stock and the returns listed in the `EFFICIENT PORTFOLIOS` table on the SHAZAM output.

RISKFREE= Specifies the risk-free rate of interest. It should be in percent. For example, if the risk-free rate is 5%, enter **RISKFREE=5.** This option can be a scalar variable or a time series variable. If a time series variable is entered the risk-free rate is set to the average value.

RISKS= Specifies a variable for saving the portfolio standard deviations for the risk-return frontier, the standard deviations for each stock return and the standard deviations for the portfolios listed in the `EFFICIENT PORTFOLIOS` table on the SHAZAM output. Also see the **RETURNS=** option.

A temporary variable available after the **PORTFOLIO** command is the error code *$ERR*.

EXAMPLES

Berndt [1991, Chapter 2] provides a data set of monthly returns for a number of companies for the period January 1978 to December 1987. From this data set, a file was prepared with returns for Mobil, IBM, Weyerhauser and Citicorp as well as the return on 30-day Treasury Bills (a measure of the risk-free return) and a value-weighted composite monthly market return based on all stocks listed at the New York and American Stock Exchanges. The SHAZAM commands below solve a portfolio selection problem.

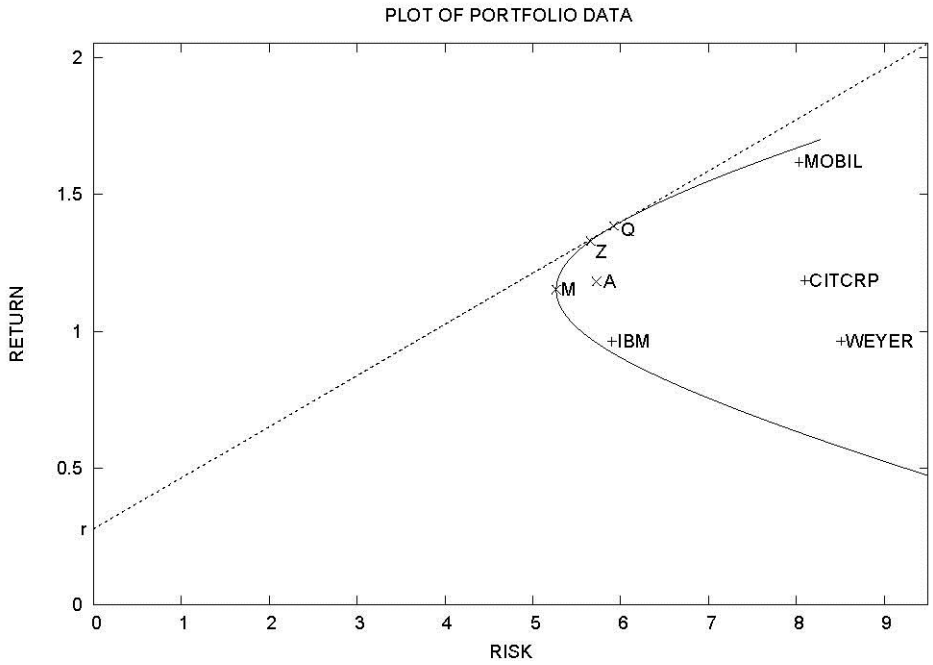
```
sample 1 120
read (p.dat) date mobil ibm weyer citcrp market rkfree /
skiplines=1

* Convert to percentages
genr mobil=100*mobil
genr ibm=100*ibm
genr weyer=100*weyer
genr citcrp=100*citcrp

* Set a risk-free rate of return
gen1 rf=100*(rkfree:120)
portfolio mobil ibm weyer citcrp / inrates riskfree=rf equalw &
          graphdata graphline
stop
```

SHAZAM output from the **PORTFOLIO** command is:

```
|_PORTFOLIO MOBIL IBM WEYER CITCRP / INRATES RISKFREE=RF EQUALW &
            GRAPHDATA GRAPHLINE
PORTFOLIO ANALYSIS - RATES OF RETURN     4 ASSETS        120 OBSERVATIONS
MEAN RISKFREE RATE OF RETURN = 0.27700
VARIABLE      MEAN        ST.DEV      SHARPE
MOBIL       1.6192       8.0308      0.16713
IBM        0.96167       5.9024      0.11600
WEYER      0.96333       8.5066      0.80682E-01
CITCRP      1.1858       8.0972      0.11224

COVARIANCE MATRIX
MOBIL       64.493
IBM         15.225       34.838
WEYER       26.403       24.694       72.363
CITCRP      20.227       20.250       37.195       65.564
              MOBIL        IBM          WEYER        CITCRP

EFFICIENT PORTFOLIOS
          MINIMUMVARIANCE   RISKFREE=ZERO       RETURN=ZERO       ACTUAL
 MEAN        1.1521            1.3300          0.44409E-15        1.1825
 VARIANCE    27.751            32.037            207.44           32.828
 STDEV       5.2680            5.6602            14.403           5.7296
 SHARPE     0.16612           0.18604          -0.19233E-01       0.15804
```

```
PORTFOLIO WEIGHTS
MOBIL       0.23486          0.48422          -1.3797          0.25000
IBM         0.59096          0.39925           1.8322          0.25000
WEYER       0.13607E-01     -0.10732           0.79655         0.25000
CITCRP      0.16057          0.22385          -0.24910         0.25000
```

The figure on the next page shows the minimum-variance risk-return frontier. The portfolios marked M and Z are the MINIMUMVARIANCE and RISKFREE=ZERO portfolios respectively. The portfolio A is the equal-weighted portfolio calculated with the **EQUALWEIGHT** option and reported on the SHAZAM output in the ACTUAL column. The straight line with an intercept at r that is tangential to the efficient frontier at point Q has a slope that is the maximum Sharpe ratio of all possible portfolios.



THE CALL AND PUT COMMANDS

A call option provides the right to buy a share of stock. A put option provides the right to sell a share of stock. A European option is only exercisable at the expiration date. In contrast, an American option is exercisable at any time before expiry.

The Black-Scholes Formula

The Black-Scholes [1973] equation gives a formula for pricing European call and European put options. This formula can also be applied for American call options on assets that do

not pay dividends since early exercise is not optimal for non-dividend paying assets. Consider an asset price S_t. The inputs to the formula are:

		CALL / PUT command option
Exercise price	K	**STRIKEPRICE=**
Standard deviation	σ	**SIGMA=**
Risk-free interest rate	r	**RISKFREE=**
Time to expiration	τ	**TIME=**
Dividend yield	δ	**DIVIDEND=**

The Black-Scholes calculation for the value of the call option at time t is:

$$C_t = S_t e^{-\delta\tau} F(d_1) - K e^{-r\tau} F(d_2)$$

where $$d_1 = \frac{\ln(S_t / K) + (r - \delta + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}$$ and $$d_2 = d_1 - \sigma\sqrt{\tau}$$

F() represents the cumulative normal distribution function. The corresponding formula for pricing puts is:

$$P_t = -S_t e^{-\delta\tau} F(-d_1) + K e^{-r\tau} F(-d_2)$$

Implied Volatility

For a call option with price C, it may be of interest to calculate the implied volatility (σ) given the current stock price (S_t), strike price (K), interest rate (r), time to maturity (τ), and dividend yield (δ). This is implemented with the **IMPVOL** option on the **CALL** command. In this case consider,

$$g(\sigma) = S_t e^{-\delta\tau} F(d_1) - K e^{-r\tau} F(d_2)$$

The problem is to find σ such that g(σ) = C. The solution can be obtained by an iterative algorithm based on the Newton-Raphson method for solving a nonlinear equation as described in Benninga [1989, pp. 147-150]. The algorithm uses a starting value for σ as:

$$\sigma^2 = \left| \ln(S_t / K) + r\,\tau \right| \frac{2}{\tau}$$

The formula for the iterations is: $\sigma(i+1) = \sigma(i) - \dfrac{g(\sigma(i)) - C}{g'(\sigma)}$

where $g'(\sigma) = \dfrac{\partial g(\sigma)}{\partial \sigma} = S_t \sqrt{\tau}\, f(d_1)$ and f() is the normal density function.

The convergence criteria is: $\left| g(\sigma(i)) - C \right| < 0.00001$

Binomial Option Pricing

The option pricing problem can be approached by viewing the asset price as a random variable with a binomial distribution. This leads to a price valuation based on a multiplicative binomial tree. An alternative assumption is that the log of the asset price follows a binomial process. In this case, the price valuation uses a general additive binomial tree. Computational details of binomial option pricing methods are available in Clewlow and Strickland [1998, Chapter 2].

CALL ***and*** **PUT** ***Command Options***

In general, the format of the **CALL** and **PUT** commands is:

CALL *stockprice / options*

or

PUT *stockprice / options*

where *stockprice* is a list of variables and *options* is a list of desired options. The default is to assume a European option.

Options as defined for most SHAZAM commands that are available are **BEG=** and **END=**. Additional options available on the **CALL** and **PUT** commands are:

AMERICAN Specifies that the stock option is American rather than European.

BLACK Use the Black-Scholes option pricing model.

EQUAL Use the equal jump model.

IMPVOL Compute the implied volatility from the Black-Scholes model. The call or put option price must be specified with **OPTIONP**=.

BARRIER= The barrier level for the asset price used in pricing an American down-and-out call option.

DIVIDEND= Continuous dividend yield of the asset.

NUMTIME= Specifies the number of time steps in the binomial tree to use for binomial option calculations.

OPTIONP= Specifies the variable containing the option prices. This is required for the **IMPVOL** option.

PREDICTP= Stores the predicted option prices in the variable specified.

PREDICTV= Stores the calculated implied volatilities in the variable specified.

RISKFREE= Specifies the risk-free interest rate. If the interest rate is 5% enter **RISKFREE=5**. Note this differs from the use in Clewlow and Strickland where in their notation a 5% interest rate is .05.

SIGMA= Specifies the standard deviation to be used which measures risk. This is commonly the historical volatility.

STRIKEPRICE= Specifies the stock option strike price. It can be specified as a single number or it could be a variable with different strike prices for each stock price.

TIME= Specifies the number of time periods until the stock option expires.

UP=
DOWN= Size of proportional upward and downward move of stochastic variable.

A temporary variable available after the **CALL** and **PUT** commands is the error code *$ERR*.

EXAMPLES

This example is from Clewlow and Strickland [1998, p. 21]. The problem is to price an at-the-money (ATM) European call option with a current asset price of $100. For an ATM option the exercise price is the same as the current price. Consider a one-year maturity (**TIME=1**) and assume the asset volatility is 20% (**SIGMA=0.2**) and the continuously compounded interest rate is 6 per cent per annum (**RISKFREE=6**). A comparison of the results for the general additive binomial tree method and the Black-Scholes method is shown in the SHAZAM output below. The binomial method provides a call option price of $11.59 and the Black-Scholes formula gives a call option price of $10.99.

```
|_SAMPLE 1 1
|_GENR S=100
|_GENR K=100
|_* Binomial option pricing - the binomial tree has 3 time steps
|_GENR N=3
|_CALL S / STRIKE=K TIME=1 RISKFREE=6 SIGMA=0.2 NUMTIME=N
GENERAL ADDITIVE BINOMIAL TREE OF EUROPEAN CALL
 OBS TIME STOCK  STRIKE RATE  N  UP  DOWN   DIV   SIG    CALL  IMPVOL   PCALL
   1 1.00 100.00 100.00  6.00  3           0.00  0.20   0.000          11.592

|_* Black-Scholes option pricing
|_CALL S / STRIKE=K TIME=1 RISKFREE=6 SIGMA=0.2 BLACK
BLACK-SCHOLES MODEL-EUROPEAN CALL
 OBS TIME STOCK  STRIKE RATE  N  UP  DOWN   DIV   SIG    CALL  IMPVOL   PCALL
   1 1.00 100.00 100.00  6.00  0           0.00  0.20   0.000          10.990
```

32. LINEAR PROGRAMMING

"Programming, both linear and nonlinear, is entirely a mathematical technique. Its economic content is therefore nil."

William J. Baumol
Professor of Economics, 1977

The **LP** command is available to compute the solution to a linear programming problem. For discussion of linear programming methods see, for example, Baumol [1977]. The purpose is to solve for the vector x and the shadow values of the constraints for the problem:

Maximize $c'x$

subject to: $Ax \leq b$

$x \geq 0$

where A is a M x N matrix, b is a M x 1 vector and c is a N x 1 vector of coefficients in the objective function. The solution algorithm is a contracted simplex algorithm based on a program written by David Ryan. Note that the second set of constraints ($x \geq 0$) is automatically imposed and does not need to be specified.

SHAZAM expects the first set of constraints ($Ax \leq b$) to be in the form of less than or equal to (≤) inequalities. However, inequalities of the form $Ax \geq b$ can be used if the signs of the elements in the A matrix and b vector are all reversed as shown in the second example below. If an exact equality constraint is required it should be specified as two inequality constraints using both ≤ and ≥ notation. If the problem is set up as a minimization rather than a maximization problem, the **MIN** option should be specified.

LP *COMMAND OPTIONS*

In general, the format of the **LP** command is:

LP *c A b / options*

where *c*, *A* and *b* are the vectors and matrices defined above. The available options are:

DUMP Prints information for SHAZAM consultants.

MIN Indicates that the problem is a **MIN**imization problem.

DSLACK= Saves the dual slack variables in the vector specified.

DUAL= Saves the dual solution in the vector specified.

ITER= Specifies the maximum number of iterations. The default is 15.

PRIMAL= Saves the primal solution in the vector specified.

PSLACK= Saves the primal slack variables in the vector specified.

Temporary variables available following the **LP** command are:

$ERR	Error code
$VAL	The value of the objective function

EXAMPLES

Consider the linear programming problem:

Maximize $x_1 + 3\,x_2$

subject to: $x_1 \le 300$

$x_2 \le 100$

$0.1\,x_1 + 0.2\,x_2 \le 40$

$x_1, x_2 \ge 0$

The SHAZAM commands to input the coefficients and solve the the maximization problem are:

```
read c / rows=2 cols=1
 1
 3
read a / rows=3 cols=2
 1  0
 0  1
.1 .2
read b / rows=3 cols=1
300
100
 40
lp c a b
```

The output is:

```
|_READ C / ROWS=2 COLS=1
   1 VARIABLES AND        2 OBSERVATIONS STARTING AT OBS       1
...SAMPLE RANGE IS NOW SET TO:         1         2
|_READ A / ROWS=3 COLS=2
      3 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1
|_READ B / ROWS=3 COLS=1
   1 VARIABLES AND        3 OBSERVATIONS STARTING AT OBS       1

|_LP C A B
NUMBER OF VARIABLES=    2 NUMBER OF CONSTRAINTS=    3
COEFFICIENTS ON VARIABLES IN OBJECTIVE FUNCTION
   1.0000       3.0000
CONSTRAINT COEFFICIENTS AND CONSTRAINT VALUES
   1.0000      0.00000E+00   300.00
  0.00000E+00   1.0000       100.00
  0.10000      0.20000       40.000

MAXIMIZED VALUE OF OBJECTIVE FUNCTION IS    500.0
PRIMAL SOLUTION
  200.00       100.00
SLACK VARIABLES
  100.00      0.00000E+00  0.00000E+00
DUAL SOLUTION
 0.00000E+00   1.0000       10.000
DUAL SLACK VARIABLES
 0.00000E+00  0.00000E+00
```

The dual linear programming problem with variables $\lambda_1, \lambda_2, \ldots$ can be constructed from the primal problem. For example, the dual solution to the above problem can be obtained as the primal solution to the following minimization problem:

Minimize $300\,\lambda_1 + 100\,\lambda_2 + 40\,\lambda_3$

subject to: $\lambda_1 + 0.1\,\lambda_3 \geq 1$

$$\lambda_2 + 0.2\,\lambda_3 \geq 3$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

Since the constraints are of the form $\geq$ rather than $\leq$ it is necessary to reverse the signs of the coefficients in the A matrix and b vector. The SHAZAM commands for solving this problem are:

```
read c / rows=3 cols=1
300
100
 40
read a / rows=2 cols=3
-1  0 -.1
 0 -1 -.2
read b / rows=2 cols=1
-1
-3
lp c a b / min
```

The output is:

```
|_READ C / ROWS=3 COLS=1
   1 VARIABLES AND        3 OBSERVATIONS STARTING AT OBS       1
...SAMPLE RANGE IS NOW SET TO:         1         3
|_READ A / ROWS=2 COLS=3
      2 ROWS AND           3 COLUMNS, BEGINNING AT ROW       1
|_READ B / ROWS=2 COLS=1
   1 VARIABLES AND        2 OBSERVATIONS STARTING AT OBS       1

|_LP C A B / MIN
NUMBER OF VARIABLES=    3 NUMBER OF CONSTRAINTS=    2
COEFFICIENTS ON VARIABLES IN OBJECTIVE FUNCTION
  300.00      100.00      40.000
CONSTRAINT COEFFICIENTS AND CONSTRAINT VALUES
  -1.0000      0.00000E+00 -0.10000      -1.0000
  0.00000E+00  -1.0000     -0.20000      -3.0000
MINIMIZATION  PROBLEM

MINIMIZED VALUE OF OBJECTIVE FUNCTION IS    500.0
PRIMAL SOLUTION
 0.00000E+00   1.0000       10.000
SLACK VARIABLES
 0.00000E+00  0.00000E+00
DUAL SOLUTION
  200.00       100.00
DUAL SLACK VARIABLES
  100.00      0.00000E+00  0.00000E+00
|_STOP
```

33. QUADRATIC PROGRAMMING

"It has been found so far that, for any computation method which seems useful in relation to some set of data, another set of data can be constructed for which that method is obviously unsatisfactory."

Tjalling Charles Koopmans
Professor of Economics, 1951

The **QP** command is available to compute the solution to quadratic programming problems, which are characterized by an objective function of quadratic form with constraints that are linear. The aim is to solve for the N variables contained in the Nx1 vector x, which maximize a given quadratic function, while satisfying M linear equality and/or inequality constraints:

Maximize $c'x + x'Qx$

subject to: $Ax \leq b$

where c is a N x 1 vector and Q is a N x N symmetric matrix of coefficients, which describe the objective function, A is a M x N matrix and b is a M x 1 vector, which contain the constraints. For a discussion of quadratic programming see, for example, Chiang [1984, Chapter 21] or Nocedal and Wright [1999]. SHAZAM provides two methods for solving quadratic programming problems.

The **POWELL** method is an algorithm written by Powell [1983], which implements the projection type dual method of Goldfarb and Idnani [1983]. The algorithm solves the problem described above, with the ability to impose upper and lower bounds on the solution vector x if required, i.e.

$$xl \leq x \leq xu,$$

where xl and xu are N x 1 vectors. Use the options **LOWER=**xl and **UPPER=** xu to specify limits in vector form. Alternatively specify scalar values using the **LOWSCAL=** or **UPSCAL=** options if all variables have the same bound. The ability to apply bounds on the solution was added to Powell's algorithm by K. Schittkowski of the University of Bayreuth [1987]. The **POWELL** method can be used for problems with any number of constraints.

The **STEP** method is based on an algorithm developed by Wang, Chukova and Lai [2004]. The quadratic programming problem is reduced to a least squares problem with equality constraints and non-negative variables, which can then be solved using a simple, stepwise algorithm. It is not possible to specify bounds on the solution vector, however the set of constraints

$$x \geq 0$$

is automatically imposed and does not need to be indicated explicitly. The **STEP** method can be used with problems where M ≤ N, and is most suited to problems where the number of constraints is fairly small.

SHAZAM expects constraints $Ax \leq b$ to be in the form of less than or equal to (≤) inequalities. Inequalities of the form $Ax \geq b$ must be rewritten with the signs of the elements in the A matrix and b vector reversed as shown in the first example below. If there are *meq* equality constraints, they must be contained in the first *meq* rows of A and b. The option **MEQ=***meq* must be specified, with **METHOD=POWELL**. Alternatively, the equality can be specified as two inequality constraints using both ≤ and ≥ notation, enabling **METHOD=STEP** to be used if desired. If the objective function is input as a minimization rather than a maximization problem:

Minimize $c'x + x'Qx$

subject to: $Ax \leq b$

the **MIN** option should be specified. If the problem is unconstrained, this must be declared by using the option **UNCONSTR** .

The matrix Q must be positive definite in a minimization problem, indicating that the objective function is convex and that the problem has a global minimum. In maximization problems, -Q must be positive definite. In both methods, the Cholesky decomposition of Q is found. If the quadratic programming problem has a non-positive definite matrix, **METHOD=POWELL** must be used (which is the default) and option **NEGDEF** specified. In this case, the algorithm forces Q (or $-Q$) to be positive definite by adding a multiple of the identity matrix. Hence the objective function becomes $c'x + x'(Q+\delta I)x$, where I is the identity matrix and δ is the multiple added. By default, δ is no larger than a factor of 1.1 times the smallest number required to make Q (or $-Q$) positive definite. This factor can be changed by using the **DIAGR=** option. The value of δ is stored in the temporary variable *$T3*. This option should be used with extreme care, as the addition of multiples of the identity matrix to matrix Q alters the objective function, and therefore quadratic programming problem, being optimized.

Where possible, the **QP** command in SHAZAM finds the dual solution to the given problem. The dual of the primal constrained quadratic programming problem

Maximize $c'x + x'Qx$

subject to: $Ax \le b$

where x, C, Q, A and b are as described above, is defined by

Minimize $-\frac{1}{4}\lambda^T(AQ^{-1}A^T)\lambda + \lambda^T(b + \frac{1}{2}AQ^{-1}c) - \frac{1}{4}c^TQ^{-1}c$

subject to: $\lambda \ge 0$

where λ is a M x 1 vector of the variables in the dual problem. For more information about the dual problem in quadratic programming see, for example, Nocedal & Wright [1999, pp. 484-485]. A warning will be thrown and the dual will not be calculated if the primal problem has any equality constraints (**MEQ**>0), or explicit bounds (using options **LOWER=**, **UPPER=**, **LOSCAL=** or **UPSCAL=**), or if any of the options **UNCONSTR**, **CHOLSPEC**, **NEGDEF** or **NOPDUAL** are specified. If the matrix $-AQ^{-1}A^T$ in the dual objective function is not positive definite, the dual solution will not be found.

When the **SET NOOUTPUT** command is in effect, all output from the **QP** command is suppressed, except for NOTE, WARNING and ERROR messages. To stop these messages being printed to the screen as well, use the **SET NOWARN** command.

QP *COMMAND OPTIONS*

In general, the format of the **QP** command is:

QP *c Q A b / options*

where *c*, Q, *A* and *b* are described above. For unconstrained problems, the command is:

QP *c Q* / **UNCONSTR** *options*

The available options are:

CHOLSPEC Used if the user wishes to supply the Cholesky decomposition of the objective matrix *Q*. In this case, the Cholesky decomposition must be contained in the lower half of the input matrix *Q*. This option is only available when both **MIN** and **METHOD=POWELL** are specified, and may not be used with the **NEGDEF** option.

DUMP Prints information for SHAZAM consultants.

MIN Indicates that the problem is a **MIN**imization problem.

NEGDEF Indicates that the quadratic matrix in the objective function, Q for a minimization problem, or –Q for a maximization problem, is not positive definite. This option is only available with **METHOD=POWELL**. The user must take extreme care when using this option, as explained above.

PDUAL Calculates and prints the dual solution. This is the default. Specify **NOPDUAL** to suppress consideration of the dual problem.

UNCONSTR Must be used if a problem is unconstrained.

CONV= As defined for the **NL** command. Used when solving the least squares sub-problems when **METHOD=STEP**. The default is **CONV=10^{-6}.**

DIAGR= If **NEGDEF** and **METHOD=POWELL** are specified, the multiple of the identity matrix added to the quadratic matrix, Q for minimization, or -Q for maximization problems, will be at most the value of this option times the smallest multiple that makes the matrix positive definite. The default is **DIAGR=1.1** and any number specified must be greater than 1.

DUAL= Saves the dual solution in the vector specified.

IFACT= Saves the indices of the final active constraints, applicable when using **METHOD=POWELL**.

ITER= Specifies the maximum number of iterations. The default number is **ITER=40(N+M)**.

LAGRANGE= Saves the Lagrange multipliers of the final active constraints, applicable when using **METHOD=POWELL**.

LOWER= Specifies the lower bounds of the solution and should be a vector of length N. **METHOD=STEP** may not be used with this option. If **METHOD=POWELL** and no lower bounds are specified using either **LOWER=** or **LOWSCAL=,** the default **LOWSCAL=-10^{12}** is applied.

LOWSCAL= Specifies a scalar which is applied as the lower bound on all variables. **METHOD=STEP** allows only **LOWSCAL=0**. If **METHOD=POWELL** and no lower bounds are specified using either **LOWER=** or **LOWSCAL=,** the default **LOWSCAL=-10^{12}** is applied. This option will be ignored if **LOWER=** is also specified.

MEQ= Indicates the number of constraints that are equality constraints. These constraints should be contained in the top rows of input matrix *A* and vector *b*. **METHOD=POWELL** is automatically used with this option.

METHOD= Specifies the method to be used for the solution of the quadratic programming problem. The default is **METHOD=POWELL**, as described by Powell [1983]. The alternative is **METHOD=STEP**, which is an implementation of the algorithm presented in Wang, Chukova and Lai [2005]. Both methods should yield nearly identical solutions, although some problems will only be solvable with **METHOD=POWELL**.

PRIMAL= Saves the primal solution in the vector specified.

UPPER= Specifies the upper bounds of the solution and should be a vector of length N. **METHOD=POWELL** is automatically used with this option. If **METHOD=POWELL** and no upper bounds are specified using either **UPPER=** or **UPSCAL=,** the default **UPSCAL=10^{12}** is applied.

UPSCAL= Specifies a scalar which is applied as the upper bound on all variables. **METHOD=POWELL** is automatically used with this option. If **METHOD=POWELL** and no upper bounds are specified using either **UPPER=** or **UPSCAL=,** the default **UPSCAL=10^{12}** is applied. This option will be ignored if **UPPER=** is also specified.

ZEROTOL = Used in both methods to set the threshold below which numbers are assumed to be zero. The default is **ZEROTOL=10^{-12}**.

Temporary variables available following the **QP** command are:

$ERR	Error code
$VAL	The value of the objective function
$T1	The number of variables in the problem, N.
$T2	The number of constraints in the problem, M.
$T3	The multiple of the identity matrix added to the objective function matrix to force it to be positive definite. It is only applicable when **METHOD=POWELL** and the **NEGDEF** option is specified.

EXAMPLES

Profit maximization problem

A firm makes two types of perfume, x and y. The managers want to know how much of each to produce in order to maximize profit, while adhering to some constraints imposed by their customers and suppliers. The quantities of perfume are measured in litres, so x and y can take any real value. The problem can be formulated as a quadratic programming problem, where the objective function represents profit:

Maximize $-3x^2 - 9y^2 + 4xy + 8y$

subject to: $3x + y \geq 4$

$2x + 5y \leq 10$

$x, y \geq 0$

The SHAZAM commands to input the objective function and constraints and solve the constrained quadratic maximization problem are:

```
read c / rows=2 cols=1
 0
 8
read q / rows=2 cols=2
 -3  2
  2 -9
read a / rows=2 cols=2
 -3 -1
  2  5
read b / rows=2 cols=1
-4
10
qp c q a b / method=step
```

The SHAZAM output is:

```
|_READ C / ROWS=2 COLS=1□
   1 VARIABLES AND        2 OBSERVATIONS STARTING AT OBS       1□ □ □
...SAMPLE RANGE IS NOW SET TO:         1         2□
|_READ Q / ROWS=2 COLS=2□
   2 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1□
|_READ A / ROWS=2 COLS=2□
      2 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1□
|_READ B / ROWS=2 COLS=1□
   1 VARIABLES AND        2 OBSERVATIONS STARTING AT OBS       1□ □

|_QP C Q A B / METHOD=STEP□
...NOTE..MAXIMIZATION  PROBLEM□
...NOTE..STEP METHOD SPECIFIED□
...NOTE..DEFAULT VALUE OF OPTION CONV IS BEING USED,   CONV = 0.1000000E-05□
...NOTE..DEFAULT VALUE OF OPTION ZEROTOL IS BEING USED, ZEROTOL = 0.1000000E-11□
...NOTE..CONSTRAINED PROBLEM□

□ NUMBER OF VARIABLES=    2 NUMBER OF CONSTRAINTS=    2□
□ MAXIMIZED VALUE OF OBJECTIVE FUNCTION IS 0.6666667□
□ PRIMAL SOLUTION□
     1.083333        0.7500000□
□ DUAL SOLUTION□
     1.166667         0.000000
```

Cost minimization problem

A distillery has a contract to supply the equivalent of ten casks of whiskey per week to a local pub. It can make two types of whiskey, x and y, and needs to minimize the costs of fulfilling the order. There are some constraints in the contract on the quantity of each type that can be supplied each week. The quadratic programming problem to minimize the cost function is as follows:

Minimize $3x^2 + 9y^2 - 4xy - 8y$

subject to: $x + y = 10$

$x - 2y \leq 0$

$x \leq 6$

$x,y \geq 1$

This problem has two variables (N=2), one equality (**MEQ**=1), and two inequality constraints (M=3). There are also bounds on the solution for x and y. The **POWELL** method must therefore be used, which is the default, but can also be set by using the option **METHOD=POWELL**. The SHAZAM commands to input the objective function, constraints and bounds, and solve the maximization problem are:

```
read c / rows=2 cols=1
  0
 -8
read q / rows=2 cols=2
  3  -2
 -2   9
read a / rows=3 cols=2
 1  1
 1 -2
 1  0
read b / rows=3 cols=1
10
 0
 6
qp c q a b / min method=powell lowscal=1 meq=1
```

The SHAZAM output is:

```
|_READ C2 / ROWS=2 COLS=1□
   1 VARIABLES AND        2 OBSERVATIONS STARTING AT OBS       1□ □
 |_READ Q2 / ROWS=2 COLS=2□
      2 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1□
 |_READ A2 / ROWS=3 COLS=2□
      3 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1□
 |_READ B2 / ROWS=3 COLS=1□
   1 VARIABLES AND        3 OBSERVATIONS STARTING AT OBS       1□ □
 |_QP C2 Q2 A2 B2 / METHOD=POWELL MIN LOWSCAL=1 MEQ=1□

...NOTE..MINIMIZATION  PROBLEM□
...NOTE..POWELL METHOD SPECIFIED□
...WARNING..NO UPPER BOUNDS HAVE BEEN SPECIFIED. USING DEFAULT VALUE FOR ALL
VARIABLES = 0.1000000E+13□
...NOTE..DEFAULT VALUE OF OPTION CONV IS BEING USED,   CONV = 0.1000000E-05□
...NOTE..DEFAULT VALUE OF OPTION ZEROTOL IS BEING USED, ZEROTOL = 0.1000000E-11□
...NOTE..CONSTRAINED PROBLEM□ □

 NUMBER OF VARIABLES=    2□
 TOTAL NUMBER OF CONSTRAINTS=    3, NUMBER OF EQUALITY CONSTRAINTS=    1□ □
 MINIMIZED VALUE OF OBJECTIVE FUNCTION IS  124.0000□ □
 PRIMAL SOLUTION□
      6.000000         4.000000□ □
 THERE ARE     2 FINAL ACTIVE CONSTRAINTS.□ □
 INDICES OF THE FINAL ACTIVE CONSTRAINTS□
      1.000000         3.000000□ □
 LAGRANGE MULTIPLIERS OF THE FINAL ACTIVE CONSTRAINTS□
     -40.00000     20.00000□

 ...WARNING..THE DUAL SOLUTION WILL NOT BE CALCULATED IF THERE ARE EQUALITY
CONSTRAINTS IN THE PRIMAL PROBLEM.□
```

34. MATRIX MANIPULATION

"I am tired of all this thing called science...We have spent millions in that sort of thing for the last few years, and it is time it should be stopped."

Simon Cameron
U.S. Senator from Pennsylvania, 1861

The **MATRIX** and **COPY** commands can be used to create and manipulate matrices in SHAZAM. The **MATRIX** command will do matrix operations and create and transform matrices instead of vectors. It is similar to the **GENR** command except matrices are used. In contrast to the **GENR** command the **MATRIX** command ignores the current **SAMPLE** command. Also, **SKIPIF** commands have no effect on the **MATRIX** command.

THE MATRIX COMMAND

In general, the format of the **MATRIX** command is:

MATRIX *mat = equation*

where *mat* is the name of the matrix to be generated from an *equation*.

Operators valid on the **MATRIX** command are:

–	Negation
*	Matrix Multiplication
+	Addition
–	Subtraction
'	Transpose
/	Hadamard Division
@	Kronecker Multiplication
|	Concatenation

Regular matrix rules apply on the **MATRIX** command. So, when multiplying with * the first matrix to be multiplied must have the same number of columns as the second matrix has rows. Addition (+), and subtraction (–), of matrices are done element by element, so

only matrices with the same dimensions may be added to and subtracted from one another. Any matrix can be multiplied by a constant. Concatenation puts two matrices together side by side so both must have the same number of rows. Kronecker multiplication can be done with matrices of any dimension. The following examples illustrate concatenation and stacking matrices. The final example shows that the Kronecker product of a 3 x 3 matrix *B* and a 2 x 2 matrix *A* results in a 6 x 6 matrix *K*.

```
|_READ A / ROWS=3 COLS=3 LIST
      3 ROWS AND           3 COLUMNS, BEGINNING AT ROW       1
 ...SAMPLE RANGE IS NOW SET TO:       1         3
    A
    3 BY     3 MATRIX
   1.000000       19.00000      -2.000000
   354.0000      0.0             28.00000
  -3.000000       15.00000       7.000000
|_READ B / ROWS=2 COLS=2 LIST
      2 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1
    B
    2 BY     2 MATRIX
   78.00000       592.0000
   4.000000      -65.00000
|_READ C / ROWS=3 COLS=2 LIST
      3 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1
    C
    3 BY     2 MATRIX
   16.00000       44.00000
  -3.000000       13.00000
  0.0             23.00000
|_* CONCATENATE A AND C SIDE BY SIDE
|_MATRIX AC=A|C
|_PRINT AC
    AC
    3 BY     5 MATRIX
   1.000000   19.00000    -2.000000     16.00000     44.00000
   354.0000  0.0           28.00000    -3.000000     13.00000
  -3.000000   15.00000     7.000000    0.0           23.00000
|_* STACK B AND C WITH B ON TOP
|_MATRIX BC=(B'|C')'
|_PRINT BC
    BC
    5 BY     2 MATRIX
   78.00000       592.0000
   4.000000      -65.00000
   16.00000       44.00000
  -3.000000       13.00000
  0.0             23.00000
|_* GET KRONECKER PRODUCT OF A AND B
|_MATRIX K=A@B
|_FORMAT(6F10.0)
|_PRINT K / FORMAT
    K
    6 BY     6 MATRIX
      78.      592.     1482.    11248.     -156.    -1184.
       4.      -65.       76.    -1235.       -8.      130.
   27612.   209568.        0.        0.     2184.    16576.
    1416.   -23010.        0.        0.      112.    -1820.
    -234.    -1776.     1170.     8880.      546.     4144.
```

```
      -12.       195.        60.      -975.        28.      -455.
```

Functions available with the **MATRIX** command are:

CHOL(*matrix*)	Cholesky's decomposition of *matrix* is performed. The *matrix* must be symmetric positive definite. For a symmetric positive definite matrix *A*, **CHOL(A)** returns a lower triangular matrix *L* such that *LL'=A*.
DET(*matrix*)	Determinant of *matrix*.
DIAG(*matrix*)	If *matrix* is N x N the DIAG function will create an N x 1 vector that consists of the diagonal elements of *matrix*. If *matrix* is an N x 1 vector the DIAG function will create an N x N matrix with zeros off the diagonal and the vector along the diagonal.
EIGVAL(*matrix*)	The eigenvalues of *matrix* are computed and sorted in descending order (that is, the largest eigenvalue is first).
EIGVEC(*matrix*)	The eigenvectors of *matrix* are computed. The eigenvectors correspond to the eigenvalues obtained with the EIGVAL function and are normalized so that, for a vector x, $x'x = 1$.
EXP(*matrix*)	The exponential operator is applied to each element of *matrix*.
FACT(*matrix*)	If the *matrix A* is symmetric positive definite, then **FACT(A)** factors the inverse of the matrix and returns a lower triangular matrix *P* such that $P'P=A^{-1}$.
IDEN(*ndim*)	An identity matrix with *ndim* rows and columns is created.
IDEN(*ndim,ndiag*)	A matrix with *ndim* rows and columns is created with a diagonal of ones on the *ndiag* lower diagonal (*ndiag*=1 gives an identity matrix).
INT(*matrix*)	Integer truncation of each element of *matrix* is performed.
INV(*matrix*)	Inverse of *matrix*.
LAG(*matrix,n*)	Each column of *matrix* is lagged *n* times.
LOG(*matrix*)	The natural log of each element of *matrix* is taken.
NCDF(*matrix*)	Normal cumulative distribution function. The probability of each element of *matrix* is taken.
NOR(*nrow,ncol*)	Generates a matrix of random numbers from a standard normal distribution. The number of rows and columns is specified by *nrow,ncol*.

RANK(*matrix*)	The rank, or the number of independent rows, of *matrix* is calculated. Calculation of the rank of a matrix is often numerically difficult especially in the case of a near singular matrix, therefore the value returned by the RANK function should be used with caution.
SAMP(*matrix,nrows*)	A new matrix with *nrows* is created from the old *matrix* by random sampling with replacement.
SEAS(*nob,nseas*)	A series of seasonal dummy variables is created. The number of observations and the number of seasons are specified by *nob* and *nseas*.
SEAS(*nob,–ncross*)	A series of cross-section dummy variables is created. The number of observations and the number of cross-sections are specified by *nob* and *ncross*. It is assumed that each cross-section has the same number of observations. The dummy variables created by this function can be used with panel data sets that are arranged in the form used by the **POOL** command.
SIN(*matrix*)	The sine of each element of *matrix* is computed.
SQRT(*matrix*)	The square root of each element of *matrix* is computed.
SVD(*matrix*)	The Singular Value Decomposition of *matrix* is performed. The singular values are returned.
SYM(*matrix*)	Creates a symmetric matrix using the lower triangle from a square full *matrix*.
TIME(*nob,x*)	A vector with *nob* observations is created with values equal to a time index plus *x*.
TRACE(*matrix*)	The trace, or the sum of the diagonal elements, of *matrix* is calculated.
TRI(*matrix*)	Creates a lower triangular matrix from a square full *matrix*.
UNI(*nrow,ncol*)	A matrix of random numbers between 0 and 1 is created. The number of rows and columns is specified by *nrow,ncol*.
VEC(*matrix*)	Stacks columns of *matrix* into a long vector.
VEC(*vector,nrows*)	Unstacks *vector* into a matrix with *nrows*.

Following a **MATRIX** command the dimension of the matrix is saved in the temporary variables *$ROWS* and *$COLS*.

As a general rule, undefined values are set to a missing value code. For example, operations like LOG or SQRT of a negative number will generate undefined values. The missing value code has a default of –99999 but can be changed with the **SET MISSVALU=** option. The temporary variable *$MISS* stores the missing value code. The LAG function sets zero values for initial observations.

When the **SET SKIPMISS** command is in effect the **MATRIX** command will assign a missing value code to results that involve a computation with a missing observation. For more details see the chapter *SET AND DISPLAY*.

Although it is not a common matrix operation, SHAZAM can do multiplication and division element-by-element. These are often called Hadamard product and division (see Rao [1973, p. 30]). For example, Hadamard division is done with the command:

```
matrix m=a/b
```

In the above example each element in the matrix *A* is divided by each element of the matrix *B* and the results of this operation are put in the matrix *M*. Of course, *A* and *B* must have the same dimensions unless either *A* or *B* is a constant.

It is also possible to perform Hadamard products or element-by-element multiplication. This is done with the command:

```
matrix m=a/(1/b)
```

Again, *A* and *B* must have the same dimensions.

If multiplying an N x 1 vector by a matrix with N rows and any number of columns, SHAZAM will do element-by-element multiplication of the N x 1 vector times each column of the matrix so that the result will have the same number of columns as the original matrix.

The example below shows how to extract pieces of a matrix.

```
|_SAMPLE 1 5
|_READ X / ROWS=5 COLS=3 LIST
                  5 ROWS AND          3 COLUMNS, BEGINNING AT ROW       1
            X
            5 BY     3 MATRIX
      1.000000    3.000000    5.000000
      1.000000    1.000000    4.000000
      1.000000    5.000000    6.000000
      1.000000    2.000000    4.000000
```

```
      1.000000    4.000000    6.000000

|_* SHOW HOW TO PICK OUT ELEMENTS, AND ROWS, OR COLUMNS OF A MATRIX
|_* CHANGE ROW 3 COLUMN 2 TO A 7
|_MATRIX X(3,2)=7
|_* GET THE SECOND COLUMN OF X
|_MATRIX XTWO=X(0,2)
|_PRINT XTWO / NOBYVAR
                  XTWO
      3.000000
      1.000000
      7.000000
      2.000000
      4.000000
|_* GET THE FOURTH ROW OF X
|_MATRIX XFOUR=X(4,0)
|_PRINT XFOUR
            XFOUR
            1 BY     3 MATRIX
      1.000000    2.000000    4.000000
|_* GET ELEMENT (4,2) OF X
|_MATRIX X42=X(4,2)
|_PRINT X42
            X42
      2.000000
```

Note that when referring to an element of a matrix there must be no embedded blanks between the variable name and the left bracket. As well, for left-hand side variables there must be no embedded blanks between the brackets.

The next example shows how to extract the upper block of a symmetric matrix.

```
* Run OLS and save the estimated covariance matrix in the symmetric matrix BVAR.
ols y x1 x2 x3 x4 x5 / cov=bvar
print bvar

* Extract the 3 x 3 upper block
matrix bup=bvar(1;3,1;3)

* Print the result
print bup
```

The next example shows the use of the **MATRIX** command to calculate the vector of OLS parameter estimates in one step. The example uses the same *X* matrix that was used above. The matrix calculation uses the ubiquitous formula $\hat{\beta} = (X'X)^{-1}X'Y$. When the transpose operator (') is used it is not necessary to use the multiplication operator (*) if multiplication between the transposed matrix and the following matrix is desired. However, without the transpose operator the multiplication operator is required. The example also shows the calculation of an F-test using matrices. This formula is given in the chapter *HYPOTHESIS TESTING AND CONFIDENCE INTERVALS*.

```
|_SAMPLE 1 5
|_READ Y / BYVAR
   1 VARIABLES AND        5 OBSERVATIONS STARTING AT OBS       1

|_READ X / ROWS=5 COLS=3
     5 ROWS AND           3 COLUMNS, BEGINNING AT ROW       1
|_MATRIX B=INV(X'X)*X'Y
|_PRINT B
    B
   4.000000       2.500000      -1.500000

|_MATRIX EE=Y'Y-(B'(X'Y))
|_PRINT EE
    EE
   1.500000
|_READ R / ROWS=1 COLS=3 LIST
      1 ROWS AND           3 COLUMNS, BEGINNING AT ROW       1
    R
    1 BY     3 MATRIX
   .0000000       1.000000       1.000000
|_READ LR / ROWS=1 COLS=1 LIST
   1 VARIABLES AND        1 OBSERVATIONS STARTING AT OBS       1
      LR
   .0000000
|_MATRIX F=((R*B-LR)'(INV(R*(INV(X'X)*R')))*(R*B-LR))/(EE/2)
|_PRINT F
    F
   2.666667
```

It is apparent from the above SHAZAM output that many calculations can be done on one **MATRIX** command. See the chapter *PROGRAMMING IN SHAZAM* for more examples of the **MATRIX** command.

THE COPY COMMAND

The **COPY** command is used to copy vectors or matrices into other matrices. It is also possible to partition matrices, delete rows and columns, and create matrices from vectors by using the **COPY** command. The format of the **COPY** command is:

COPY *fromvar(s) tovar / options*

where *fromvar(s)* is either a list of vectors or a single matrix, *tovar* is the variable into which the *fromvar(s)* are to be copied and *options* is a list of desired options. The available options on the **COPY** command are:

FROW=*beg;end* Specifies the rows of the *fromvar(s)* that are to be copied into the *tovar* . If this option is not specified the current **SAMPLE** command will be used.

FCOL=*beg;end* Specifies the columns of the *fromvar* that are to be copied into the new variable. If the old variables are a list of vectors the **FCOL=** option need not be used as SHAZAM will automatically treat each vector as a column. Therefore, this option is only used if the old variable is a matrix.

TROW=*beg;end* Specifies the rows of the *tovar* into which the *fromvar(s)* are to be copied. If this option is not specified the current **SAMPLE** command will be used.

TCOL=*beg;end* Specifies the columns of the *tovar* into which the old variables are to be copied.

If no options are specified on the **COPY** command all the *fromvar(s)* will be copied into the *tovar*. It is impossible to copy from vectors and a matrix simultaneously. It is very important to specify consistent options on the **COPY** command, that is, the dimensions specified with the **COPY** options must be compatible with the dimensions of the *fromvar(s)* and *tovar*. It is also important to remember to use a semicolon (;) only to separate *beg* and *end* and no more than 8 characters are allowed.

If **SKIPIF** commands or the expanded form of the **SAMPLE** have been used the *tovar* matrix will be reduced in size. This is useful in deleting rows of a matrix to create a new matrix. For example, if *A* is a 15 x 5 matrix, the following commands will create a new matrix *B* which is 4 x 12 because rows 4, 10, 11 and column 3 of *A* have been deleted:

```
|_PRINT A
    A
   15 BY     5 MATRIX
   40.05292        1170.600        97.80000        2.528130        191.5000
   54.64859        2015.800        104.4000        24.91888        516.0000
   40.31206        2803.300        118.0000        29.34270        729.0000
   84.21099        2039.700        156.2000        27.61823        560.4000
   127.5724        2256.200        172.6000        60.35945        519.9000
   124.8797        2132.200        186.6000        50.61588        628.5000
   96.55514        1834.100        220.9000        30.70955        537.1000
   131.1601        1588.000        287.8000        60.69605        561.2000
   77.02764        1749.400        319.9000        30.00972        617.2000
   46.96689        1687.200        321.3000        42.50750        626.7000
   100.6597        2007.700        319.6000        58.61146        737.2000
   115.7467        2208.300        346.0000        46.96287        760.5000
   114.5826        1656.700        456.4000        57.87651        581.4000
   119.8762        1604.400        543.4000        43.22093        662.3000
   105.5699        1431.800        618.3000        22.87143        583.8000
|_SAMPLE 1 3 5 9 12 15
|_COPY A:1 A:2 A:4 A:5 B
...NOTE..SOME OBSERVATIONS MAY BE SKIPPED
|_PRINT B
    B
   12 BY     4 MATRIX
```

40.05292	1170.600	2.528130	191.5000
54.64859	2015.800	24.91888	516.0000
40.31206	2803.300	29.34270	729.0000
127.5724	2256.200	60.35945	519.9000
124.8797	2132.200	50.61588	628.5000
96.55514	1834.100	30.70955	537.1000
131.1601	1588.000	60.69605	561.2000
77.02764	1749.400	30.00972	617.2000
115.7467	2208.300	46.96287	760.5000
114.5826	1656.700	57.87651	581.4000
119.8762	1604.400	43.22093	662.3000
105.5699	1431.800	22.87143	583.8000

35. PRICE INDEXES

"When the U.S. government stops wasting our resources by trying to maintain the price of gold, its price will sink to...$6 an ounce rather than the current $35 an ounce."

Henry Reuss
U.S. Senator from Wisconsin, 1967

The **INDEX** command computes price indexes from a set of price and quantity data on a number of commodities. To calculate an index, it is necessary to have prices and quantities for at least two commodities. Let p_{it} and q_{it} be the price and quantity for variable i in period t for i = 1, 2, . . . , K and t = 1, 2, . . . , N. The period t price and quantity vectors that are to be aggregated into scalars are given by:

$$q_t' = (q_{1t}, q_{2t}, \cdots, q_{Kt}) \qquad \text{and} \qquad p_t' = (p_{1t}, p_{2t}, \cdots, p_{Kt})$$

Weighted Aggregate Price Indexes

Let p_{i0} and q_{i0} be the price and quantity for variable i in the base period. Alternative price index calculations are:

Laspeyres index	$L_t = (p_t' q_0)/(p_0' q_0)$
Paasche index	$P_t = (p_t' q_t)/(p_0' q_t)$
Fisher index	$F_t = \sqrt{L_t P_t}$

Chained Price Indexes

Chained price indexes are computed when the **CHAIN** option is specified on the **INDEX** command. The level of prices in period t relative to period t–1, for t = 2, 3, . . . , N, for the alternative price index formulas are:

Laspeyres index	$L_t = (p_t' q_{t-1})/(p_{t-1}' q_{t-1})$
Paasche index	$P_t = (p_t' q_t)/(p_{t-1}' q_t)$

Fisher index $F_t = \sqrt{L_t P_t}$

Discrete approximation to the Divisia (Törnqvist or Translog)

$$D_t = \exp\left[.5 \sum_{i=1}^{K} (s_{it} + s_{i,t-1}) \log(p_{it} / p_{i,t-1}) \right]$$

where $s_{it} = (p_{it} q_{it}) / (p'_t q_t)$

These are chain links that are used in constructing the final price index series. For example, for a base period at observation 1, the Laspeyres price index series computed with the **CHAIN** option is:

$$1, \quad L_2, \quad L_2 L_3, \ldots, \prod_{t=2}^{N} L_t .$$

For the Paasche, Fisher or Divisia price indexes, the L_t are replaced by P_t, F_t or D_t respectively. If this new price series is denoted by R, then the corresponding quantity series, Q, is computed as:

$$Q_t = (p'_t q_t) / R_t$$

These index number formulas are explained more fully in Diewert [1978].

INDEX *COMMAND OPTIONS*

In general, the format of the **INDEX** command is:

INDEX *p1 q1 p2 q2 p3 q3 . . . / options*

where the *p* and *q* are names of the variables for the prices and quantities of the commodities.

The **OLS** options available are: **BEG=** and **END=**. Additional options available on the **INDEX** command are:

CHAIN **CHAIN**s the Laspeyres, Paasche or Fisher indexes using the method described above. The Divisia index is always chained.

EXPEND Indicates that the quantity variables measure **EXPEND**itures rather than quantities. SHAZAM will first divide each **EXPEND**iture variable by its respective price to get the quantities.

NOALTERN Normally, variables are listed as described above, with p and q alternating in the list. Sometimes, it is more convenient to list all the prices followed by all the quantities. This would be the case if the prices were in one matrix and the quantities in another matrix. In this case, **NOALTERN** should be specified. The default is **ALTERN**.

NOLIST With this option the price and quantity indexes are not printed.

BASE= Specifies the observation number to be used as the **BASE** period for the index. The value of the index in the base period will be 1.0. If the **BASE=** option is not specified SHAZAM will use the first available observation as the base period. Use of **SKIPIF** commands is not recommended when computing Divisia indexes since the index needs to be chained.

DIVISIA=
PAASCHE=
LASPEYRES=
FISHER=
QDIVISIA=
QPAASCHE=
QLASPEYRES=
QFISHER=

These options specify the variable where the index will be stored for this SHAZAM run. Note that the options beginning with the letter **Q**, (**QDIVISIA=**, **QPAASCHE=**, etc.) tell SHAZAM to store the *quantity* in the variable specified. Those options not preceded by **Q** tell SHAZAM to store the *price index* in the variable specified. The quantity is computed residually in this option, i.e. according to the method described above. To compute quantity indexes and prices residually, interchange the role of prices and quantities in the **INDEX** command.

EXAMPLES

An example of the **INDEX** command is:

```
index pfood food pclothes clothes phouse house / expend base=23
```

In this example the quantity data is in expenditure form and observation 23 is to be used as the base year.

A problem arises if there is a zero price or quantity in any year. SHAZAM handles this problem by ignoring any commodity whenever a zero price or quantity would have to be used in a calculation. The remaining quantities are then assumed to exhaust the set for that year. A good reference for treatment of zero price and quantities is Diewert [1980].

Some users may wish to compute quantity indexes directly. To do this, reverse the p and q variables on the **INDEX** command.

Using data for price and quantity of cars for various corporations found in Newbold [1984, Chapter 16] the SHAZAM output obtained from the **INDEX** command is:

```
|_INDEX P1 Q1 P2 Q2 P3 Q3 P4 Q4 / QDIVISIA=QD PAASCHE=PA

BASE PERIOD IS OBSERVATION     1
PAASCHE  WILL BE STORED AS VARIABLE: PA
QDIVISIA WILL BE STORED AS VARIABLE: QD
                    PRICE INDEX                    QUANTITY
  DIVISIA PAASCHE LASPEYRES FISHER  DIVISIA  PAASCHE  LASPEYRES  FISHER
  1  1.000  1.000  1.000  1.000     1509.    1509.    1509.    1509.
  2  0.994  0.998  0.989  0.994     816.0    812.8    819.8    816.3
  3  0.979  0.977  0.980  0.978     733.3    735.4    732.9    734.1
  4  0.999  0.997  0.999  0.998     747.9    749.4    748.1    748.7
  5  1.045  1.044  1.045  1.044     1133.    1135.    1134.    1134.
  6  1.048  1.044  1.043  1.044     930.6    933.4    934.3    933.9
  7  1.029  1.025  1.025  1.025     637.5    639.9    639.6    639.8
  8  1.075  1.077  1.069  1.073     1213.    1211.    1219.    1215.
  9  1.110  1.106  1.106  1.106     923.6    926.6    926.9    926.8
 10  1.109  1.104  1.101  1.102     1210.    1215.    1218.    1217.
 11  1.198  1.195  1.188  1.191     1540.    1544.    1553.    1548.
 12  1.153  1.148  1.148  1.148     1178.    1184.    1184.    1184.
|_PRINT QD PA
      QD             PA
   1509.375       1.000000
   816.0446      0.9979699
   733.3490      0.9765596
   747.8928      0.9969307
   1133.374       1.043727
   930.5681       1.044271
   637.4855       1.024789
   1212.867       1.076632
   923.5981       1.106354
   1210.000       1.103586
   1540.261       1.194918
   1178.282       1.147695
```

As the above example shows, all the price indexes and quantities are computed and printed. Note that the Divisia price index is computed as a chained price index but the other price indexes are computed using the weighted aggregate price index method. The Divisia quantity and the Paasche price index are saved. The **BASE=** option is not specified, so the base period is the first period.

36. PRINCIPAL COMPONENTS AND FACTOR ANALYSIS

"That's an amazing invention, but who would ever want to use one of them?"
Rutherford B. Hayes
U.S. President, 1876
(after seeing the telephone)

The **PC** command is available to extract principal components from a set of data and, as an option, do a varimax rotation for factor analysis. A good reference on principal components is Jolliffe [1986]. Output may include the eigenvalues, eigenvectors, components, factor matrix, and rotated factor matrix. It is possible to specify conditions under which factors are retained. Multicollinearity diagnostics including condition numbers, condition indexes, and variance proportions may also be printed. These diagnostics are discussed in Belsley, Kuh and Welsch [1980]; Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 22.3] and Judge, Hill, Griffiths, Lütkepohl and Lee [1988, pp. 872-4].

Consider a data set in the (N x K) matrix X. Transformed data is:

PC option	(N x K) matrix S
default	$S_{kt} = X_{kt} - \overline{X}_k$
COR	$S_{kt} = (X_{kt} - \overline{X}_k) \Big/ \sqrt{\sum_{t=1}^{N} (X_{kt} - \overline{X}_k)^2}$
RAW	$S_{kt} = X_{kt}$
SCALE	$S_{kt} = X_{kt} \Big/ \sqrt{\sum_{t=1}^{N} X_{kt}^2}$

For the matrix $C = S'S$, a_k is a (K x 1) eigenvector of C. The corresponding eigenvalue is λ_k. The eigenvalues are arranged in descending order. The (N x 1) vectors of principal components are obtained as:

$$Z_k = S \cdot a_k \qquad \text{for } k = 1, \ldots, K$$

The principal components can be normalized four different ways with the use of the **NC=** option on the **PC** command. There does not appear to be much agreement in other computer programs on how the components should be normalized. SHAZAM options are:

NC=	Method	$Z_k' Z_k =$
1	default method	λ_k
2	Theil's method	1
3	BMDP method	$\lambda_k (N - 1)$
4	standard normal	$N - 1$

The value: $\lambda_1 \Big/ \sum_{i=1}^{K} \lambda_i$

gives the proportionate contribution of the first principal component to the total variation in the variables. The cumulative percentage of eigenvalues is:

$$\sum_{i=1}^{k} \lambda_i \Big/ \sum_{i=1}^{K} \lambda_i \qquad \text{for } k = 1, \ldots, K$$

The variance reduction benchmark function (see Judge et al. [1985, p. 912]) is:

$$100 \sum_{i=k}^{K} \left(\frac{1}{\lambda_i} \right) \Bigg/ \sum_{i=1}^{K} \left(\frac{1}{\lambda_i} \right) \qquad \text{for } k = 1, \ldots, K$$

An inspection of the eigenvectors and eigenvalues can aid in the detection of multicollinearity. The condition numbers are λ_1 / λ_k and the condition indexes are $\sqrt{\lambda_1 / \lambda_k}$. Belsley, Kuh and Welsch [1980] suggest that condition indexes around 5 or 10 reveal weak dependencies and condition indexes around 30 to 100 demonstrate strong linear dependencies. The finding of several large condition indexes indicates more than one near exact dependency. The **PCOLLIN** option reports a table of variance-decomposition proportions. The values in the table are computed as:

$$\phi_{kj} = \frac{a_{jk}^2 \Big/ \lambda_k}{\sum_{i=1}^{K} a_{ki}^2 \Big/ \lambda_i} \qquad \text{for } k, j = 1, \ldots, K$$

The columns in the variance proportions table sum to one. Multicollinearity is indicated if a *row* that is associated with an eigenvalue that has a high condition index contains two or more values of ϕ_{kj} that are greater than the rule of thumb value of 0.50. This diagnostic procedure is discussed in Belsley et al. [1980, p. 112].

The model for factor analysis has the form: $X_k = \Lambda f_k + \varepsilon_k$ for $k = 1, \ldots, K$

where Λ is an (N x M) factor loading matrix and the f_k are (M x 1) vectors of factors. The ε_k is a zero mean error vector. As an estimate, the columns of Λ are constructed from the first M principal components as $z_j / \sqrt{\lambda_j}$ for j = 1, . . . , M. The factors f_k are then computed from OLS estimation and are listed with the **PFM** option on the **PC** command.

PC *COMMAND OPTIONS*

In general, the format of the **PC** command is:

PC *vars* / *options*

where *vars* is a list of the variables desired in the analysis, and *options* is a list of desired options. The available options are:

COR Specifies that the analysis is to be done on the **COR**relation matrix. If this option is not specified, the analysis is done on the deviations from the means cross-product matrix. Since the components are sensitive to transformations, it is important that the user be sure this option is needed. If all the variables are measured in the same units, it is probably better to use the cross-product matrix. This is often the case for economists. If the variables are all measured differently, it may be more appropriate to use the correlation matrix which, in effect, normalizes all variables. See also the **SCALE** option.

LIST **LIST**s the matrix of principal components. If there are many observations the list of all the components will be very long.

MAX Sets the **PEVEC**, **PFM** and **PRM** options.

PCOLLIN **P**rints the table of variance-decomposition proportions that can be used for the detection of multi**COLLIN**earity.

PEVEC **P**rints all the **E**igen**VEC**tors for the retained components. If there are many variables this option could yield a lot of costly output which may have little value.

PFM **P**rints the **F**actor **M**atrix for the retained factors.

PRM Computes and **P**rints the **R**otated factor **M**atrix by a varimax rotation. The method is described in Kaiser [1959]. The varimax rotation is one of the most common types of rotation.

RAW Specifies that the analysis is to be done on the **RAW** cross-product matrix. If this option is not specified, the analysis is done on the deviations from the means cross-product matrix. The same warnings specified for the **COR** option apply here.

SCALE Specifies that the analysis is to be done on a scaled cross-product matrix. The scaled matrix transforms a **RAW** cross-product matrix into one where the data vectors all have unit length so that the diagonals of the cross-product matrix are all equal to 1. Note that this is ***not*** the same as the **COR** option. If the **SCALE** option is not specified, the analysis is done on the deviations from the means cross-product matrix. The same warnings specified for the **COR** option apply here.

BEG=, END= Specifies the **BEG**inning and **END**ing observations to be used in the analysis. This option overrides the **SAMPLE** command and defaults to the sample range in effect.

EVAL= Saves the **E**igen**VAL**ues in the vector specified. The values are sorted in descending order (that is, the largest eigenvalue is first).

EVEC= Saves the **E**igen**VEC**tors in the (K x K) matrix specified. The first column of the matrix is the eigenvector that corresponds to the largest eigenvalue.

MAXFACT= Specifies the **MAX**imum number of **FACT**ors to be retained. If no value of **MINEIG** is specified **MAXFACT** will be the actual number retained. If **MAXFACT** is not specified, all will be included.

MINEIG= Specifies the **MIN**imum **EIG**envalue allowed to be retained. If the **COR** option is specified, the eigenvalues will range from 0 to the number of variables in the analysis. Sometimes a convenient value is **MINEIG=1**. This rule of thumb does not work if the analysis is done on the cross-product matrix. The default is **MINEIG=0**.

NC= Specifies the **N**ormalization **C**ode. The options are described above and the default is **NC=1**.

PCINFO= Saves a matrix of **INFO**rmation needed for regressions on **P**rincipal **C**omponents.

PCOMP= Saves the matrix of **P**rincipal **COMP**onents in the (NxK) matrix specified. The first column of the matrix is the principal component that corresponds to the largest eigenvalue.

PRINCIPAL COMPONENTS REGRESSION

A consequence of multicollinearity is that the OLS estimators may have large standard errors. A solution is to consider a restricted least squares estimator. One approach is to use principal components analysis to reduce the dimensionality of the data set. A set of principal components is generated and a sub-set is selected to include as regressors in an OLS regression. The estimators are then transformed to obtain estimators for the coefficients of the original model. The resulting estimator has an interpretation as a restricted least squares estimator and therefore has smaller sampling variance compared to the unrestricted OLS estimator. Discussion with further references is available in Judge, Griffiths, Hill, Lütkepohl, and Lee [1985, Chapter 22.5]; Mundlak [1981]; Jolliffe [1986, Chapter 8] and Maddala [1992, Chapter 7.6].

The general command format for principal components regression is:

PC *vars* / **PCOMP=***pc* **PCINFO=***info options*
OLS *depvar indeps* / **PCOMP=***pc* **PCINFO=***info options*

Note that the **PCOMP=** and the **PCINFO=** options are used in both the **PC** and the **OLS** commands. In the **PC** command, they are used to store the information for later use in the **OLS** command. On the **OLS** command *indeps* is a list of variables that must contain a sub-set of the principal components saved in the matrix *pc*.

The next example illustrates principal components regression using the data set described in Judge, Griffiths, Hill, Lütkepohl, and Lee [1985, p. 930]. First, the **PC** command is used to obtain principal components and some multicollinearity diagnostics. The output `CUMULATIVE PERCENTAGE OF EIGENVALUES` shows that the first two principal components account for 97.340% of the total variation in the variables. These two components are selected for the principal components regression implemented with the **OLS** command.

```
|_SAMPLE 1 20
|_READ (JUDGE22.DAT) X1 X2 X3 X4 X5 Y1
   6 VARIABLES AND       20 OBSERVATIONS STARTING AT OBS       1
```

```
|_PC X2 X3 X4 X5 / PCOMP=PC PCINFO=INFO PCOLLIN
 PRINCIPAL COMPONENTS ON    4 VARIABLES     MAXIMUM OF    4 FACTORS RETAINED

EIGENVALUES
  33.645       5.7812       .95354       .12366
SUM OF EIGENVALUES =   40.504

CUMULATIVE PERCENTAGE OF EIGENVALUES
  .83067       .97340       .99695       1.0000

VARIANCE REDUCTION BENCHMARK FUNCTION
  100.00       99.682       97.829       86.599

CONDITION NUMBERS
  1.0000       5.8197       35.285       272.08
CONDITION INDEXES
  1.0000       2.4124       5.9401       16.495

VARIANCE PROPORTIONS
      X2       X3       X4       X5
  1   .00149   .00059   .00250   .02602
  2   .00540   .11700   .00730   .05538
  3   .00349   .53393   .08059   .64291
  4   .98962   .34848   .90960   .27570

  4 COMPONENTS STORED IN MATRIX PC

|_OLS Y1 PC:1 PC:2 / PCINFO=INFO PCOMP=PC
 OLS ESTIMATION
      20 OBSERVATIONS     DEPENDENT VARIABLE = Y1
...NOTE..SAMPLE RANGE SET TO:    1,   20

 R-SQUARE =    .8913     R-SQUARE ADJUSTED =    .8786
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .91550
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .95682
SUM OF SQUARED ERRORS-SSE=   15.564
MEAN OF DEPENDENT VARIABLE =   20.107
LOG OF THE LIKELIHOOD FUNCTION = -25.8708

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      17 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
PC         1.8892      .1650       11.45      .000  .941      .9156      .0000
PC        -1.1455      .3979      -2.878      .010 -.572     -.2301      .0000
CONSTANT   20.107      .2140       93.98      .000  .999      .0000     1.0000

ORIGINAL COEFFICIENTS TRANSFORMED BACK FROM COMPONENTS

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      17 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
X2          .48574      .1745       2.783      .013  .559      .1226      .0375
X3         1.0811       .2936       3.682      .002  .666      .1820      .0680
X4          .55642      .1686       3.300      .004  .625      .1472      .0497
X5         1.7796       .2011       8.849      .000  .906      .6186      .2994
CONSTANT   10.968       .8434       13.00      .000  .953      .0000      .5455
```

Note that the **OLS** command uses a special format for the variable names. The principal components are saved in the matrix variable *PC*. The first and second columns of this

matrix (corresponding to the first and second largest eigenvalues) are identified by *PC:1* and *PC:2* respectively.

37. PROBABILITY DISTRIBUTIONS

"A severe depression like that of 1920-21 is outside the range of probability."
Harvard Economic Society
November 16, 1929

The **DISTRIB** command provides functions of probability distributions. For a continuous random variable X the cumulative distribution function (CDF) is defined for a value x as:

$$F(x) = Pr(X \le x)$$

and the probability density function (PDF) satisfies

$$f(x) = \frac{dF(x)}{dx} \ge 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(x)\,dx = 1$$

The **DISTRIB** command can provide critical values for use if adequate statistical tables are unavailable. Since approximation formulas are often used to compute probabilities users may find that the numbers printed may not exactly match those found in statistical tables which are usually computed with far greater precision. However, the approximation formulas are usually accurate to at least two significant digits. In some cases either the PDF or CDF is difficult to compute and so is not calculated. A handbook for statistical distributions is Evans, Hastings and Peacock [1993].

DISTRIB *COMMAND OPTIONS*

In general, the format of the **DISTRIB** command is:

DISTRIB *vars* / *options*

where *vars* is a list of variables and *options* is a list of the options that are required on the specified type of distribution. The available options are:

INVERSE Computes the inverse survival function. The data in *vars* must contain probabilities. With a probability α the inverse survival function Z(α) is such that $Pr[X > Z(\alpha)] = \alpha$. The relation to the inverse cumulative distribution function is $Z(\alpha) = F^{-1}(1-\alpha)$. The **INVERSE** option is useful

for obtaining critical values. For example, when the **TYPE=CHI** option is also specified the values tabulated in chi-square tables will be obtained. The **INVERSE** option is available with **TYPE=BETA**, **CAUCHY**, **CHI**, **ERLANG**, **EXPONENTIAL**, **EXTREME**, **F**, **GAMMA**, **GEOMETRIC**, **IG2**, **LOGISTIC**, **NORMAL**, **PARETO**, **POWER** and **T**.

LLF Computes the **L**og of the **L**ikelihood **F**unction for the data and prints it and stores it in the temporary variable *$LLF*. This option may not be used with **TYPE=IMHOF**, non-central distributions, or the **INVERSE** option.

NOLIST Suppresses the listing of probability densities or critical values for each observation. It would normally be used only if these numbers were saved in temporary or permanent variables for later use and the listing was not required.

ACCURACY= Specifies the level of accuracy required for the **TYPE=DAVIES** option. The default is **ACCURACY=1E-6**. Computational time can be reduced by specifying a lower value, for example **ACCURACY=.001** (that is, 1E–3).

BEG=, END= Specifies the **BEG**inning and **END** observations to be used for the given **DISTRIB** command. If none are specified the current **SAMPLE** range is used.

BIGN= Specifies the population size (N) for the Hypergeometric distribution. It must be specified along with the **BIGX=** and **N=** options.

BIGX= Specifies the population number of successes (X) for the hypergeometric distribution. It must be specified along with the **BIGN=** and **N=** options.

C= Specifies a parameter value for **TYPE=F**, **BURRII**, **BURRIII**, **BURRXII**, **PARETO** or **POWER**.

CDF= Saves the **C**umulative **D**istribution **F**unction values in the variable specified.

CRITICAL= Saves the **CRITICAL** values in the variable specified when the **INVERSE** option is being used.

DF= Specifies the **D**egrees of **F**reedom. This option is used only with **TYPE=T** and **TYPE=CHI**.

DFVEC= Specifies a vector of N elements containing the **D**egrees of **F**reedom for the **TYPE=DAVIES** option.

DF1=, DF2= Specifies the **D**egrees of **F**reedom for the numerator (**DF1**) and the denominator (**DF2**) respectively. This option is used only with **TYPE=F**.

EIGENVAL= Specifies the vector of **EIGENVAL**ues to be used with the **TYPE=IMHOF** option.

H= Specifies the Precision Parameter for the t-distribution. The default is **H=1** which corresponds to the most common use of the t-distribution. For examples of situations where **H≠1** see Zellner [1971] or Chapters 4 and 7 of the *Judge Handbook*.

K= Specifies the parameter **K** for **TYPE=BURRII, BURRIII, BURRXII** .

KURTOSIS= Specifies the population excess **KURTOSIS** parameter for **TYPE=EDGE**. The coefficient of excess kurtosis is equal to 0 for a normal distribution.

LAMBDA= Specifies the name of an N element λ vector to be used with **TYPE=DAVIES**.

LIMIT= Specifies the maximum number of integration terms to be used with **TYPE=DAVIES**. The default is 10000.

MEAN= Specifies the population **MEAN** value for use with **TYPE=BETA**, **EDGE**, **EXPONENTIAL**, **NORMAL** or **POISSON**.

N= Specifies the sample size (**N**) for use with **TYPE=BINOMIAL** or **TYPE=HYPERGEO**. With **TYPE=DAVIES** this option specifies the number of chi-square variables.

NEIGEN= Specifies the **N**umber of **EIGEN**values to be used with **TYPE=IMHOF** if the entire vector is not required.

NONCEN= Specifies a vector of non-centrality parameters with the **TYPE=DAVIES** option. An example of the use of this option is given later in this chapter.

P= Specifies a parameter value for **TYPE=BERNOULLI**, **BETA**, **BINOMIAL**, **ERLANG**, **GAMMA**, **GEOMETRIC**, **NEGBIN**, **PASCAL** or **WEIBULL**. This is not needed with **TYPE=BETA** if **MEAN=** and **VAR=** are used.

PDF= Saves the **P**robability **D**ensity **F**unction for each observation in the variable specified. It may not be used with **TYPE=IMHOF**.

Q= Specifies a parameter value for **TYPE=BETA**, **ERLANG**, **GAMMA** or **WEIBULL**. This is not needed with **TYPE=BETA** if **MEAN=** and **VAR=** are used.

S= Specifies a parameter value for **TYPE=IG2**. For **TYPE=DAVIES**, the **S=** option specifies the value of σ and the default value is zero.

SKEWNESS= Specifies the population **SKEWNESS** coefficient for **TYPE=EDGE**. The coefficient of skewness is equal to 0 for a normal distribution.

TYPE= Specifies the **TYPE** of distribution. If the type is not specified SHAZAM assumes **TYPE=NORMAL**. The other choices, described in detail later in this chapter, are **BERNOULLI**, **BETA**, **BINOMIAL**, **BURRII, BURRIII, BURRXII, CAUCHY**, **CHI**, **DAVIES, EDGE**, **ERLANG**, **EXPONENTIAL**, **EXTREME**, **F**, **GAMMA**, **GEOMETRIC**, **HYPERGEO**, **IG2**, **IMHOF**, **LOGISTIC**, **LOGNORMAL**, **NEGBIN**, **PARETO**, **PASCAL**, **POISSON**, **POWER**, **T** and **WEIBULL**.

V= Specifies the degrees for freedom for **TYPE=IG2**.

VAR= Specifies the population **VAR**iance. This option is only used with **TYPE=NORMAL**, **TYPE=EDGE** or **TYPE=BETA**.

X= Specifies a parameter value for **TYPE=NEGBIN** or **PASCAL**.

There are several temporary variables available from the last observation of the previous **DISTRIB** command. These are *$CDF*, *$CRIT* and *$PDF*. For more information on temporary variables see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.

TYPES OF DISTRIBUTIONS

The distributions available on the **TYPE=** option are described below. See Evans, Hastings and Peacock [1993] for more facts and formulas.

BERNOULLI *Bernoulli Distribution*

Required option: **P=**p (the probability parameter, $0 < p < 1$). The two values are x=1 (success) or x=0 (failure).

The probability function is: $f(0) = 1 - p\ ;\ f(1) = p$

The cumulative probability function is: $F(0) = 1 - p\ ;\ F(1) = 1$

BETA *Beta Distribution*

Required options: **P=**p and **Q=**q ($p > 0$; $q > 0$) or **MEAN=**μ and **VAR=**σ^2. The range is $0 \le x \le 1$.

The probability density function is:

$$f(x) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)}$$ where B is the beta function.

When the **MEAN=**μ and **VAR=**σ^2 options are specified the parameters p and q are estimated by the method of moments as:

$$p = \mu \cdot [\mu(1-\mu)/\sigma^2 - 1] \quad \text{and}$$

$$q = (1-\mu) \cdot [\mu(1-\mu)/\sigma^2 - 1]$$

BINOMIAL *Binomial Distribution*

Required options: **N=**n and **P=**p ($0 \le p \le 1$); x is an integer and the range is $0 \le x \le n$.

The probability function is: $$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

BURRII *Burr Distribution*
BURRIII
BURRXII

The Burr family of distributions is quite general and contains many other distributions. The type must be specified as **TYPE=BURRII**, **TYPE=BURRIII**, or **TYPE=BURRXII** (see Burr [1968] and Johnson and Kotz [1970, pp. 30-31]).

Required options: **C=**c and **K=**k (c and k are positive parameters).

The cumulative distribution functions are:

(II) $F(x) = (\exp(-x) + 1)^{-k}$

(III) $F(x) = (x^{-c} + 1)^{-k}$ $(0 < x)$

(XII) $F(x) = 1 - (1 + x^{c})^{-k}$ $(0 < x)$

CAUCHY *Cauchy Distribution*

Same as the t-distribution with 1 degree of freedom.

CHI *Chi-Squared Distribution*

Required options: **DF**=v. The range is $0 \le x < \infty$.
The probability density function for v degrees of freedom is:

$$f(x) = \frac{x^{(v-2)/2} \exp(-x/2)}{2^{(v/2)} \Gamma(v/2)}$$

DAVIES *Davies Method*

Davies [1980] describes an algorithm to compute the distribution of a linear combination of chi-square random variables. Consider:

$$Q = \sum_{j=1}^{N} \lambda_j Z_j + \sigma Z_0$$

where Z_j are independent random variables each having a chi-square distribution with n_j degrees of freedom and non-centrality parameter δ_j^2 and Z_0 has a standard normal distribution. The algorithm computes the cumulative distribution function $F(x) = Pr(Q \le x)$. The Davies algorithm can be used to compute a wide variety of distributions including non-central chi-square, non-central F and the distribution of the Durbin-Watson statistic (that is, the algorithm can be used for obtaining the distribution function of quadratic forms as computed with **TYPE=IMHOF**).

Required options: **N**=N (the number of variables), **LAMBDA=** (an $N \times 1$ λ vector), **DFVEC=** (an $N \times 1$ vector of degrees of freedom) and **NONCEN=** (an $N \times 1$ vector of non-centrality parameters). Other options are: **S=**σ (the default value of σ is zero), **LIMIT=** and **ACCURACY=**.

The algorithm can be used to find the distribution of the ratio of two quadratic forms. To calculate the F distribution with the Davies method the ratio of two χ^2 variables must be expressed as the sum of two χ^2 variables. That is,

$$F_{v,w} = \frac{(\chi^2_v / v)}{(\chi^2_w / w)} \qquad \text{is stated as} \qquad 0 = \left(\frac{\chi^2_v}{v}\right) - F_{v,w}\left(\frac{\chi^2_w}{w}\right)$$

An example of how this works is given at the end of this chapter.

EDGE *Edgeworth Approximation*

The Edgeworth expansion is a method for obtaining approximations to many distributions and is described in Bickel and Doksum [1977, p. 33]. Required options: **MEAN**=μ, **VAR**=σ^2, **SKEWNESS**=γ_1, and **KURTOSIS**=γ_2. The defaults with **TYPE=EDGE** are **MEAN=0, VAR=1, SKEWNESS=0** and **KURTOSIS=0**. γ_1 and γ_2 are the coefficient of skewness and excess kurtosis of the standardized variable.

The Edgeworth expansion for the cumulative distribution function of the standardized variable is given by:

$$F(x) = \Phi(x) - \phi(x)\left[\frac{1}{6}\gamma_1 H_2(x) + \frac{1}{24}\gamma_2 H_3(x) + \frac{1}{72}\gamma_1^2 H_5(x)\right]$$

where $\phi(x)$ and $\Phi(x)$ are the standard normal probability density and cumulative distribution functions respectively and H_2, H_3 and H_5 are Hermite polynomials defined by:

$$H_2(x) = x^2 - 1, \quad H_3(x) = x^3 - 3x \quad \text{and} \quad H_5(x) = x^5 - 10x^3 + 15x$$

An example of the Edgeworth approximation to the χ^2 distribution is given in Bickel and Doksum [1977, p. 33].

ERLANG *Erlang Distribution*

Required options: **P**=p (the scale parameter p > 0) and **Q**=q (the shape parameter, q is a positive integer). The range is $0 \le x < \infty$. For distributional properties see the gamma distribution.

EXPONENTIAL *Exponential Distribution*

Required option: **MEAN**=μ. The range is $x \ge 0$.
The probability density function is: $f(x) = \exp(-x/\mu)/\mu$

The cumulative distribution function is: $F(x) = 1 - \exp(-x/\mu)$

For a probability α the inverse survival function is:

$$Z(\alpha) = \mu \cdot \log(1/\alpha)$$

EXTREME *Extreme Value Distribution*

The range is $-\infty < x < \infty$.

The probability density function is: $f(x) = \exp(-x) \cdot \exp[-\exp(-x)]$

The cumulative distribution function is: $F(x) = \exp[-\exp(-x)]$

For a probability α the inverse survival function is:

$$Z(\alpha) = -\log[\log(1/(1-\alpha))]$$

F *F-Distribution*

Required options: **DF1**=v and **DF2**=w. The range is $0 \le x < \infty$. For the non-central F-distribution, the **C**= option specifies the non-centrality parameter.

The probability density function for the central F-distribution with v and w degrees of freedom is:

$$f(x) = \frac{\Gamma[(v+w)/2](v/w)^{(v/2)} x^{(v-2)/2}}{\Gamma(v/2)\Gamma(w/2)((1+v/w)x)^{((v+w)/2)}}$$

GAMMA *Gamma Distribution*

Required options: **P**=p (the scale parameter p > 0) and **Q**=q (the shape parameter q > 0). The range is $0 \le x < \infty$.

The probability density function is: $f(x) = \frac{(x/p)^{q-1} \exp(-x/p)}{p\,\Gamma(q)}$

When q is an integer the distribution is the Erlang distribution. When q=1 the distribution reduces to the exponential distribution with mean p.

GEOMETRIC *Geometric Distribution*

Required option: **P**=p (the probability parameter, 0 < p < 1). The range is $x \ge 0$, x an integer (the number of trials required before the first success).

The probability function is: $f(x) = p(1-p)^n$

The cumulative probability function is: $F(x) = 1 - (1-p)^{n+1}$

For a probability α the inverse survival function is:

$$Z(\alpha) = \log(\alpha)/\log(1-p) - 1$$

Note that the first success occurs on trial x + 1. An alternative form of the geometric distribution considers the number of trials up to and including the first success.

HYPERGEO *Hypergeometric Distribution*

Required options: **BIGN**=N (number of elements in the population), **N**=n (the sample size) and **BIGX**=X (number of successes in the population). The range is $\max[\ 0, n - N + X] \le x \le \min[\ X, n]$.
The probability of exactly x successes is:

$$f(x) = \binom{X}{x}\binom{N - X}{n - x} \Big/ \binom{N}{n}$$

IG2 *Inverted Gamma Distribution - Type 2*

Required options: **S**=$\hat{\sigma}$ (the estimate of σ) and **V**=v (the degrees of freedom).
The probability density function is:

$$f(\sigma) = \left(\frac{2}{\Gamma(v/2)}\right)\left(\frac{v\hat{\sigma}^2}{2}\right)^{v/2} \frac{1}{\sigma^{v+1}} \exp\left(-\frac{v\hat{\sigma}^2}{2\sigma^2}\right)$$

The inverted-gamma probability density function is shown in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Equation 4.2.5]. See Chapters 4 and 7 of the *Judge Handbook* for examples of the **TYPE=IG2** option.

IMHOF *Imhof Method*

The **TYPE=IMHOF** option computes the cumulative distribution function for the quadratic form in normal variables: Q=X′AX. The method is described in Imhof [1961] and Koerts and Abrahamse [1968 and 1969].
Required options: **EIGENVAL=**. The **NEIGEN=** option may also be used.

The cumulative distribution function is obtained by using the result:

$$F(x) = \Pr(\ Q \le x) = \Pr(\ R < 0) \qquad \text{where} \quad R = \sum_{j=1}^{n} (\lambda_j - x)\, Z_j^2$$

and λ_j are the nonzero eigenvalues of the matrix A and the Z_j are standard normal variables. The cumulative distribution function is:

$$F(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\sin\ \varepsilon(u)}{u\ \gamma(u)} du \qquad \text{where}$$

$$\varepsilon(u) = \frac{1}{2}\sum_{j=1}^{n} \arctan\{\ (\lambda_j - x)\,u\ \} \quad \text{and} \quad \gamma(u) = \prod_{j=1}^{n} \{1 + (\lambda_j - x)^2 u^2\}^{\frac{1}{4}}$$

The integral is computed by numerical integration.

An example of the use of **TYPE=IMHOF** for the computation of an exact p-value for the Durbin-Watson test statistic is given in the chapter *PROGRAMMING IN SHAZAM*. An alternative method is obtained with the **TYPE=DAVIES** option.

LOGISTIC *Standard Logistic Distribution*

The range is $-\infty < x < \infty$.

The probability density function is: $f(x) = \exp(\ x)\big/[1 + \exp(\ x)]^2$

The cumulative distribution function is: $F(x) = 1 - 1/[1 + \exp(\ x)]$

For a probability α the inverse survival function is:

$$Z(\alpha) = \log[(\ 1 - \alpha)\ /\ \alpha\ ]$$

LOGNORMAL *Lognormal Distribution*

The range is $0 \le x < \infty$.

The probability density function is:

$$f(x) = \frac{1}{x\,(2\pi)^{1/2}} \exp\left(-\frac{[\log(\ x)]^2}{2}\right)$$

The cumulative distribution function is:

$$F(x) = F_Z(\log(\ x)) \qquad \text{where } F_Z \text{ is the standard normal CDF.}$$

NEGBIN *Negative Binomial Distribution*

Required options: **P**=p (the probability of success at each trial) and **X**=r ($0 < r < \infty$). x is an integer and the range is $0 \le x < \infty$.

The probability function is: $f(x) = \dfrac{\Gamma(r + x)}{\Gamma(r)\,x!} p^r (1 - p)^x$

The mean is: $r \cdot (1 - p)/p$

When r is an integer the distribution is the Pascal distribution. When r = 1 the distribution reduces to the geometric distribution.

Suppose the random variable P has a Pascal distribution and the random variable B has a binomial distribution with parameters n = x + r and p. The cumulative probability functions are related as follows:

$$F_B(r-1;\, n,\, p) = 1 - F_P(x;\, r,\, p)$$

NORMAL *Normal Distribution*

TYPE=NORMAL is the default distribution.
Required options: **MEAN**=μ and **VAR**=σ^2 (the default is **MEAN=0** and **VAR=1**). The range is $-\infty < x < \infty$.
The probability density function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

PARETO *Pareto Distribution*

Required option: **C**=c (c > 0). The default is **C**=1. The range is $1 \le x < \infty$.

The probability density function is: $f(x) = c / x^{c+1}$

The cumulative distribution function is: $F(x) = 1 - (1/x)^c$

For a probability α the inverse survival function is: $Z(\alpha) = \alpha^{-1/c}$

PASCAL *Pascal Distribution*

Required options: **P**=p (the probability of success at each trial) and **X**=r, r = 1, 2, . . . , x is an integer and the range is $0 \le x < \infty$. For distributional properties see the negative binomial distribution.

POISSON *Poisson Distribution*

Required option: **MEAN**=λ where $\lambda \ge 0$. The number $x \ge 0$ is an integer.
The probability function is: $P(x) = \exp(-\lambda)\lambda^x / x!$

POWER *Standard Power Function Distribution*

Required option: **C**=c. The default is **C**=1. The range is $0 \le x \le 1$.

The probability density function is: $f(x) = c\,x^{c-1}$

The cumulative distribution function is: $F(x) = x^c$

For a probability α the inverse survival function is: $Z(\alpha) = (1-\alpha)^{1/c}$

T *Student's t-Distribution*

Required option: **DF**=v (the degrees of freedom). The range is $-\infty < x < \infty$.

The probability density function is:

$$f(x) = \frac{\Gamma((v+1)/2)}{\sqrt{v\,\pi}\,\Gamma(v/2)} \cdot \frac{1}{(1+x^2/v)^{(v+1)/2}}$$

WEIBULL *Weibull Distribution*

Required options: **P**=b (the scale parameter b>0) and **Q**=c (the shape parameter c > 0). The range is $0 \le x < \infty$.

The probability density function is: $f(x) = (c\,x^{c-1}/b^c)\exp[-(x/b)^c]$

The cumulative distribution function is: $F(x) = 1 - \exp[-(x/b)^c]$

The mean is: $b\,\Gamma((c+1)/c)$

For c = 1, the Weibull distribution is the exponential distribution with mean b.

EXAMPLES

The **DISTRIB** command can return p-values for test statistics. This example uses the Theil textile data set and the t-statistics for significance tests from an **OLS** regression are saved in the variable *TR*. The degrees of freedom for the **OLS** regression is available in the temporary variable *$DF*. The **DISTRIB** command is then used to get the p-values for the t-statistics.

```
|_OLS CONSUME INCOME PRICE / TRATIO=TR
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17
 R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE     -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT   130.71      27.09       4.824      .000  .790      .0000      .9718
```

```
|_SAMPLE 1 3
|_GENR TR=ABS(TR)
|_DISTRIB TR / TYPE=T DF=$DF CDF=CDF
T DISTRIBUTION DF=   14.000
VARIANCE=   1.1667       H=   1.0000

               DATA        PDF        CDF        1-CDF
  TR
 ROW     1     3.9813      .13395E-02  .99932      .68261E-03
 ROW     2     16.501      .57984E-10  1.0000      .71625E-10
 ROW     3     4.8241      .25335E-03  .99986      .13502E-03
|_* Now get the p-value for a 2-sided test.
|_GENR P_VAL=2*(1-CDF)
|_PRINT TR P_VAL
      TR             P_VAL
   3.981302       .1365217E-02
   16.50060       .1432510E-09
   4.824137       .2700430E-03
```

In the above example, with the absolute value of the t-test statistic $|t|$ the p-value for a test of significance is calculated as:

$$2(1 - \Pr[t_{(14)} < |t|])$$

With the **DISTRIB** command the values $\Pr[t_{(14)} < |t|]$ are saved in the variable *CDF*. The p-values computed and reported following the **DISTRIB** command are identical to the values listed in the column labelled P-VALUE on the OLS estimation output.

The next example shows how to compute probability values for the chi-square distribution using the option **TYPE=CHI**. An alternative calculation is done using the Davies algorithm with **TYPE=DAVIES**.

```
|_* Program to compute the chi-square distribution function.
|_* Find the probability (CHI-SQUARE<3) with 1 degree of freedom
|_SAMPLE 1 1
|_GEN1 C=3
|_DISTRIB C / TYPE=CHI DF=1
CHI-SQUARE PARAMETERS- DF=   1.0000
MEAN=   1.0000     VARIANCE=   2.0000     MODE=   .00000

               DATA        PDF        CDF        1-CDF
  C
 ROW     1     3.0000      .51393E-01  .91674      .83265E-01
|_*
|_* Now use the Davies algorithm
|_GEN1 DF=1
|_GEN1 LAMB=1
|_DISTRIB C / TYPE=DAVIES N=1 DFVEC=DF LAM=LAMB
DAVIES ALGORITHM N=   1 ACCURACY=   .1000000E-05 LIMIT= 10000 S=   .00000
DF
  1.0000
LAMB
  1.0000
```

```
                  DATA          CDF       1-CDF
   C
  ROW     1     3.0000       .91674       .83265E-01
```

The next example computes probability values for the non-central F distribution using the option **TYPE=F**. An alternative calculation is also done using the Davies algorithm with **TYPE=DAVIES**.

```
|_* PROGRAM FOR NON-CENTRAL F(1,4,5) DISTRIBUTION
|_SAMPLE 1 1
|_GEN1 CRIT=16
|_GEN1 DF1=1
|_GEN1 DF2=4
|_GEN1 C=5
|_DISTRIB CRIT / TYPE=F DF1=DF1 DF2=DF2 C=C

                  DATA          CDF       1-CDF
  CRIT
 ROW     1     16.000     0.81373     0.18627

|_* Now use the Davies algorithm.
|_* Since the Davies algorithm works on chi-square and F is the ratio of
|_* two chi-square variables, you must convert the statistic to the
|_* sum of chi-squares by moving the denominator to the left.
|_* That is, F=(CHI1/df1)/(CHI2/df2) becomes 0=(1/df1)CHI1-(F/df2)CHI2
|_DIM LAMB 2 DF 2 NC 2
|_GENR DF=DF1
|_GENR LAMB=1/DF
|_GENR NC=C
|_SAMPLE 2 2
|_GENR DF=DF2
|_GENR LAMB=-CRIT/DF2
|_GENR NC=0
|_SAMPLE 1 1
|_GEN1 C=0
|_DISTRIB C / TYPE=DAVIES N=2 DFVEC=DF LAMBDA=LAMB NONCEN=NC
DAVIES ALGORITHM N=   2 ACCURACY=  0.1000000E-05 LIMIT= 10000 S=  0.00000
DF
  1.0000       4.0000
LAMB
  1.0000      -4.0000
NC
  5.0000      0.00000

                  DATA          CDF       1-CDF
  C
 ROW     1    0.00000     0.81375     0.18625
```

38. SORTING DATA

"Where a calculator on the ENIAC is equipped with 18,000 vacuum tubes and weighs 30 tons, computers in the future may have only 1,000 vacuum tubes and perhaps weigh only 1.5 tons."

Popular Mechanics
March 1949

The **SORT** command allows the user to sort data. A variable which will be used to sort the data must be specified. When completed, all observations of the specified variables will be rearranged in ascending order according to the ranking in the sort variable.

SORT *COMMAND OPTIONS*

In general, the format of the **SORT** command is:

SORT *sortvar vars / options*

where *sortvar* is the variable name of the sorting variable, *vars* is a list of the variables to be sorted and *options* is a list of desired options. Note that only the variables listed in *sortvars* and *vars* will be sorted, that only the observations for the currently defined **SAMPLE** will be sorted, and that **SKIPIF** commands are not in effect.

The available options on the **SORT** command are:

DESC Specifies that the variables are to be to be sorted in **DESC**ending (rather than ascending) order according to the ranking in the sort variable.

LIST **LIST**s the sorted data in the *vars* and the *sortvar* on the SHAZAM output.

BEG=, END= Specify the sample range for the given **SORT** command. If these are not specified, the current **SAMPLE** range is used.

EXAMPLES

In this example Theil's [1971, p. 102] textile data is prepared with the most current observation first. The **SORT** command reverses the time series order of the data set.

```
|_SAMPLE 1 17
|_READ(11) YEAR CONSUME INCOME PRICE
    4 VARIABLES AND       17 OBSERVATIONS STARTING AT OBS 1
|_PRINT YEAR CONSUME INCOME PRICE
  YEAR           CONSUME           INCOME         PRICE
  1939.000       165.5000       103.8000       61.30000
  1938.000       149.0000       101.6000       59.50000
  1937.000       154.3000       102.4000       59.70000
  1936.000       168.0000       97.60000       52.60000
  1935.000       136.2000       96.40000       63.60000
  1934.000       140.6000       95.40000       62.50000
  1933.000       158.5000       101.7000       61.30000
  1932.000       153.6000       105.3000       65.40000
  1931.000       154.2000       109.3000       70.10000
  1930.000       136.0000       112.3000       82.80000
  1929.000       121.1000       110.8000       90.60000
  1928.000       117.6000       109.5000       89.70000
  1927.000       122.2000       104.9000       86.50000
  1926.000       111.6000       104.9000       90.60000
  1925.000       100.0000       100.0000       100.0000
  1924.000       99.00000       98.10000       100.1000
  1923.000       99.20000       96.70000       101.0000
|_SORT YEAR CONSUME INCOME PRICE
DATA HAS BEEN SORTED BY VARIABLE YEAR
|_PRINT YEAR CONSUME INCOME PRICE
  YEAR           CONSUME           INCOME         PRICE
  1923.000       99.20000       96.70000       101.0000
  1924.000       99.00000       98.10000       100.1000
  1925.000       100.0000       100.0000       100.0000
  1926.000       111.6000       104.9000       90.60000
  1927.000       122.2000       104.9000       86.50000
  1928.000       117.6000       109.5000       89.70000
  1929.000       121.1000       110.8000       90.60000
  1930.000       136.0000       112.3000       82.80000
  1931.000       154.2000       109.3000       70.10000
  1932.000       153.6000       105.3000       65.40000
  1933.000       158.5000       101.7000       61.30000
  1934.000       140.6000       95.40000       62.50000
  1935.000       136.2000       96.40000       63.60000
  1936.000       168.0000       97.60000       52.60000
  1937.000       154.3000       102.4000       59.70000
  1938.000       149.0000       101.6000       59.50000
  1939.000       165.5000       103.8000       61.30000
```

It is important to realize that once the data has been sorted it can only be unsorted back to its original state if, prior to the sort, there was a variable which was in either ascending or descending order. This type of variable may be created on a **GENR** command with the **TIME(0)** function.

39. SET AND DISPLAY

"Gone With The Wind is going to be the biggest flop in Hollywood history. I'm just glad it'll be Clark Gable who's falling flat on his face and not Gary Cooper."

Gary Cooper
Actor, 1938

This chapter describes the **SET** command and the **DISPLAY** command.

SET *COMMAND OPTIONS*

SET commands make it possible to turn certain options on or off. In general, the format of the **SET** command to turn options on is:

SET *option*

To turn options off, the format of the **SET** command is:

SET NO*option*

where *option* is the desired option. The available options on the **SET** command are:

BATCH Used in **BATCH** mode or when more extensive output is desired or when the OUTPUT unit is assigned to a file. Most modern operating systems can detect a batch run, so this option is rarely used. If the operating system is not able to do this, **BATCH** is the default.

BYVAR Turns on the **BYVAR** option for the **PRINT** and **WRITE** commands but not for the **READ** command.

CC/ NOCC Used to turn on/off carriage control. When **SET CC** is in effect, some commands (like **OLS**) will skip to a new page. The default is **SET NOCC**. For more information on carriage control in SHAZAM see the chapter *MISCELLANEOUS COMMANDS AND INFORMATION*.

CONTINUE/ NOCONTINUE An error may terminate a **DO**-loop. The **SET CONTINUE** command forces calculations to continue.

CPUTIME The **DISPLAY CPUTIME** command will print the amount of computer time used in the current SHAZAM run. The **SET CPUTIME** command resets the timer to zero.

DELETE/ NODELETE In distributed-lag models, before estimation, SHAZAM will automatically delete the number of observations equal to the longest lag. If you do not want SHAZAM to do this and prefer to adjust this yourself with the **SAMPLE** command, use **SET NODELETE**. The default is **SET DELETE**.

DOECHO Commands are normally printed for each cycle through a SHAZAM **DO**-loop. To prevent these commands from being printed, **SET NODOECHO**.

DUMP **DUMP**s a lot of output which is primarily of interest to SHAZAM consultants.

ECHO Causes commands to be printed in the output. In interactive mode it may be necessary to use **NOECHO** to prevent the repetition of each command, however most modern operating systems can set this automatically. The default is **ECHO** in BATCH and TERMINAL modes and **NOECHO** in TALK mode.

GRAPH/ NOGRAPH The **SET NOGRAPH** command suppresses the display of gnuplot graphs that are generated with some commands. The gnuplot command and data files with the extension **.GNU** will still be created.

LASTCOM Creates a variable called *C$* which will contain the previous command typed. This variable can then be printed at any time after the **LASTCOM** option is **SET** by typing **PRINT C$**. This option is only useful in TALK mode.

LCUC/ NOLCUC When this option is specified, all lower case characters will be converted to upper case before processing by SHAZAM. This is the default on most computers. If you wish to distinguish upper and lower case variable names and file names in SHAZAM then use **SET NOLCUC.**

MAX The **MAX** option can be **SET** to turn on the **MAX** option on each command. This eliminates the need to use the **MAX** option on every individual command if it is always desired.

OPTIONS Displays the value of all command **OPTIONS** in subsequent commands. This option is primarily of interest to SHAZAM consultants.

OUTPUT/ NOOUTPUT These options are used to turn off or on the output for all following commands. **SET NOOUTPUT** is equivalent to putting a "?" in front of every command.

PAUSE Causes a pause to occur after each command or screen of output. This is useful when the user is working on a machine which has no pause control on its keyboard and therefore much of the output is missed as it appears on the screen. The default is **SET NOPAUSE**. On some systems the user can simply press *RETURN* on the keyboard to resume execution. However, this may not be the case on all operating systems. This option has no effect in BATCH mode.

RANFIX When **RANFIX** is **SET** the random number generator is not set by the system clock. Thus, the same set of random numbers will be obtained in repeated jobs.

SAMPLE/ NOSAMPLE These options allow the use of the omitted observations in the expanded form of the **SAMPLE** command to be turned on and off.

SCREEN/ NOSCREEN Works on some (but not all computers) to turn on and off the display of the SHAZAM output on the terminal screen when the output has been redirected to a file assigned with the **FILE SCREEN** command. Due to the peculiarities of various operating systems the option may or may not work. Try it and see.

SKIP/ NOSKIP These options allow **SKIPIF** commands to be turned on and off. For an example of these options, see the chapter *GENERATING VARIABLES*.

SKIPMISS This option is used to turn on automatic deletion of missing observations in any data analysis or estimation command such as **OLS**, **NL**, **ROBUST**, **STAT**, **FC** etc. If any observation has a missing value code for any of the dependent or independent variables on that command, then the observation will be omitted for that particular command. This is less general than the **SKIPIF** command which will delete an observation for all subsequent commands. The **SET SKIPMISS** command deletes observations only if the variable is actually used. The option can be turned off with **SET NOSKIPMISS**.

Missing values are checked by scanning whether a variable is equal to the missing value code. The missing value code has a default of –99999 but can be changed with the **SET MISSVALU=** option. The temporary variable *$MISS* stores the missing value code.

When the **SET SKIPMISS** command is in effect **GENR** commands and **MATRIX** commands will assign a missing value code to results that involve a computation with a missing observation.

The **AUTO** command should also specify the **MISS** option when estimation with missing observations is considered. Estimation with missing observations in time series may not be appropriate with the **POOL**, **ARIMA**, or **COINT** commands or ARCH estimation with the **HET** command.

STATUS Used to display a status line for some SHAZAM operations. The status line differs among operating systems and can include the index of the current **DO** loop, or the name of the current command. If this status line is not desired it can be turned off with **SET NOSTATUS.**

TALK **TALK** is **SET** at the beginning of an interactive session when typing commands in interactive mode. If all commands are in a file then the user is not "talking" and should use the **TERMINAL** option instead. Most modern operating systems can detect an interactive run, so it may not be necessary to **SET** this option.

TIMER Times each command and the CPU time is printed out after execution of each command.

TRACE This option is primarily of interest to SHAZAM consultants. It prints the name of each subroutine as it is executed.

WARN/ NOWARN Warning messages are normally printed for illegal operations in **GENR**, **MATRIX**, **IF**, **SKIPIF** and **ENDIF** statements. These messages can be turned off with **SET NOWARN**.

WARNMISS/ NOWARNMISS Messages about missing observations can be turned off with **SET NOWARNMISS**.

WARNSKIP/ NOWARNSKIP A warning message is printed for every observation skipped by a **SKIPIF** command. There may be a large number of these warnings if the sample is large. These warnings can be turned off with **SET NOWARNSKIP**.

WIDE/ NOWIDE The **WIDE** option is used to control the line length of printed output. **WIDE** will assume that up to 120 columns are available. **NOWIDE** will try to fit all output in 80 columns. This option mainly affects the **PRINT** and **PLOT** commands. The default is **WIDE** in Batch operation and **NOWIDE** in Talk mode at a terminal.

COMLEN= The maximum number of characters in a command line is 255 on some computers but on others the maximum length is only 80. You can see the length for your computer with the command **DISPLAY COMLEN**. This option can be used to prevent SHAZAM from reading commands beyond a certain column. For example, for a file with sequence numbers in columns 73-80, **COMLEN**=72 would be appropriate. **COMLEN=** should never be specified at a value of greater than 255 under any circumstance. Note that on all computers, commands can be continued onto additional lines if the continuation symbol (&) is used at the end of a line. **The use of continuation lines allows a total command length on all computers of 16,384 characters.**

MAXCOL= When using the colon (:) function to specify a row of a matrix or a number in a vector, the default maximum number is at least 1000 or twice the maximum number of variables specified on the **SIZE** command. If a larger number is required a **SET** command such as **SET MAXCOL=1500** should be used.

MISSVALU= Used with the **SET SKIPMISS** option to specify a missing value code. The default is –99999.

OUTUNIT= The **OUTUNIT=** option is **SET** when the SHAZAM output for the run is to be put into more than one file. In this case, the output files are assigned to appropriate units and addressed with the **OUTUNIT=** option. Of course, if one of the files is originally assigned to Unit 6, SHAZAM automatically outputs into this file and continues until another file is specified on a **SET OUTUNIT=** command. This option is rarely used.

RANSEED= If you wish to initialize the random number generator with a particular integer number use the **SET RANSEED=xxx** option where xxx is a positive integer. This can be used to obtain the same set of random numbers in different runs. This option should be used before any random numbers are generated in the run. **SET RANSEED=0** is the same as **SET RANFIX.**

DISPLAY *COMMAND OPTIONS*

The **DISPLAY** command displays the current value of any option that has been specified with the **SET** command. The format of the **DISPLAY** command is:

DISPLAY *option*

where *option* is the option or a list of the options previously set on a **SET** command.

In addition to the **DISPLAY** command, the **DUMP** command described in the chapter *MISCELLANEOUS COMMANDS AND INFORMATION* is useful for SHAZAM consultants who wish to see technical debugging output.

40. MISCELLANEOUS COMMANDS AND INFORMATION

"If God had wanted a Panama Canal, he would have put one here."
King Philip II of Spain
1552

This is a very important chapter that all users should read. There are various SHAZAM commands available that do not require separate chapters, but are very useful.

The CHECKOUT *Command*

The **CHECKOUT** command prints information about the machine being used. This information is primarily of interest to SHAZAM consultants. The format of the **CHECKOUT** command is:

CHECKOUT

Comment Lines

Comment lines are permitted in SHAZAM. The first column of the line must have an asterisk (*), but the rest of the line may contain anything the user desires. Thus, a comment line might look like the following:

```
* ORDINARY LEAST SQUARES REGRESSION USING TEXTILE DATA
```

This comment line will appear in the output for the run and can be used to identify which regressions were run. Comment lines may not be placed inside data to be read.

The COMPRESS *Command*

The **COMPRESS** command is used to retrieve the space of deleted variables. The **DELETE** command for deleting variables does not automatically free previously occupied space. Since there is a limited amount of memory available in a SHAZAM run the **COMPRESS** command is useful after **DELETE** commands have been used. The format of the **COMPRESS** command is:

COMPRESS

The **COMPRESS** command is not allowed in **DO**-loops or procedures.

Continuation Lines

Continuation lines are permitted in SHAZAM. An ampersand (&) is used at the end of the line to be continued. For example, if a long and complicated equation were to be given on a **GENR** command, it could be continued onto the following line in this way:

```
GENR Y=LOG(X)+P**16*T/(X-2/P)*203 &
-6042
```

SHAZAM will remove the & from the equation and put the two pieces together. The continued line need not start in the first column. Any space typed before the ampersand will be retained in the equation. The maximum length of a command including continuation lines is 16,384 characters. Also see the **SET COMLEN=** command described in the chapter *SET AND DISPLAY*.

The **DELETE** *Command*

The **DELETE** command is used to delete variables. The format of the **DELETE** command is:

DELETE *vars*

where *vars* is a list of variables to be deleted. All variables can be deleted with the command:

DELETE / ALL

The special **ALL_** option can be used to delete all variables with an underscore as a final character (see the chapter *SHAZAM PROCEDURES*). The command is:

DELETE / ALL_

The **ALLDATA** option can be used to delete all variables with the exception of character variables. In particular, FORMAT will not be deleted. The command is:

DELETE / ALLDATA

After **DELETE** commands, a **COMPRESS** command (described above) can be used to get back the space.

The **DEMO** *Command*

The **DEMO** command is used to teach beginners the basic commands in SHAZAM. To see a SHAZAM **DEMO**nstration type **DEMO** and follow the instructions displayed at the terminal. The **DEMO** can be restarted at any time with: **DEMO START**.

The **DIM** *Command*

The **DIM** command dimensions a vector or matrix before any data is defined. This is useful if the data for a given variable or matrix comes from several sources. In this case, **COPY** commands are useful for filling the previously dimensioned vector or matrix. The format of the **DIM** command is:

DIM *var size var size ...*

where *var* is the name of the vector or matrix to be dimensioned, and *size* is either one or two numbers separated by a space to indicate the size of the *var* to be dimensioned. The *size* parameter can also be a scalar variable name. If only one number is given, SHAZAM assumes the *var* is a vector. If two numbers are given, SHAZAM assumes that the *var* specified is a matrix and that the first number given specifies the rows of the matrix, and the second number specifies the columns. More than one vector or matrix can be dimensioned with a single **DIM** command, as shown above. In the example below, the vector *V* is dimensioned to 12 rows (or observations).

```
DIM V 12
```

Similarly, if a matrix *M* were to be dimensioned to 4 rows and 5 columns, the following **DIM** command would be appropriate (matrices may have a maximum of 2 dimensions):

```
DIM M 4 5
```

DO-*loops*

DO-loops perform repeat operations. For instructions on the use of **DO**-loops see the chapter *PROGRAMMING IN SHAZAM*.

The DUMP *Command*

The **DUMP** command prints out information that is primarily of interest to SHAZAM consultants. The format of the **DUMP** command is:

DUMP *options*

where *options* is a list of desired options.

There are many options available on the **DUMP** command, although most of these options are only useful for those who have a source listing of SHAZAM. **DUMP DATA** prints a chart of all the current variables, their addresses, increment, type, their number of observations and their second dimension. **DUMP VNAME** lists all the current variable names. **DUMP KADD** prints the first address of each variable. The following common block options are also available for SHAZAM consultants: **ADDCOM**, **DATCOM**, **FCOM**, **GENCOM**, **INPCOM**, **IOCOM**, **LODCOM**, **MACOM**, **NLCOM**, **OCOM**, **OLSCOM**, **OPTCOM**, **OSCOM**, **RANCOM**, **SCNCOM**, **SYSCOM**, **TEMCOM**, **VCOM**, **VLCOM**, **VPLCOM**, **VTCOM**, **VTECOM** and **VTICOM**. Finally, any range of data in the SHAZAM workspace can be dumped by typing the first and last word desired. For example, the command **DUMP 30 50** would print words 30 through 50 of SHAZAM memory. This would normally be used in conjunction with the information obtained from the **DUMP DATA** command.

DUMP ASCII displays hex codes for characters in the ASCII character set.

The FILE *Command*

The **FILE** command is used to specify a particular action for a file, or assign a search path, or assign units to files. This command is described in the chapter *DATA INPUT AND OUTPUT*.

The **HELP** *Command*

Help on various SHAZAM commands and options is available on the **HELP** command. The format of the **HELP** command is:

HELP *command*

where *command* is any SHAZAM command name.

The **MENU** *Command*

The **MENU** command is useful when you are running SHAZAM interactively and you need a list of available commands. At any point in time some commands may not be valid. These are indicated by an * in the menu list.

The **NAMES** *Command*

The **NAMES** command will print out the **NAMES** of all the variables that have been read or generated in a particular SHAZAM run. It is mainly used in TALK mode to see which variables are currently defined. The format of the **NAMES** command is:

NAMES

or

NAMES *

which will print a table of names, the type of variable and the size of the variable.

The **PAR** *Command*

The **PAR** command sets the **PAR** value. The **PAR** value specifies the amount of memory (in batches of 1 Kilobyte) that is needed for the SHAZAM run. The format of the **PAR** command is:

PAR *number*

where *number* specifies the amount of memory that is needed. This command may have different machine implementations. Check the installation instructions for details on raising **PAR** for your version.

The RENAME *Command*

The **RENAME** command is used to rename variables that already exist in the current run. The format of the **RENAME** command is:

RENAME *oldname newname*

where *oldname* is the name of the variable whose name is to be changed, and *newname* is the new name to be given to that variable.

The REWIND *Command*

The **REWIND** command is used to rewind any unit, usually the **WRITE** unit. **REWIND** is also available as an option on the **WRITE** command. After a **REWIND** command, anything written to the specified unit will overlay what was already there. The format of the **REWIND** command is:

REWIND *unit*

where *unit* is the unit assigned to the **WRITE** file.

The SIZE *Command*

Normally, SHAZAM allows at least 300 variables. This can be changed with the **SIZE** command on most (but not all) systems. The format of the **SIZE** command is:

SIZE *maximum*

For example:

```
size 500
```

The **SIZE** command should be placed at the beginning of the command file. This command may have different machine implementations. Check the installation instructions for your version.

The **STOP** *Command*

The **STOP** command is used to indicate that the SHAZAM run is finished. If no **STOP** command is included SHAZAM will read to the end of the command file.

Temporary Variables

Temporary variables are scalar variables that contain values from current computations. They are redefined each time a new estimation is performed. For example, temporary variables that are generally available are:

$N	The number of observations
$ERR	An error code
$PI	The value for π (*$PI* = 3.1415926535898).
$MISS	The missing value code

Note that the value for *$PI* may be printed in a shortened form on the SHAZAM output even though the number printed above is used in the calculations.

The matrix dimensions from a **MATRIX** command are in:

$ROWS and *$COLS*.

The current values of each index from the **DO** command are in:

$DO and *$DO2 -$DO8.*

Temporary variables available following the **TEST** command are:

$CHI, $DF1, $DF2, $F, $STES, $T, $VAL, $VAL1, $VAL2, $CT11, $CT22, $CT12

Temporary variables available following the **DISTRIB** command are:

$CDF, *$CRIT*, *$LLF* and *$PDF*.

Some temporary variables available following the **AUTO**, **BOX**, **GLS**, **MLE**, **OLS**, **POOL** or **2SLS** regression commands are:

$ADR2, *$ANF*, *$DF*, *$DW*, *$ERR*, *$K*, *$LLF*, *$N*, *$R2*, *$R2OP*, *$RAW*, *$RHO*, *$SIG2*, *$SSE*, *$SSR*, *$SST*, *$ZANF*, *$ZDF* and *$ZSSR*.

Further description is in the chapter *ORDINARY LEAST SQUARES* and other chapters.

Temporary variables are useful for subsequent calculations. For example, suppose the adjusted R^2 (described in the chapter *A CHILD'S GUIDE TO RUNNING REGRESSIONS*) is needed. This variable could be calculated in the following way:

```
ols consume income price
gen1 ar2=1-($n-1)/($n-$k)*(1-$r2)
```

where *$N* is the number of observations, *$K* is the number of parameters, and *$R2* is the R-SQUARE statistic. However, the adjusted R^2 is also automatically available in the temporary variable *$ADR2*.

The TIME *Command*

The **TIME** command specifies the beginning year and frequency for a time series so that an alternate form of the **SAMPLE** command can be used. The format is:

TIME *beg freq var*

where *beg* is the beginning year, *freq* is the frequency of the data (for example, use 1 for annual data, 4 for quarterly data and 12 for monthly data), and *var* is an optional variable name to store dates. An example is:

```
TIME 1981 12
SAMPLE 1982.3 1984.10
```

When a dot "." is included, as in the example above, it is assumed to be a date according to the specification of the **TIME** command. This sets the **SAMPLE** from March of 1982 to October of 1984 for monthly data. The date can also be saved in a variable by specifying a variable name in *var* if a **SAMPLE** range for *var* has previously been defined. However, this variable should only be used for labelling output and not in calculations as it often has no useful numerical meaning.

If yearly data is used a decimal point must be included. For example, for Theil's [1971, p. 102] Textile data you could use:

```
sample 1 17
time 1923 1 year
sample 1930.0 1939.0
```

A **SAMPLE** command without a decimal point is interpreted to be the observation number.

The **TITLE** *Command*

The **TITLE** command prints the specified title at the top of selected pages of output. The **TITLE** may be changed at any time, and as many times as desired. The format of the **TITLE** command is:

TITLE *title*

where *title* is any title the user requires.

Suppressing Output

To suppress the output from SHAZAM commands a question mark (?) may be placed in the first column of the line on which the command is typed. For example, to suppress the output from an **OLS** command, use:

```
?ols a b c / rstat
```

This is useful when a particular statistic is needed (say the Durbin-Watson statistic) for a subsequent test, but no other output is needed. In this case the Durbin-Watson statistic would have been stored in the temporary variable *$DW*, and can easily be retrieved when required.

The command **SET NOOUTPUT** (see the chapter *SET AND DISPLAY*) is equivalent to using the ? prefix on every command. This is recommended for Monte Carlo studies (an example of a Monte Carlo study is given in the chapter *PROGRAMMING IN SHAZAM*).

To suppress the printing of the command itself, but not the output an equal sign (=) may be placed in the first column of the line on which the command is typed. For example, to suppress the **OLS** command, type:

```
=ols a b c
```

To suppress both the command and the output from that command an equal sign and a question mark are placed in the first and second columns of the line on which the command is typed. To suppress both an **OLS** command and its output, use:

```
=?ols a b c
```

Error Codes

When an error occurs in executing a SHAZAM command an error code will often be set in the temporary variable *$ERR*. This variable can then be examined following each command to keep track of the type of error. If *$ERR* is a zero then an error code was not set. An example is:

```
het consume income price  / stdresid=res
if($err.eq.0)
arima res
```

A list of error codes is available with the command **HELP ERROR**.

41. PROGRAMMING IN SHAZAM

"Computers are useless. They can only give you answers."
Pablo Picasso

SHAZAM provides many features to aid users who wish to write their own algorithms or procedures. This chapter provides a few programming examples. More examples are available on the internet at the SHAZAM webpage: http://www.econometrics.com/

DO-LOOPS

DO-loops provide repeat operations. The general format of **DO**-loops is:

DO *dovar=start,stop,inc*
commands
. . .

ENDO

The statements between the **DO** and **ENDO** commands are repeatedly executed. The *dovar* variable is the loop variable and this must be set as a symbol such as #, %, ! or ?. Other symbols on the keyboard can also be used (provided it does not have another purpose). The $ symbol must not be used as a loop variable when a **DELETE SKIP$** command is used in the do-loop commands.

The **DO**-loop facility provides a numeric character substitution for the *dovar* variable. The *dovar* is incremented by the value of *inc*. If *inc* is not specified then an increment of one is set.

The **ENDIF** command is a 1 line conditional statement that can be used inside the **DO**-loop to terminate the **DO**-loop and then continue execution with the command after the **ENDO**. This is described in the chapter *GENERATING VARIABLES* and shown in the example on iterative Cochrane-Orcutt estimation later in this chapter.

The **DO**-loop execution can generate a large amount of repetitive output and this can be suppressed by using the command **SET NODOECHO** before the **DO** command. For applications like Monte Carlo studies, the command **SET NOOUTPUT** may also be useful.

DO-loops can be nested up to 18 levels. Each level must use a different **DO**-loop symbol. The value of the **DO**-loop index is contained in the temporary variable *$DO*. The value of the second loop and following loops are contained in the temporary variables *$DO2 - $DO18*.

An example of the use of a **DO**-loop is:

```
read(11) var1-var10
do # = 1,10
  genr lvar# = log(var#)
  plot lvar# var#
endo
```

This example will create 10 new variables and 10 plots of the log of each variable against each original variable. (It is assumed that the variables *VAR1, VAR2,...VAR10* are in a file.) This example also shows how a series of variables with the same initial letters (in this case, *VAR*) can be easily specified.

The next example shows how a second level **DO**-loop can be used to run a total of 6 regressions. In this case the dependent variables *VAR1, VAR2,* and *VAR3,* are each run with *VAR4* and then run with *VAR5* as independent variables. Note that when a second level is used the first **ENDO** closes the **DO %=4,5** loop and the second **ENDO** closes the **DO #=1,3** loop.

```
sample 1 10
read(11) var1-var5
do #=1,3
  do %=4,5
    ols var# var%
  endo
endo
```

The next example takes the variables *X1, X2* and *X3* and divides them by 2 to get *X21, X22, X23* and then by 3 to get *X31, X32, X33*.

```
do # = 2,3
  do % = 1,3
    genr x#% = x%/#
  endo
endo
```

It is also possible to increment the **DO**-loop by any integer. For example, **DO # = 1,9,2** would set # to the numbers: 1, 3, 5, 7, 9.

EXAMPLES

Splicing Index Number Series

Suppose you have 2 overlapping price indexes where the base year has changed from 1971=100 to 1976=100 and you want to create a new spliced index with 1976=100. You can use the overlapping year, 1976, to adjust the 1971 data as follows:

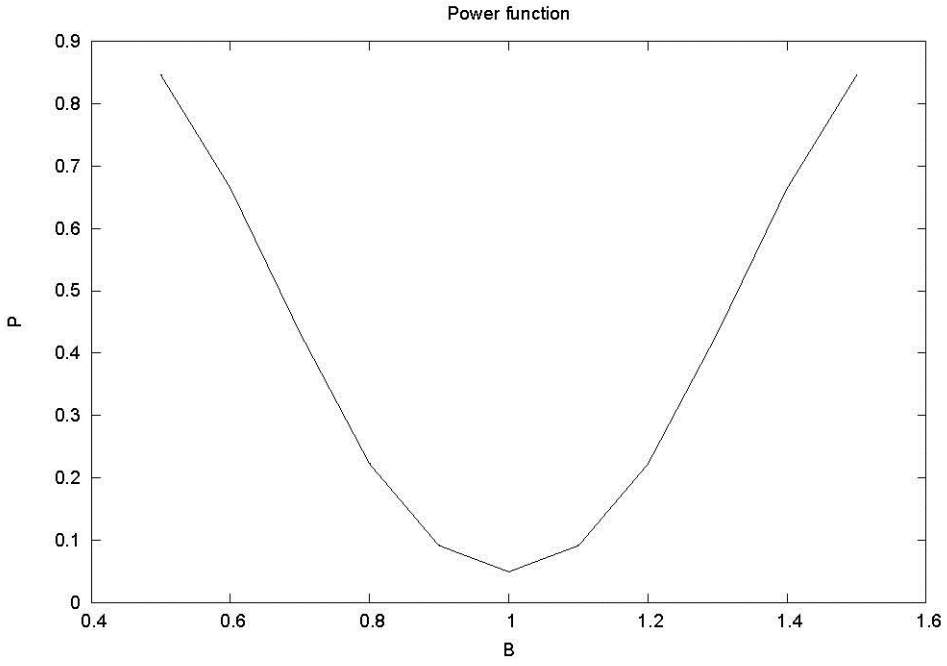
```
|_* Data set from: Newbold, Statistics for Business and Economics,
|_*    Fourth Edition, 1995, Table 17.8, p. 685.
|_* First READ the data for the 1971 and 1976 based indexes.
|_SAMPLE 1 10
|_READ YEAR  P71 P76 / LIST
   3 VARIABLES AND       10 OBSERVATIONS STARTING AT OBS       1
   1971.000       100.0000      0.0
   1972.000       92.20000      0.0
   1973.000       131.2000      0.0
   1974.000       212.0000      0.0
   1975.000       243.0000      0.0
   1976.000       198.5000       100.0000
   1977.000      0.0             94.00000
   1978.000      0.0             86.70000
   1979.000      0.0             94.90000
   1980.000      0.0             107.0000
|_SAMPLE 6 10
|_* Copy the last 5 years of P76 into the SPLICE index.
|_GENR SPLICE=P76
|_SAMPLE 1 5
|_* Compute the first 5 years of P71 using 1976 base.
|_GENR SPLICE=P71*P76:6/P71:6
|_SAMPLE 1 10
|_* Now PRINT all 10 years of the SPLICED INDEX.
|_PRINT YEAR SPLICE
      YEAR           SPLICE
   1971.000       50.37783
   1972.000       46.44836
   1973.000       66.09572
   1974.000       106.8010
   1975.000       122.4181
   1976.000       100.0000
   1977.000       94.00000
   1978.000       86.70000
   1979.000       94.90000
   1980.000       107.0000
```

Computing the Power of a Test

This example shows the use of **DO**-loops. Regression coefficients are estimated for the Theil textile data. The power function is calculated for testing the null hypothesis that the coefficient on log income is one against a 2-sided alternative. The power function is then plotted.

```
|_sample 1 17
|_* Convert data to base 10 logs as implemented by Theil.
|_genr lconsume=log(consume)/2.3026
|_genr lincome=log(income)/2.3026
|_genr lprice=log(price)/2.3026
|_* Run OLS and save the coefficients. Assume these are the true coefficients.
|_ols lconsume lincome lprice / coef=beta
 OLS ESTIMATION
       17 OBSERVATIONS     DEPENDENT VARIABLE= LCONSUME
...NOTE..SAMPLE RANGE SET TO:      1,     17

 R-SQUARE =   0.9744     R-SQUARE ADJUSTED =   0.9707
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.18340E-03
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.13542E-01
SUM OF SQUARED ERRORS-SSE=  0.25675E-02
MEAN OF DEPENDENT VARIABLE =   2.1221
LOG OF THE LIKELIHOOD FUNCTION =  50.6612

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
LINCOME    1.1432     0.1560       7.328     0.000 0.891     0.3216     1.0839
LPRICE   -0.82884     0.3611E-01  -22.95     0.000-0.987    -1.0074    -0.7314
CONSTANT   1.3739     0.3061       4.489     0.001 0.768     0.0000     0.6474

|_* Assume the OLS sigma**2 is the true value.
|_gen1 sig2=$sig2
..NOTE..CURRENT VALUE OF $SIG2=  0.18340E-03
|_sample 1 1
|_* Set a 5% significance level.
|_genr alpha=.05
|_* Find the critical value for the rejection of the null hypothesis.
|_distrib alpha / type=f df1=1 df2=14 inverse
F DISTRIBUTION- DF1=   1.0000     DF2=   14.000
MEAN=   1.1667     VARIANCE=   3.5389     MODE=   0.0000

              PROBABILITY CRITICAL VALUE   PDF
  ALPHA
 ROW     1    0.50000E-01  4.6001     0.21696E-01
|_genr cr=$crit
..NOTE..CURRENT VALUE OF $CRIT=   4.6001

|_* Dimension room for 11 values of the power function.
|_dim  p 11 b 11
|_* Turn off the DO-loop echoing of commands.
|_set nodoecho nooutput
|_do #=1,11
|_* Let the "true" income coefficient (beta:1) vary between .5 and 1.5.
|_* The other coefficients are unchanged.
|_genr beta:1=.4+.1*#
|_* Store the hypothesised values of the income coefficient in the vector b.
```

```
|_genr b:#=beta:1
|_sample 1 17
|_* Calculate OLS regression results using the "true" beta and sig2.
|_* These results are then used as input for the TEST command that follows.
|_ols lconsume lincome lprice / incoef=beta insig2=sig2
|_test lincome=1
|_* Temporary variables available from the TEST command are the
|_* degrees of freedom saved in $df1 and $df2 and the chi-square
|_* test statistic saved in $chi. The latter is used as the
|_* non-centrality parameter.
|_sample 1 1
|_distrib cr / type=f df1=$df1 df2=$df2 c=$chi cdf=cdf
|_* p is the power (the probability of rejecting the false null hypothesis).
|_genr p:#=1-cdf
|_endo
|_* List the power function
|_sample 1 11
|_print b p
      B              P
  0.5000000      0.8459213
  0.6000000      0.6648456
  0.7000000      0.4330300
  0.8000000      0.2230227
  0.9000000      0.9189548E-01
   1.000000      0.5000000E-01
   1.100000      0.9189548E-01
   1.200000      0.2230227
   1.300000      0.4330300
   1.400000      0.6648456
   1.500000      0.8459213
|_graph p b / lineonly
```



In the above example, a **DO**-loop is set up in which 11 hypothetical coefficients are generated and 11 power values (in the variable *P*) are generated in conjunction with the hypothetical coefficients. Notice that the variables for the hypothetical coefficients and the power values are **DIM**ensioned before the **DO** -loop to be vectors of 11 observations so that their values will be saved as they are calculated in the **DO**-loop. Several commands are used within the **DO**-loop. The **OLS** command is used in order to allow the subsequent **TEST** command since **TEST** commands must directly follow an estimation command. The **TEST** command results in the calculation of the chi-square statistic which is stored in the temporary variable *$CHI* and used in the **DISTRIB** command which follows. The **DISTRIB** command calculates the CDF (Cumulative Distribution Function) which is used to compute the power value for the particular hypothetical coefficient. Finally, the power values are plotted against the coefficient values.

The values of the power in the variable *P* are the probabilities of rejecting the null hypothesis when the alternative is true. The hypothetical values of the *INCOME* coefficient are the alternatives. Thus, we would hope that the probability of rejecting the null hypothesis (that the coefficient on *INCOME* is 1) would increase as the alternative (*B*) moves away from 1. This turns out to be the case as can be seen from the plotted power function. Of course, the ideal power function is one in which the probability of rejecting the null hypothesis when it is true is extremely low and in which the probabilities increase rapidly as the alternatives (the true values) move away from the null value.

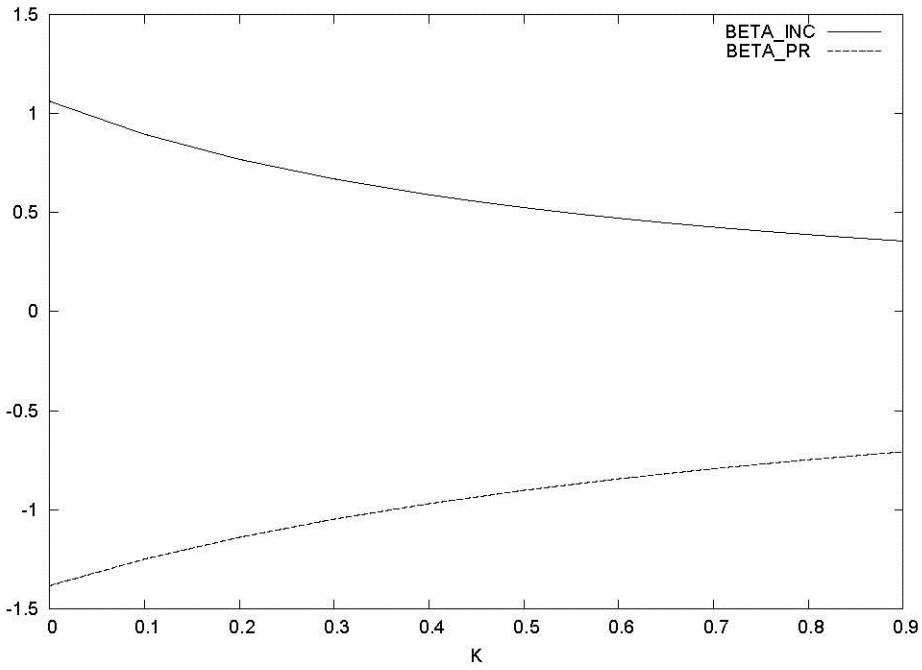
A good reference on the non-central F-distribution and power functions can be found in Graybill [1976, Chapter 4.3].

Ridge Regression

The output below shows the use of **DO**-loops for computing a ridge trace from ridge regressions for the Theil textile data. A reference is Watson and White [1976].

```
|_sample 1 17
|_* Make room for 10 different sets of coefficients
|_dim beta 3 10   k 10
|_* Put OLS coefficients in the first column, ridge k=0
|_gen1 k:1=0
|_* Use the ? prefix to suppress printing of the OLS output.
|_?ols consume income price / coef=beta:1
|_* Let ridge k go from 0.1 to 0.9.
|_do #=2,10
|_gen1 k:#=(#-1)/10
|_?ols consume income price / ridge=k:# coef=beta:#
|_endo
```

```
 _do #=2,10
****** EXECUTION BEGINNING FOR DO LOOP  # =       2
#_         gen1 k:2=(2-1)/10
#_         ?ols consume income price / ridge=k:2 coef=beta:2
#_         endo
#_         gen1 k:3=(3-1)/10
#_         ?ols consume income price / ridge=k:3 coef=beta:3
#_         endo
#_         gen1 k:4=(4-1)/10
#_         ?ols consume income price / ridge=k:4 coef=beta:4
#_         endo
#_         gen1 k:5=(5-1)/10
#_         ?ols consume income price / ridge=k:5 coef=beta:5
#_         endo
#_         gen1 k:6=(6-1)/10
#_         ?ols consume income price / ridge=k:6 coef=beta:6
#_         endo
#_         gen1 k:7=(7-1)/10
#_         ?ols consume income price / ridge=k:7 coef=beta:7
#_         endo
#_         gen1 k:8=(8-1)/10
#_         ?ols consume income price / ridge=k:8 coef=beta:8
#_         endo
#_         gen1 k:9=(9-1)/10
#_         ?ols consume income price / ridge=k:9 coef=beta:9
#_         endo
#_         gen1 k:10=(10-1)/10
#_         ?ols consume income price / ridge=k:10 coef=beta:10
#_         endo
****** EXECUTION FINISHED FOR DO LOOP  #=      10
|_* Transpose the beta matrix.
|_matrix b=beta'
|_matrix results=(k|b)
|_print results
    RESULTS
   10 BY     4 MATRIX
   0.000000       1.061710      -1.382985       130.7066
  0.1000000      0.8957645      -1.248780       137.5545
  0.2000000      0.7689530      -1.138775       142.2192
  0.3000000      0.6695782      -1.046880       145.4404
  0.4000000      0.5900597     -0.9689098       147.6794
  0.5000000      0.5252973     -0.9018889       149.2343
  0.6000000      0.4717498     -0.8436397       150.3036
  0.7000000      0.4268921     -0.7925306       151.0229
  0.8000000      0.3888814     -0.7473141       151.4868
  0.9000000      0.3563458     -0.7070192       151.7624
|_sample 1 10
|_* Plot the coefficients for income (b:1) and price (b:2) against k.
|_* This gives the ridge trace for these coefficients.
|_genr beta_inc=b:1
|_genr beta_pr=b:2
|_graph beta_inc beta_pr k / lineonly
```



In the above program, two variables, *BETA* and *K* are dimensioned to be a 3 x 10 matrix and a vector of 10 observations, respectively. *BETA* is used in the subsequent **DO**-loop to store the values of the coefficients for each ridge regression. *K* is used to store the ridge parameters used in the ridge regressions. The ridge parameter k goes from 0.1 to 0.9 in increments of 0.1. Note that the ? symbol typed before the **OLS** command is used to suppress the output of regression results. In all, 10 regressions are run and the intercept and the coefficients on each of *INCOME* and *PRICE* for each of these regressions are saved in the variable *BETA*. *BETA* is then transposed to facilitate the plotting of the estimated coefficients on *INCOME* and *PRICE* against *K*. This plot is known as the ridge trace.

An Exact p-value for the Durbin-Watson Test

The next SHAZAM run computes (the hard way) an exact p-value value for the Durbin-Watson test. This is illustrated with the Theil textile data. The SHAZAM commands reproduce the method used by SHAZAM with the **DWPVALUE** option on the **OLS** command. A useful discussion is in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, p. 322]. The Imhof technique applied in Koerts and Abrahamse [1968 and 1969] is used to obtain the distribution of the Durbin-Watson test statistic. Other methods are discussed in Durbin and Watson [1971] and Pan, Jei-jian [1968].

The Durbin-Watson test is based on the statistic: $$d = \sum_{t=2}^{N} (e_t - e_{t-1})^2 \Big/ \sum_{t=1}^{N} e_t^2$$

where the e_t are OLS residuals. In matrix notation this can be stated as:

$$d = e'Ae \,/ e'e = \varepsilon'MAM\ \varepsilon / \varepsilon'M\varepsilon$$

where $M = I - X(X'X)^{-1}X'$ and A is a matrix with 2 on the main diagonal (except in the extreme corners where there is a 1), the upper and lower off diagonals have the value –1, and 0 is everywhere else.

The computations are shown in the SHAZAM output:

```
|_READ(11) YEAR CONSUME INCOME PRICE
...SAMPLE RANGE IS NOW SET TO:         1        17
|_* Create X.
|_GENR ONE=1
|_COPY INCOME PRICE ONE X
|_* Compute M.
|_MATRIX M=IDEN(17)-X*INV(X'X)*X'
|_* Generate the "A" matrix.
|_* The Diagonal has 2 everywhere except 1 in the corners.
|_GENR D=1
|_SAMPLE 2 16
|_GENR D=2
|_SAMPLE 1 17
|_* Put 1 on the off-diagonals.
|_MATRIX A=IDEN(17,2)
|_* Turn off diagonal to -1 and add the diagonal.
|_MATRIX A=-(A+A')+DIAG(D)
|_* Compute the eigenvalues of "MA" (Eigenvalues are sorted so largest is first)
|_MATRIX MA=M*A
|_MATRIX E=EIGVAL(MA)
|_* Run OLS just to get the DW statistic from $DW when RSTAT is used.
|_OLS CONSUME INCOME PRICE / RSTAT
OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE     -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT   130.71      27.09       4.824      .000  .790      .0000      .9718

DURBIN-WATSON = 2.0185    VON NEUMAN RATIO = 2.1447    RHO = -0.18239
RESIDUAL SUM =  0.96212E-12  RESIDUAL VARIANCE =   30.951
SUM OF ABSOLUTE ERRORS=   72.787
R-SQUARE BETWEEN OBSERVED AND PREDICTED = 0.9513
```

```
|_* Use the DISTRIB command to compute the exact DW probability.
|_DISTRIB $DW / EIGENVAL=E TYPE=IMHOF NEIGEN=14
                  DATA         CDF      1-CDF
  $DW
 ROW     1     2.0185     0.30127     0.69873
```

First, A and M are generated in matrix form and stored in the variables *A* and *M*. The generation of *M* is straightforward; the independent variables, *INCOME* and *PRICE*, and a column of ones for the intercept, are copied into a matrix, *X*, with a **COPY** command and *M* is then generated with a **MATRIX** command. The generation of *A*, however, takes several steps. First, to create the diagonal of twos with ones in the extreme corners, a vector of twos is generated with a **GENR** command and this vector is then modified to have ones in the first and last rows (which will later become the extreme corners of the *A* matrix) with the use of two **IF** commands. The *A* matrix is then generated as an identity matrix with the ones starting on the second row. The *A* matrix is then transformed again to its final form with another **MATRIX** command. This is done by adding *A* and *A*′ and multiplying the result by –1 (thus creating the upper and lower off-diagonals of –1) and adding *D* to be the diagonal of the *A* matrix. Next, the N–K=14 eigenvalues of the product of *M* and *A* are computed.

The Durbin-Watson statistic from the **OLS** regression is saved in a temporary variable called *$DW*. This value is then used with the **DISTRIB** command to compute the exact Durbin-Watson p-value. The **TYPE=IMHOF, EIGENVAL=** and **NEIGEN=** options must be used for correct results.

In this example, d = 2.0185. For a test of the null hypothesis of no autocorrelation in the residuals against positive autocorrelation the critical region is in the left hand tail of the distribution of d. The output of the **DISTRIB** command reports this probability in the `CDF` column. The p-value of 0.30127 suggests that the null hypothesis cannot be rejected.

Iterative Cochrane-Orcutt Estimation

The easy way to implement iterative Cochrane-Orcutt estimation of the model with first-order autoregressive errors is to use the **AUTO** command. However, the example below shows how the method can also be programmed in SHAZAM. The final results are compared to the SHAZAM **AUTO** command:

```
|_* Program to perform Iterative Cochrane-Orcutt estimation of AR(1) Model.
|_READ (11) CONSUME INCOME PRICE
...SAMPLE RANGE IS NOW SET TO:         1        17
|_* Initialize RHO to first estimate OLS.
|_GEN1 RHO=0
|_* Initialize LASTRHO to a high number.
|_GEN1 LASTRHO=999
```

```
|_* Turn off useless output in DO-loop.
|_SET NODOECHO NOWARN
|_* Try up to 20 iterations.
|_DO #=1,20
|_* Transform all but first observation including CONSTANT.
|_SAMPLE 2 17
|_GENR C=CONSUME-RHO*LAG(CONSUME)
|_GENR I=INCOME-RHO*LAG(INCOME)
|_GENR P=PRICE-RHO*LAG(PRICE)
|_GENR CONS=1-RHO
|_* Transform first observation.
|_SAMPLE 1 1
|_GENR C=SQRT(1-RHO**2)*CONSUME
|_GENR I=SQRT(1-RHO**2)*INCOME
|_GENR P=SQRT(1-RHO**2)*PRICE
|_GENR CONS=SQRT(1-RHO**2)
|_* Run OLS on transformed data, suppress OLS output with "?".
|_SAMPLE 1 17
|_* Check for convergence, if so jump out of loop.
|_ENDIF(ABS(RHO-LASTRHO).LT.0.01)
|_?OLS C I P CONS / NOCONSTANT  COEF=BETA
|_* Take OLS coefficients and get RHO using original data.
|_?FC CONSUME INCOME PRICE / COEF=BETA
|_* Save RHO to check for convergence next time.
|_GEN1 LASTRHO=RHO
|_PRINT $DO $SSE $RHO
|_* Put latest value of $RHO into RHO.
|_GEN1 RHO=$RHO
|_ENDO
****** EXECUTION BEGINNING FOR DO LOOP  # =  1
   $DO       1.000000
   $SSE      433.3130
   $RHO    -0.1823932
   $DO       2.000000
   $SSE      419.9997
   $RHO    -0.1947947
   $DO       3.000000
   $SSE      419.8044
   $RHO    -0.1953525
...ENDIF IS TRUE AT OBSERVATION         1
...DO LOOP ENDED AT  #=       4
|_* Print final results.
|_OLS C I P CONS / NOCONSTANT
  OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = C
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =    .9709     R-SQUARE ADJUSTED =    .9668
VARIANCE OF THE ESTIMATE-SIGMA**2 =   29.986
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.4759
SUM OF SQUARED ERRORS-SSE=   419.80
MEAN OF DEPENDENT VARIABLE =   158.77
LOG OF THE LIKELIHOOD FUNCTION = -51.3777
RAW MOMENT R-SQUARE =    .9991

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
I          1.0650      .2282       4.667      .000  .780      .3237      .8170
P         -1.3751      .7105E-01  -19.35      .000 -.982     -.8615     -.7830
CONST      129.62      23.05       5.624      .000  .833      .2247      .9656
```

```
|_* Compare to SHAZAM AUTO command.
|_AUTO CONSUME INCOME PRICE /  CONV=.01
DEPENDENT VARIABLE =  CONSUME
..NOTE..R-SQUARE,ANOVA,RESIDUALS DONE ON ORIGINAL VARS

LEAST SQUARES ESTIMATION             17 OBSERVATIONS
BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.01000

    ITERATION          RHO              LOG L.F.            SSE
        1             0.0            -51.6471             433.31
        2            -0.18239        -51.3987             420.00
        3            -0.19479        -51.3972             419.80
        4            -0.19535        -51.3972             419.80

 LOG L.F. =   -51.3972       AT RHO =    -0.19535

                    ASYMPTOTIC  ASYMPTOTIC  ASYMPTOTIC
          ESTIMATE    VARIANCE    ST.ERROR     T-RATIO
RHO       -0.19535     0.05658     0.23786    -0.82128

 R-SQUARE =    .9528     R-SQUARE ADJUSTED =    .9461
VARIANCE OF THE ESTIMATE-SIGMA**2 =   29.986
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.4759
SUM OF SQUARED ERRORS-SSE=   419.80
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.3972

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.0650      .2282       4.667      .000  .780      .2394      .8154
PRICE     -1.3751      .7105E-01  -19.35      .000 -.982     -.9837     -.7802
CONSTANT   129.62      23.05       5.624      .000  .833      .0000      .9637
```

Note that the output from the programmed procedure matches that of the **AUTO** command except for certain statistics like R-SQUARE, LOG OF THE LIKELIHOOD FUNCTION, STANDARDIZED COEFFICIENT and ELASTICITY AT MEANS which are not properly computed with an OLS algorithm. The **AUTO** output has the correct values of these statistics.

Nonlinear Least Squares by the Rank One Correction Method

The next example illustrates how to write a SHAZAM program to do least squares by the rank one correction (ROC) method described in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, p. 959]. This method merely minimizes the sum of squared residuals and thus produces the same results as the **OLS** run. The following commands assume use of the data from Table B.2 on page 956 of the above reference.

```
sample 1 20
read (tableb.2) y one x1 x2
genl nobs=20
copy x1 x2 one x
matrix yy=y'y
matrix xx=x'x
```

```
matrix xy=x'y
matrix h=2*xx
matrix hinv=inv(h)
matrix p=iden(3)
sample 1 3
read b / byvar list
  1  1  1
matrix s=yy-2*b'xy+b'xx*b
matrix g=-2*xy+2*xx*b

* Print starting value info.
print b s g yy xx xy h hinv

* Program to do least squares by rank one correction (ROC) method.
* Allow up to 10 iterations.
do #=1,10
  * First compute the gradient.
  matrix glast=g
  * Now get next round betas.
  matrix blast=b
  matrix b=b-p*g
  matrix g=-2*xy+2*xx*b
  * Now compute S.
  matrix s=yy-2*b'xy+b'xx*b
  matrix eta=(b-blast)-p*(g-glast)
  matrix m=(eta*eta')/(eta'(g-glast))
  matrix p=p+m
  print b s g p m
  matrix gg=g'g
  * Now check for convergence.
  endif(gg.lt.0.0000001)
endo

* End of loop
matrix vb=2*(s/nobs)*p
matrix se=sqrt(diag(vb))
matrix t=b/se
print b se t
print vb

* Now compare to regular OLS.
sample 1 nobs
ols y x1 x2 / dn pcov
```

As can be seen in the above example, the ROC method involves computing the gradient at each iteration to see if it is close to zero and thus minimized. To do this, first *S* is computed, which is the objective function to be minimized. *G*, the gradient, is then computed from *S*.

Next, a **DO**-loop is initiated to perform the necessary computations until convergence, i.e. until *G'G* is close to zero. The starting value for *B* (beta) is one, but this value is transformed on each iteration, thus transforming the values for *G* and *P*. *P* is the inverse of the Hessian and is transformed on each iteration by *M*, a correction matrix. When convergence occurs, or when the **DO**-loop is performed 10 times, the current values of *S* and *P* are used to compute the variance of beta and ultimately to compute a t-ratio. In the commands above, a **PRINT** command is used to print out the final values of beta, the standard error and the t-ratio. These results can then be compared to the **OLS** results which will be generated by the final **OLS** command. The results should be the same.

Monte Carlo Experiments

This example is from an exercise in Judge, Hill, Griffiths, Lütkepohl and Lee [1988, Section 9.6.2, pp. 411-412]. A Monte Carlo experiment is designed to compare the parameter estimates obtained with OLS, GLS, and feasible GLS estimation of a model with AR(1) errors. A feature of the SHAZAM program to note is the use of the **GENR** command to generate the AR(1) errors by recursive calculations. The **SET RANFIX** command is used to ensure that the same set of random numbers will be used in repeated runs of the program so that the results can be replicated.

```
* Design Matrix
sample 1 20
read x1 x2 / list
14.53  16.74
15.30  16.81
15.92  19.50
17.41  22.12
18.37  22.34
18.83  17.47
18.84  20.24
19.71  20.37
20.01  12.71
20.26  22.98
20.77  19.33
21.17  17.04
21.34  16.74
22.91  19.81
22.96  31.92
23.69  26.31
24.82  25.93
25.54  21.96
25.63  24.05
28.73  25.66
```

```
* Generate 1000 samples with true model:  Y = 10 + X1 + X2 + e
* where e is an AR(1) process with:       e = 0.8*e(-1) + v
* and v is independent normally distributed with 0 mean and
* var.=6.4.
gen1 se=sqrt(6.4)

* Allocate arrays to hold the estimates
dim bols 4 1000 stdols 4 1000 bgls 4 1000 stdgls 4 1000
dim begls 4 1000 stdeg 4 1000

* Request the same set of random numbers in repeated runs
set ranfix

* Suppress useless output
set nodoecho
do #=1,1000
  sample 1 20
  * Generate v
  genr v=nor(se)
  * Set an initial condition for e
  genr e=0
  * Generate e - recursive calculations
  sample 2 20
  genr e=0.8*lag(e)+v
  sample 1 20
  * Generate Y given X and e.
  genr y=10+x1+x2+e
  * OLS estimation, suppress the output with ?
  ?ols y x1 x2 / coef=bols:# stderr=stdols:#
  * GLS estimation (RHO is known)
  ?auto y x1 x2 / rho=0.8 coef=bgls:# stderr=stdgls:#
  * Estimated GLS (RHO is estimated - iterated Cochrane-Orcutt)
  ?auto y x1 x2 / coef=begls:# stderr=stdeg:#
endo

* Transpose the arrays so that the STAT command can be used
matrix bols=bols'
matrix stdols=stdols'
matrix bgls=bgls'
matrix stdgls=stdgls'
matrix begls=begls'
matrix stdeg=stdeg'
sample 1 1000
stat bols   bgls   begls / mean=b stdev=ase
stat stdols stdgls stdeg / mean=ese stdev=stdse
sample 1 12
```

```
format(4f12.4)
print b ase ese stdse / format
```

Annotated results obtained from the above program follow. *B* is the average parameter estimate, *ASE* is the standard deviation of the parameter estimate, *ESE* is the average standard error and *STDSE* is the standard deviation of the standard errors.

```
          B           ASE           ESE        STDSE
OLS ESTIMATION
       .9957        .4091        .2092        .0540
       .9986        .2093        .1768        .0456
     10.1006      8.8212      3.8956      1.0047
       .0000        .0000        .0000        .0000
GLS estimation (with RHO=0.8)
       .9889        .3080        .3802        .0649
      1.0032        .1345        .1291        .0220
     10.1398      5.7728      8.3551      1.4261
       .8000        .0000        .0000        .0000
EGLS estimation (by iterative Cochrane-Orcutt)
       .9905        .3424        .2700        .0907
      1.0051        .1433        .1348        .0242
     10.0649      6.5016      5.6791      2.1816
       .5031        .2589        .0000        .0000
```

The results show that the parameter estimates from the three estimation methods are unbiased with the exception of the parameter estimate for the autocorrelation parameter RHO that is obtained by iterative Cochrane-Orcutt estimation. The average parameter estimate for RHO is 0.5031 which is less than the true parameter value of 0.8. The standard deviations of the parameter estimates are smaller for EGLS (Estimated GLS or feasible GLS) estimation compared to OLS estimation to demonstrate that EGLS is efficient.

Bootstrapping Regression Coefficients

This example shows how to approximate the distribution of an estimator by using the Efron [1979] bootstrapping method that is discussed in Freedman and Peters [1984]. This is illustrated with an OLS regression using the Theil textile data set. A more automatic way of getting the results produced in the SHAZAM program below is with the commands:

```
set ranfix
ols consume income price
diagnos / bootsamp=1000
```

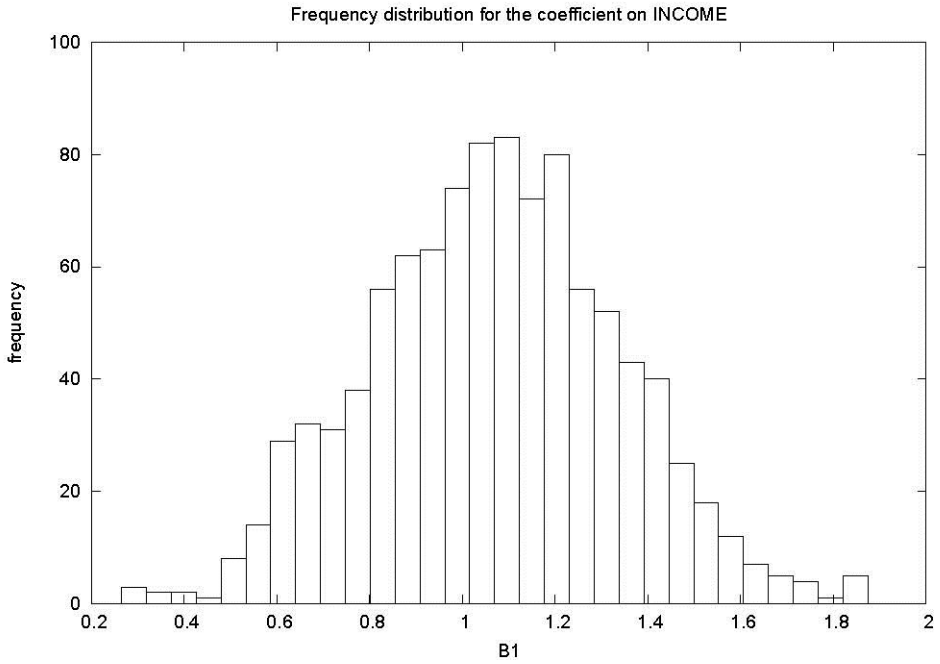
Note that the bootstrap method is a computationally slow and inaccurate way of getting OLS standard errors, but might be useful on other kinds of models. A valuable way to view the results is with a graphical presentation. The commands show two methods of obtaining a graphical display. First, a histogram is produced with the **GRAPH** command. Second, the

NONPAR command is used to construct a nonparametric kernel density estimate. The **GRAPH** option specifies that the plots are produced with the GNUPLOT interface.

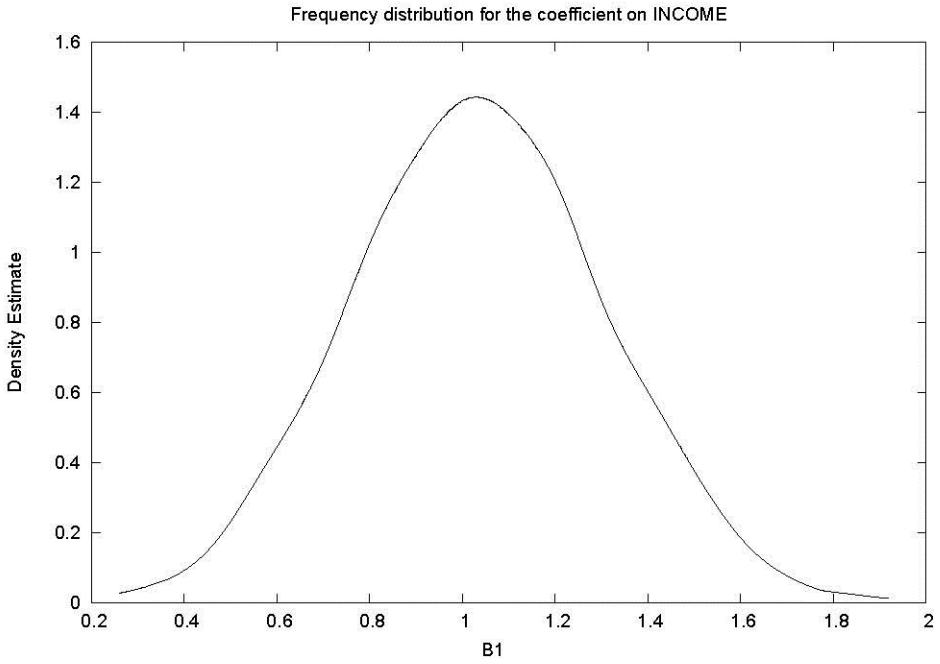
```
* Read the Theil textile data set
read(theil.dat) year consume income price

* Program to get OLS standard errors by Bootstrapping.
* Warning: This is a computationally expensive run.
* Run the original regression, save residuals and predicted values.
ols consume income price / resid=e predict=yhat
gen1 n=$n
gen1 k=$k
gen1 nrep=1000

* Create space to hold vectors of bootstrapped coefficients.
dim beta k nrep

* Turn off DO-loop printing or you will get lots of output.
set nodoecho
set nooutput
set ranfix

do #=1,nrep
  * Draw a random sample of errors with replacement.
  genr newe=samp(e)*sqrt(n/(n-k))
  * Generate new dependent variable by using NEWE.
  genr y=yhat+newe
  ols y income price / coef=beta:#
endo

* Transpose the BETA matrix for use in STAT and PLOT commands.
* This is needed to get the numbers in column order.
matrix beta=beta'
set output

* Set the sample size to number of replications.
sample 1 nrep

* Get the statistics on the replications.
stat beta

* Look at the frequency distribution for the INCOME coefficient.
genr b1=beta:1

* Plot a histogram
```

```
graph b1 / histo groups=30

* Now get a nonparametric density estimate
nonpar b1 / density graph
```

The histogram presentation of the distribution for the coefficient on *INCOME* is:



The above histogram can be compared with the nonparametric density estimate that is generated with the **NONPAR** command. The graph of this estimate is:



Heteroskedastic Consistent Covariance Matrices

The easy way to get White's [1980] heteroskedastic consistent covariance matrix is to use the **HETCOV** option on **OLS**. However, this example shows how to program it and some variations discussed in MacKinnon and White [1985]. The example also shows how to use the estimator proposed by Cragg [1983].

```
|_READ(11) YEAR CONSUME INCOME PRICE
...SAMPLE RANGE IS NOW SET TO:         1        17
|_* First run OLS, save residuals
|_OLS CONSUME INCOME PRICE / RESID=U STDERR=OSE
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME      1.0617      .2667       3.981      .001  .729      .2387      .8129
PRICE      -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
CONSTANT    130.71      27.09       4.824      .000  .790      .0000      .9718

|_* Next square the residuals and copy the independent variables
|_* into the X matrix.
|_GENR U2=U**2
```

```
|_GENR ONE=1
|_COPY INCOME PRICE ONE X
|_* Now HC is White's(1980) covariance matrix for heteroskedasticity.
|_MATRIX HC=INV(X'X)*X'DIAG(U2)*X*INV(X'X)
|_* Now get the corrected standard errors and print them out.
|_MATRIX HSE=SQRT(DIAG(HC))
|_PRINT OSE
OSE
  0.2666740      0.8381426E-01   27.09429
|_PRINT HSE
HSE
  0.2172196      0.7422455E-01   23.66507
|_* Note that the corrected standard errors are smaller.
|_* Now get HC1, the Hinkley method of estimation.
|_MATRIX HC1=($N/$DF)*HC
..NOTE..CURRENT VALUE OF $N   =   17.000
..NOTE..CURRENT VALUE OF $DF  =   14.000
|_MATRIX KTT=DIAG(X*INV(X'X)*X')
|_MATRIX SIG2=U2/(1-KTT)
|_* HC2 is the Horn and Duncan estimate.
|_MATRIX HC2=INV(X'X)*X'DIAG(SIG2)*X*INV(X'X)
|_MATRIX USTAR=U/(1-KTT)
|_MATRIX OM=DIAG(USTAR**2)
|_* HC3 is the MacKinnon and White(1985) Jackknife estimate.
|_MATRIX HC3=(($N-1)/$N)*INV(X'X)*(X'OM*X-(1/$N)*(X'USTAR*USTAR'X))*INV(X'X)
|_* Now print out the 4 different covariance matrix estimates.
|_PRINT HC HC1 HC2 HC3
HC
    3 BY     3 MATRIX
  0.4718433E-01  0.2076371E-03  -4.957136
  0.2076371E-03  0.5509284E-02 -0.4802344
  -4.957136     -0.4802344       560.0357
HC1
    3 BY     3 MATRIX
  0.5729526E-01  0.2521307E-03  -6.019380
  0.2521307E-03  0.6689845E-02 -0.5831418
  -6.019380     -0.5831418       680.0433
HC2
    3 BY     3 MATRIX
  0.6044015E-01 -0.5570915E-03  -6.295172
 -0.5570915E-03  0.6942529E-02 -0.5138845
  -6.295172     -0.5138845       704.1588
HC3
    3 BY     3 MATRIX
  0.7333148E-01 -0.1782186E-02  -7.563165
 -0.1782186E-02  0.8325627E-02 -0.4927166
  -7.563165     -0.4927166       836.8303
|_* Now do the Cragg (1983) estimator using X**2 as auxiliary variables.
|_GENR INCOME2=INCOME**2
|_GENR PRICE2=PRICE**2
|_COPY INCOME PRICE ONE INCOME2 PRICE2 Q
|_* The coefficient vector BA is Cragg's equation (13), p. 753.
|_MATRIX BA=INV(X'Q*INV(Q'DIAG(U2)*Q)*Q'X)*X'Q*INV(Q'DIAG(U2)*Q)*Q'CONSUME
|_* The covariance matrix VBA is Cragg's equation (14), p. 754.
|_MATRIX VBA=INV(X'Q*INV(Q'DIAG(U2)*Q)*Q'X)
|_* Note that BA and VBA have many similar terms. It would have
|_* been cheaper to compute them separately first.
|_PRINT BA VBA
BA
  0.9725258      -1.367748       138.8088
VBA
```

```
     3 BY     3 MATRIX
   0.3068722E-01  0.1126381E-02  -3.324788
   0.1126381E-02  0.5418412E-02 -0.5683495
   -3.324788     -0.5683495       398.3224
```

Hausman Specification Test

This example shows how to use Hausman's [1978] specification test in an errors in variables model. The model is a regression of consumption on income where it is suspected that income is measured with error. The investment variable is used as an instrumental variable.

```
|_* Hausman specification test of error in variables
|_* EXAMPLE:   The data set on investment (I), consumption (C) and
|_* income (Y) is from Griffiths, Hill and Judge (1993, Table 14.2, p. 464).
|_SAMPLE 1 20
|_READ I C Y
   3 VARIABLES AND       20 OBSERVATIONS STARTING AT OBS       1

|_* Estimation using the consistent estimator (IV) under both
|_* the null and the alternative hypotheses;
|_* in SHAZAM, the 2SLS command is used for instrumental variable estimation
|_2SLS C Y (I) / DN COEF=B1 PCOV COV=V1
 TWO STAGE LEAST SQUARES - DEPENDENT VARIABLE = C
  1 EXOGENOUS VARIABLES
  2 POSSIBLE ENDOGENOUS VARIABLES
       20 OBSERVATIONS
DN OPTION IN EFFECT - DIVISOR IS N

 R-SQUARE =    .9893     R-SQUARE ADJUSTED =    .9887
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .67175E-01
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .25918
SUM OF SQUARED ERRORS-SSE=   1.3435
MEAN OF DEPENDENT VARIABLE =   21.747

                            ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
Y          .79036      .2074E-01   38.12      .000  .994      .9560      .8955
CONSTANT   2.2734      .5142       4.421      .000  .722      .0000      .1045

VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
Y          .42995E-03
CONSTANT  -.10594E-01   .26437
             Y            CONSTANT

|_GEN1 SIGIV=$SIG2
..NOTE..CURRENT VALUE OF $SIG2=   .67175E-01

|_* Estimation using the efficient estimator (OLS) under the null
|_OLS C Y / DN COEF=B0 COV=V0 NOMULSIGSQ
 OLS ESTIMATION
      20 OBSERVATIONS     DEPENDENT VARIABLE = C
...NOTE..SAMPLE RANGE SET TO:    1,   20

 R-SQUARE =    .9908     R-SQUARE ADJUSTED =    .9903
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .57482E-01
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .23975
```

```
SUM OF SQUARED ERRORS-SSE=   1.1496
MEAN OF DEPENDENT VARIABLE =   21.747
LOG OF THE LIKELIHOOD FUNCTION =  .184015

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
Y          .82289      .7389E-01   11.14      .000  .934      .9954      .9323
CONSTANT   1.4718      1.834       .8024      .422  .186      .0000      .0677

|_* Use the error variance estimate obtained from IV estimation.
|_MATRIX V0=SIGIV*V0
|_PRINT V0
    V0
    2 BY     2 MATRIX
   .3667221E-03  -.9035666E-02
  -.9035666E-02   .2259885

|_* Compute the Hausman specification test statistic using the method
|_* illustrated in Griffiths, Hill and Judge (1993, p. 465)
|_SAMPLE 1 2
|_GENR Q=B1-B0
|_MATRIX VQ=V1-V0
|_MATRIX M=(Q(1)**2) / VQ(1,1)
|_* The statistic M is distributed chi-square with 1 degree of freedom
|_* under the null hypothesis. The 5% critical value is 3.84.
|_PRINT M
    M
   16.73798
```

A general version of the Hausman specification test is described in Griffiths, Hill and Judge [1993, Appendix 14A.4, pp. 475-6].

Non-Nested Model Testing

This example shows how to apply the nonnested Cox test and the Davidson-MacKinnon [1981] J-test described in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, pp. 883-884]. Theil's textile data is used to test the two linear regressions *CONSUME* on *INCOME* and *CONSUME* on *PRICE*. The test procedure first tests H1 (*INCOME* is the correct variable) and then H2 (*PRICE* is the correct variable). It should be noted that we have strong theoretical reasons to believe that both *INCOME* and *PRICE* belong in the regression in this example.

In the output below the Cox test statistics indicate that the *INCOME* model is rejected, but the *PRICE* model is not rejected. However, the J-test statistics reject both models.

```
|_SAMPLE 1 17
|_READ(11) YEAR CONSUME INCOME PRICE
   4 VARIABLES AND       17 OBSERVATIONS STARTING AT OBS       1
|_* Non-nested Cox Test, H1: Consume on income, H2: Consume on price.
|_*  See BIG JUDGE pp. 883-884.
```

```
|_OLS CONSUME INCOME / COEF=B1 DN PREDICT=Y1HAT
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =    .0038     R-SQUARE ADJUSTED =   -.0626
VARIANCE OF THE ESTIMATE-SIGMA**2 =   521.19
STANDARD ERROR OF THE ESTIMATE-SIGMA =   22.830
SUM OF SQUARED ERRORS-SSE=   8860.3
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -77.2990

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     .27473      1.077       .2552      .799  .066      .0618      .2103
CONSTANT   106.21      111.0       .9567      .339  .240      .0000      .7897
|_GEN1 S1=$SIG2
..NOTE..CURRENT VALUE OF $SIG2=   521.19
|_OLS CONSUME PRICE / COEF=B2 DN PREDICT=Y2HAT
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =    .8961     R-SQUARE ADJUSTED =    .8892
VARIANCE OF THE ESTIMATE-SIGMA**2 =   54.348
STANDARD ERROR OF THE ESTIMATE-SIGMA =   7.3721
SUM OF SQUARED ERRORS-SSE=   923.91
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -58.0829

                              ASYMPTOTIC
VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR   ***** DF   P-VALUE CORR. COEFFICIENT  AT MEANS
PRICE     -1.3233      .1093      -12.11      .000 -.952     -.9466     -.7508
CONSTANT   235.49      8.528       27.61      .000  .990      .0000     1.7508
|_GEN1 S2=$SIG2
..NOTE..CURRENT VALUE OF $SIG2=   54.348
|_* Create the X and Z matrices, X for H1, Z for H2.
|_GENR ONE=1
|_COPY INCOME ONE X
|_COPY PRICE ONE Z
|_* Create the M matrices, M1 for H1, M2 for H2.
|_MATRIX M2=IDEN(17)-Z*INV(Z'Z)*Z'
|_MATRIX M1=IDEN(17)-X*INV(X'X)*X'
|_* Test H1: The hypothesis that income is the true model.
|_* The test statistic H1 is asymptotically normally distributed.
|_MATRIX S21=S1+1/$N*(B1'X'M2*X*B1)
..NOTE..CURRENT VALUE OF $N   =   17.000
|_MATRIX C12=$N/2*LOG(S2/S21)
..NOTE..CURRENT VALUE OF $N   =   17.000
|_MATRIX VC12=(S1/S21**2)*(B1'X'M2*M1*M2*X*B1)
|_MATRIX H1=C12/SQRT(VC12)
|_PRINT C12 VC12 H1
    C12
  -19.24761
    VC12
  0.2001174E-02
    H1
  -430.2633
|_* Now test H2: The hypothesis that price is the true model.
```

```
|_* The test statistic H2 is asymptotically normally distributed.
|_MATRIX S12=S2+1/$N*(B2'Z'M1*Z*B2)
..NOTE..CURRENT VALUE OF $N   =   17.000
|_MATRIX C21=$N/2*LOG(S1/S12)
..NOTE..CURRENT VALUE OF $N   =   17.000
|_MATRIX VC21=(S2/S12**2)*(B2'Z'M1*M2*M1*Z*B2)
|_MATRIX H2=C21/SQRT(VC21)
|_PRINT C21 VC21 H2
    C21
  0.2147064
    VC21
  0.5193263E-01
    H2
  0.9421603
|_* Now try the Davidson-MacKinnon J Test.
|_* To test H1, see if coef on Y2HAT is zero.
|_OLS CONSUME INCOME Y2HAT
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =    .9513     R-SQUARE ADJUSTED =    .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME     1.0617      .2667       3.981      .001  .729      .2387      .8129
Y2HAT      1.0451      .6334E-01   16.50      .000  .975      .9893     1.0451
CONSTANT  -115.40      30.20      -3.821      .002 -.714      .0000     -.8580

|_* To test H2, see if coef on Y1HAT is zero.
|_OLS CONSUME PRICE Y1HAT
 OLS ESTIMATION
      17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
...NOTE..SAMPLE RANGE SET TO:    1,   17

 R-SQUARE =    .9513     R-SQUARE ADJUSTED =    .9443
VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
SUM OF SQUARED ERRORS-SSE=   433.31
MEAN OF DEPENDENT VARIABLE =   134.51
LOG OF THE LIKELIHOOD FUNCTION = -51.6471

VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      14 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
PRICE     -1.3830      .8381E-01  -16.50      .000 -.975     -.9893     -.7846
Y1HAT      3.8645      .9707       3.981      .001  .729      .2387     3.8645
CONSTANT  -279.75      129.6      -2.159      .049 -.500      .0000    -2.0799
```

Solving Nonlinear Sets of Equations

Consider the task of finding a solution X to the equation f(X) = 0 (where X may be a vector). For example, in the single variable case,

$$X^2 - 4X = -4$$

This can be expressed in implicit function form as:

$$X^2 - 4X + 4 = 0$$

A solution can be obtained by a numerical iterative method with the use of the **SOLVE** option on the **NL** command. The **SAMPLE** command must be set to one observation. The implicit function form of the equation (excluding the = sign) is specified on the **EQ** command. This is shown in the SHAZAM output below.

```
|_SAMPLE 1 1
|_NL 1 / NCOEF=1 SOLVE PITER=50
...NOTE..SAMPLE RANGE SET TO:      1,      1
|_EQ (X*X)-4*X+4
|_COEF X 5
   0 VARIABLES IN  1 EQUATIONS WITH   1 COEFFICIENTS
         1 OBSERVATIONS
COEFFICIENT STARTING VALUES
X          5.0000
      100 MAXIMUM ITERATIONS, CONVERGENCE =  .000010

INITIAL STATISTICS :
TIME =        .250 SEC.   ITER. NO.      0   FUNCT. EVALUATIONS      1
FUNCTION VALUE=   81.00000     FUNCTION VALUE/N =   81.00000
COEFFICIENTS
   5.000000
GRADIENT
   108.0000

FINAL STATISTICS :
TIME =        .340 SEC.   ITER. NO.     34   FUNCT. EVALUATIONS     35
FUNCTION VALUE=   .8261134E-15 FUNCTION VALUE/N =   .8261134E-15
COEFFICIENTS
   2.000128
GRADIENT
   .1949125E-10
      COEFFICIENT
X          2.0001
|_END
```

The solution of the equation is near X = 2. The `FUNCTION VALUE` (in this case the value reported is $f^2(X)$) and the `GRADIENT` should be very close to zero if a solution has been found.

This technique can be generalized to solve a set of equations simultaneously. For example, consider two equations with variables X1 and X2. The implicit function form in SHAZAM notation is:

X1**2 + 3*X1*X2 – 22

X2**2 + 2*X1*X2 – 21

The solution is shown in the output:

```
|_SAMPLE 1 1
|_NL 2 / NCOEF=2 SOLVE
...NOTE..SAMPLE RANGE SET TO:      1,      1
|_EQ X1**2 + 3*X1*X2 - 22
|_EQ X2**2 + 2*X1*X2 - 21
|_COEF X1 1 X2 1
   0 VARIABLES IN  2 EQUATIONS WITH   2 COEFFICIENTS
         1 OBSERVATIONS

COEFFICIENT STARTING VALUES
X1          1.0000      X2          1.0000

      100 MAXIMUM ITERATIONS, CONVERGENCE =   0.100000E-04

INITIAL STATISTICS :
TIME =      0.190 SEC.   ITER. NO.      0   FUNCT. EVALUATIONS      1
FUNCTION VALUE=   648.0000     FUNCTION VALUE/N =   648.0000
COEFFICIENTS
   1.000000       1.000000
GRADIENT
  -252.0000      -252.0000

FINAL STATISTICS :
TIME =      0.190 SEC.   ITER. NO.      8   FUNCT. EVALUATIONS     14
FUNCTION VALUE=  0.7009970E-14 FUNCTION VALUE/N =  0.7009970E-14
COEFFICIENTS
   2.000000       3.000000
GRADIENT
  0.1618480E-05  0.1900579E-05

      COEFFICIENT
X1         2.0000
X2         3.0000
|_END
```

The solution to the equations is near X1=2 and X2=3. Remember that there may be multiple answers, so you may need to experiment with different starting values.

Multinomial Logit Models

An introduction to multinomial choice models is available in Judge, Griffiths, Hill, Lütkepohl and Lee [1985, Chapter 18.3]. Suppose that J+1 alternatives are available. For the multinomial logit model consider that the probability that individual (or firm) t will select alternative j for j = 0, . . . , J is:

$$P_{t0} = 1 \Big/ [1 + \sum_{j=1}^{J} \exp(X_t'\beta_j)] \quad \text{and} \quad P_{tj} = \exp(X_t'\beta_j) \Big/ [1 + \sum_{j=1}^{J} \exp(X_t'\beta_j)] \quad \text{for } j = 1, \ldots, J$$

where X_t is a vector of variables and the β_j are vectors of unknown parameters. For individual t let Y_{tj} be a binary variable that is 1 if alternative j is chosen and 0 otherwise. The log-density for observation t can be stated as:

$$L_t = \sum_{j=1}^{J} Y_{tj}(X_t'\beta_j) - \log[\ 1 + \sum_{j=1}^{J} \exp(X_t'\beta_j)]$$

With the assumption of independence the log-likelihood function is obtained by summing the individual log-densities. Maximum likelihood estimation of this model can be implemented with the **LOGDEN** option on the **NL** command. The **EQ** statement is used to give the formula for the log-density of a single observation.

To interpret the results it is useful to evaluate how the choice probabilities change in response to changes in the independent variables. For predicted choice probabilities $\hat{P}_{tj}$ the elasticities are calculated as:

$$(X_{tk} / \hat{P}_{tj})(\partial \hat{P}_{tj} / \partial X_{tk})$$

The **DERIV** command can be used to compute the required derivatives and then the estimated elasticities can be obtained. Elasticities can be computed as weighted aggregated elasticities as described in the chapter *PROBIT AND LOGIT REGRESSION* and given in Hensher and Johnson [1981, Equation 3.44].

The SHAZAM program that follows shows the steps involved in the estimation and analysis of a multinomial logit model. It is assumed that three alternatives are available and a data file **`FIRMS.DAT`** contains binary variables *Y1* and *Y2* that indicate choices for alternatives 1 and 2. The variable calculated as 1−*Y1*−*Y2* is then 1 if the third alternative is chosen and 0 otherwise. The data file also contains independent variables *X1*, *X2* and *X3*.

The first task is to determine sensible starting values for the multinomial logit estimation. The approach illustrated here is to estimate a "null model" with constant terms only. The estimates are used to initialize starting values in the variable *BETA*. The value of the log-likelihood function for the null model is computed in the variable *LL0*. The multinomial logit model is then specified and estimated with the **NL** command. Note that the value of the log-likelihood function at the first iteration should be identical to the value of *LL0*.

A number of goodness-of-fit measures can be considered (see, for example, Maddala [1983, Chapter 3]). A measure that is easily computed is a likelihood ratio test statistic. The value of the log-likelihood function at the final iteration of the **NL** procedure is available in the temporary variable *$LLF*. The commands below show the calculation of the likelihood ratio test statistic in the variable *LR*. Predicted choice probabilities are then computed. For each observation, the alternative with the highest probability can be determined and a prediction success table can be formed. This is left to the user as an exercise. Finally, derivatives and elasticities are calculated.

The commands demonstrated here can be modified to estimate nested logit models as described in Amemiya [1985, Chapter 9]. That is, the main requirement for maximum likelihood estimation of these models in SHAZAM is to write an expression for the log-density of a single observation.

```
* Read and Generate data
read (firms.dat) y1 y2 x1 x2 x3
genr y0=1-y1-y2
?stat y1 y2 y0 / sums=stot
gen1 ntot=stot:1+stot:2+stot:3
gen1 a1=log(stot:1/stot:3)
gen1 a2=log(stot:2/stot:3)

* Value of log-likelihood function for "null" model
gen1 ll0=stot:1*a1+stot:2*a2-ntot*log(1+exp(a1)+exp(a2))
print stot ntot a1 a2 ll0

* Set starting values
dim beta 8
gen1 beta:1=a1
gen1 beta:5=a2

* Specify the parameterization of the model.
eq1: (b10+b11*x1+b12*x2+b13*x3)
eq2: (b20+b21*x1+b22*x2+b23*x3)
```

```
* Multinomial logit estimation by maximum likelihood
nl 1 / logden ncoef=8 start=beta piter=100 genrvar
eq  y1*[eq1] + y2*[eq2] -log(1+exp([eq1])+exp([eq2]))
end

* A goodness-of-fit measure is the likelihood ratio test.
* The test can be compared with a Chi-square distribution with
* 6 d.f.
gen1 lr=2*($llf-ll0)
print lr

* Calculate the predicted choice probabilities
genr p1=exp([eq1]) / (1+exp([eq1])+exp([eq2]))
genr p2=exp([eq2]) / (1+exp([eq1])+exp([eq2]))

* Calculate derivatives and elasticities
set nodoecho
do #=1,3
  deriv x# vp1=exp([eq1]) / (1+exp([eq1])+exp([eq2]))
  deriv x# vp2=exp([eq2]) / (1+exp([eq1])+exp([eq2]))
do %=1,2
genr ve=vp%*x#
?stat ve p% / sums=tot
gen1 ve#%=tot:1/tot:2
endo
endo

* Weighted Aggregate Elasticities
print ve11 ve21 ve31  ve12 ve22 ve32
```

42. SHAZAM PROCEDURES

"It will take six months or more for the colonial secretary to deal with the matter and months more before we learn of his decision. But you will not be interested in what he decides for you are to be hanged on Monday morning."

Matthew B. Begbie
Canadian Judge, 19th Century

SHAZAM provides many features to aid users who wish to write their own programs. SHAZAM PROCS require the knowledge of three commands: **PROC**, **PROCEND**, and **EXEC** as well as the concept of SHAZAM character strings.

SHAZAM *CHARACTER STRINGS*

A character string is actually a SHAZAM variable which contains a set of characters rather than numeric data. Character strings can be used at any time and are not restricted for use in SHAZAM procedures. They can simplify the use of repeated character sequences in a SHAZAM program. For example, if you always use the list of variable names: `CONSUME INCOME PRICE` in many parts of your SHAZAM program, you can define a character variable which contains these characters. Note that the character string contains only the characters and not the actual data. To define a character string, simply chose a variable name that is not used in your SHAZAM program and place a colon ":" after the name and follow it with the desired character string, as in:

```
mylist:consume income price
```

In this case the SHAZAM variable *MYLIST* is created and it contains only the characters `CONSUME INCOME PRICE`. If any spaces are included immediately following the ":" they will be included in the character string.

Next, you could use the contents of the variable *MYLIST* in any SHAZAM command by enclosing the character variable in left and right brackets as in:

```
ols [mylist] / rstat
```

SHAZAM would then interpret the above command as:

```
ols consume income price / rstat
```

If you wish to see the actual characters in the variable *MYLIST*, just print it with:

```
print mylist
```

The output from the above SHAZAM commands would look like:

```
|_MYLIST:CONSUME INCOME PRICE
|_OLS CONSUME INCOME PRICE / RSTAT
  OLS ESTIMATION
       17 OBSERVATIONS     DEPENDENT VARIABLE = CONSUME
 ...NOTE..SAMPLE RANGE SET TO:    1,   17

  R-SQUARE =     .9513     R-SQUARE ADJUSTED =     .9443
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   30.951
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   5.5634
 SUM OF SQUARED ERRORS-SSE=   433.31
 MEAN OF DEPENDENT VARIABLE =   134.51
 LOG OF THE LIKELIHOOD FUNCTION = -51.6471

 VARIABLE   ESTIMATED  STANDARD   T-RATIO         PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      14 DF  P-VALUE  CORR. COEFFICIENT  AT MEANS
 INCOME     1.0617      .2667       3.981    .001  .729      .2387      .8129
 PRICE     -1.3830     0.8381E-01  -16.50    .000 -.975     -.9893     -.7846
 CONSTANT   130.71      27.09       4.824    .000  .790      .0000      .9718

 DURBIN-WATSON = 2.0185    VON NEUMANN RATIO = 2.1447    RHO =  -.18239
 RESIDUAL SUM =  0.53291E-14  RESIDUAL VARIANCE =   30.951
 SUM OF ABSOLUTE ERRORS=   72.787
 R-SQUARE BETWEEN OBSERVED AND PREDICTED =  .9513
 RUNS TEST:    7 RUNS,    9 POSITIVE,    8 NEGATIVE, NORMAL STATISTIC = -1.2423
 |_PRINT MYLIST
  CONSUME INCOME PRICE
```

Character strings are used heavily in SHAZAM procedures to define lists of variables at execution time rather than in the procedure itself. Examples will be detailed below.

While character strings are frequently used to contain a list of variable names, they could also be used to contain any set of characters including an entire equation. For example suppose you wanted to square a set of variables, you could first define the character variable:

```
mymath:**2
genr y=x[mymath]
genr z=w[mymath]
```

This would be equivalent to:

```
genr y=x**2
genr z=w**2
```

If you later decide to divide by 3 instead of squaring the variable it would only be necessary to change one line of the program and use:

```
mymath:/3
```

and this would be equivalent to:

```
genr y=x/3
genr x=w/3
```

WRITING A SHAZAM PROCEDURE

A SHAZAM procedure is a set of SHAZAM commands contained within **PROC** and **PROCEND** commands. If the procedure is maintained in a separate file the **FILE PROC** or **FILE PROCPATH** command must be specified. The commands are executed with the SHAZAM **EXEC** command. Every procedure must have a name which has not been used as a previous variable. The procedure name is specified on the **PROC** command and also on the **EXEC** command.

The **READ** command followed by data is not allowed in a SHAZAM procedure.

A simple SHAZAM procedure is:

```
proc olsyx
  sample 1 20
  genr y=nor(1)
  genr x=nor(1)
  ols y x
procend
```

The above commands will simply define two random variables using the normal random number generator and then run an OLS regression. The procedure is executed with the **EXEC** command. Suppose you want to run the procedure three times and hence obtain three different OLS regressions since different random numbers would be generated each time. The command file would look like:

```
proc olsyx
  sample 1 20
  genr y=nor(1)
  genr x=nor(1)
  ols y x
procend
exec olsyx
```

```
exec olsyx
exec olsyx
```

You might decide that you like your *OLSYX* procedure so much that you want to keep it in your library of procedures. In that case you would simply create a file with a filename identical to the PROC name. Use a file with the name **OLSYX** and put the commands from **PROC** to **PROCEND** in the file. After the file has been saved, your SHAZAM command file only requires the commands:

```
exec olsyx
exec olsyx
exec olsyx
```

SHAZAM will search your current default directory for a file with the name **OLSYX** and load the procedure and then execute it three times.

It is also possible to use SHAZAM **DO**-loops to avoid repetitive commands as in:

```
do #=1,3
  exec olsyx
endo
```

Most users like to keep their SHAZAM procedures in a separate folder. For example, if all procedures are in the folder **C:**SHAZAM****SHAZPROCS**** then you would include the **FILE PROCPATH** command in your SHAZAM command file so that the proper folder is searched as in:

```
file procpath c:\shazam\shazprocs\
exec olsyx
exec olsyx
exec olsyx
```

Alternatively, you can use the **FILE PROC** command to load the PROC from any folder as in:

```
file proc c:\procs\olsyx
exec olsyx
exec olsyx
exec olsyx
```

Other systems should have a **FILE PROCPATH** corresponding to filename conventions for that system.

While the *OLSYX* procedure will certainly prove valuable in your research you might find that it is too restrictive because sometimes you would like to add additional variables to the regression. This is where character variables are used. However, first the *OLSYX* proc will be modified to reduce potential confusion in a SHAZAM program. It will be convenient to use an underscore "_" in variable names inside a procedure so they would not get confused with variable names outside the procedure. For example you might already have a variable with the name *Y* and it could be unexpectedly redefined if the *OLSYX* procedure were used in its current form. Hence, the procedure will use the variables *Y_* and *X_* instead of *Y* and *X*. The **ALL_** option on the **DELETE** command can be used to delete all variables with an underscore at the end. The second modification to the procedure will allow you to include additional variables and change them each time the procedure is executed. This feature will require a character string which will be given the name *MOREVARS* in the revised procedure below. In addition, a character string with the name *OPTS* will be used to pass options for the **OLS** command down to the revised *OLSYX* procedure.

```
proc olsyx
  sample 1 20
  genr y_=nor(1)
  genr x_=nor(1)
  ols y x [morevars] / [opts]
procend
```

Now consider the SHAZAM commands:

```
sample 1 20
genr z=uni(1)
genr w=uni(1)
morevars:z w
opts:anova
exec olsyx
morevars:z
opts:
exec olsyx
morevars:
opts:max
exec olsyx
```

It should be easy to figure out that the above program would be equivalent to the following three **OLS** commands:

```
ols y x z w / anova
ols y x z
```

```
ols y x / max
```

In summary, the general format for using SHAZAM procedures is:

FILE PROCPATH *pathname* or **FILE PROC** *filename*
charname: string
EXEC *proc_name*

The general format for the PROC is:

PROC *proc_name*
. . .
(SHAZAM commands)
. . .
PROCEND

CONTROLLING PROCEDURE OUTPUT

SHAZAM procedures can often become quite long and you will not want to see the commands printed when the procedure is loaded or when it is executed. It is easy to stop the procedure from printing when loaded by placing the commands **SET NOECHO** before the procedure and then include **SET ECHO** after the procedure to allow commands to appear again. In addition it is sometimes useful to begin certain commands with an "=" which suppresses the printing of that particular command. Commands that begin with "?" will suppress the output of the command. Furthermore, the **SET NODOECHO** command will suppress the echoing of commands within a **DO**-loop and the **SET NOOUTPUT** command will turn off most (but not all output). These features are used in the example below. You will want to experiment with these features to obtain desired results. Comments that begin with a single * will not be printed in the output but comments that begin with a double ** will be printed.

EXAMPLES

Square Root of a Matrix

For a given matrix A it is sometimes necessary to find the square root of A. The **SQRT( )** function on the **MATRIX** command simply takes the square root of each element of the matrix but it may be necessary to find the matrix X such that XX=A. The **SQRT()** function does not do this. Various algorithms have been proposed from time to time and they

usually involve matrix decompositions. For example, if A is a real, symmetric matrix then an eigenvalue-eigenvector decomposition can be used such that new matrices are defined where V'V=I, AV=VD, D is a diagonal matrix, A–VQV'VQV', QQ=D, and XX=A, where X=VQV'. A procedure named *SQRTA* to compute X is:

```
proc sqrta
  * [AMATRIX]=INPUT MATRIX
  * [XMATRIX]=OUTPUT MATRIX
  * GET SQRT OF A MATRIX
  matrix a_=[amatrix]
  matrix v_=eigvec(a_)
  matrix d_=eigval(a_)
  matrix q_=sqrt(d_)
  matrix x_=v_*diag(q_)*v_'
  matrix [xmatrix]=x_
  delete / all_
procend
```

However, another method is an iterative procedure described in Golub and Van Loan [1983, p.395]. This method says to simply iterate on the formula:

$$X_{k+1} = \frac{1}{2}(X_k + AX_k^{-1})$$

where the initial matrix X_0 is an identity matrix. A SHAZAM procedure to do this for up to 20 iterations and check for convergence is:

```
set noecho

proc sqrtm
  set nodoecho nooutput

  * [AMATRIX]=INPUT MATRIX
  * [XMATRIX]=OUTPUT MATRIX

  * Procedure for obtaining the square root of a matrix
  matrix a_=[amatrix]
  gen1 n_=$rows
  matrix x_=iden(n_)

  * Allow up to 20 iterations (but we probably don't need
  * that many)
  do #=1,20
    matrix x_=.5*(x_+a_*inv(x_))
    * See how close we are by first turning the matrix
    * into a vector
```

```
    matrix c_=vec(x_*x_)-vec(a_)
    gen1 nc_=$rows
    * Get the sum of squared discrepancies
    matrix cc_=c_'c_
    * If we have converged then stop the iterations
    sample 1 nc_
    endif(c_.lt.1e-15)
  endo

  atitle_: Number of iterations
  print atitle_ $do / noname
  matrix [xmatrix]=x_

  delete / all_
  set doecho output
procend

set echo
```

The input to this procedure is the matrix *A_* and the solution will be in the matrix *X_*. An example is to find the square root of the matrix:

$$\begin{bmatrix} 10 & 5 & 3 \\ 5 & 12 & 2 \\ 3 & 2 & 11 \end{bmatrix}$$

Assume that the *SQRTM* procedure is contained in the file **SQRTM.** SHAZAM will automatically search for the *SQRTM* procedure if it has not been previously defined in the command file. SHAZAM will search the directory specified by the **FILE PROCPATH** statement or look in the default directory if there is no defined PROCPATH. For example, if the file **SQRTM** were in an alternate folder such as **C:\SHAZAM\PROCS\SQRTM** then the **PROCPATH** should be defined as:

```
file procpath c:\shazam\procs\
```

The SHAZAM command file will be:

```
sample 1 3
read  a / rows=3 cols=3
10 5 3
5 12 2
3 2 11
amatrix:a
xmatrix:x
```

```
exec sqrtm
print a x

* Check the result
matrix xx=x*x
print xx
```

The output from this exercise will be:

```
 |_SAMPLE 1 3
 |_READ A / ROWS=3 COLS=3 LIST
       3 ROWS AND           3 COLUMNS, BEGINNING AT ROW       1
 |_AMATRIX:A
 |_XMATRIX:X
 |_EXEC SQRTM
 ...DO LOOP ENDED AT  #=       7
 #_         PROCEND
 |_PRINT A
    10.00000       5.000000       3.000000
    5.000000       12.00000       2.000000
    3.000000       2.000000       11.00000
  |_PRINT X
    3.036379       .7631512       .4449762
    .7631512       3.369748       .2498038
    .4449762       .2498038       3.277132
 |_* Check the result
 |_MATRIX XX=X*X
 |_PRINT XX
    10.00000       5.000000       3.000000
    5.000000       12.00000       2.000000
    3.000000       2.000000       11.00000
```

Black-Scholes Option Pricing Model

The Black-Scholes [1973] equation gives a formula for pricing call options. The formula gives the value of the call option at time t as a function of the current price of the stock, the exercise price of the option, the time to expiration, the risk free interest rate and the standard deviation of the stock's return. A formula can also be applied for pricing put options. The Black-Scholes option pricing formula can be implemented with the **CALL** and **PUT** commands as described in the chapter *FINANCIAL TIME SERIES*. A demonstration of the option pricing calculations for assets with no dividends is shown in the procedure below.

```
proc bs
  * INPUTS: [K] EXERCISE PRICE
  *         [S] STOCK PRICE
  *         [SIG] STANDARD DEVIATION
  *         [TAU] TIME TO EXPIRATION
  *         [R] RISK FREE INTEREST RATE
```

```
  * OUTPUTS: [C] CALL PRICE
  *          [P] PUT PRICE
  genr d1_=(log([s]/[k])+([r]+([sig]**2)/2)*[tau])/([sig]*sqrt([tau]))
  genr d2_=d1_-[sig]*sqrt([tau])
  genr [c]=[s]*ncdf(d1_)-[k]*exp(-[r]*[tau])*ncdf(d2_)
  genr [p]=-[s]*ncdf(-d1_)+[k]*exp(-[r]*[tau])*ncdf(-d2_)
  print [s] [k] [c] [p]
procend
```

To run the *BS* PROC, one needs only to set the inputs as illustrated in the SHAZAM commands:

```
sample 1 1
read exprice sprice
50 50
sig:.35
r:.08
tau:.25
k:exprice
s:sprice
c:call
p:put
exec bs
print [c] [p]
```

This produces the output below. Note that a listing of the PROC and all commands occur because no options were used to suppress this output.

```
|_PROC BS
|_* INPUTS: [K] EXERCISE PRICE
|_*         [S] STOCK PRICE
|_*         [SIG] STANDARD DEVIATION
|_*         [TAU] TIME TO EXPIRATION
|_*         [R] RISK FREE INTEREST RATE
|_* OUTPUTS: [C] CALL PRICE
|_*          [P] PUT PRICE
|_GENR D1_=(LOG([S]/[K])+([R]+([SIG]**2)/2)*[TAU])/([SIG]*SQRT([TAU]))
|_GENR D2_=D1_-[SIG]*SQRT([TAU])
|_GENR [C]=[S]*NCDF(D1_)-[K]*EXP(-[R]*[TAU])*NCDF(D2_)
|_GENR [P]=-[S]*NCDF(-D1_)+[K]*EXP(-[R]*[TAU])*NCDF(-D2_)
|_PRINT [S] [K] [C] [P]
|_PROCEND
|_SAMPLE 1 1
|_READ EXPRICE SPRICE
   2 VARIABLES AND        1 OBSERVATIONS STARTING AT OBS       1
|_SIG:.35
|_R:.08
|_TAU:.25
|_K:EXPRICE
|_S:SPRICE
|_C:CALL
|_P:PUT
|_EXEC BS
 _PROC BS
```

```
 _         GENR D1_=(LOG(SPRICE/EXPRICE)+(.08+(.35**2)/2)*.25)/(.35*SQRT(.25))
 _         GENR D2_=D1_-.35*SQRT(.25)
 _         GENR CALL=SPRICE*NCDF(D1_)-EXPRICE*EXP(-.08*.25)*NCDF(D2_)
 _         GENR PUT=-SPRICE*NCDF(-D1_)+EXPRICE*EXP(-.08*.25)*NCDF(-D2_)
 _         PRINT SPRICE EXPRICE CALL PUT
   50.00000       50.00000       3.969272       2.979205
 _         PROCEND
|_PRINT CALL PUT
    3.969272       2.979205
```

From the output one can see that the Black-Scholes model predicts a call option price of $3.96 and a put option price of $2.97.

It is also possible to write a procedure to compute the Black-Scholes implied volatility given the current stock price, strike price, interest rate, time to maturity and call price. This requires an iterative solution algorithm. The computational details are described in the chapter *FINANCIAL TIME SERIES*. The SHAZAM PROC is:

```
proc impvol

  set nodoecho
  * program to compute black-scholes implied volatility
  * inputs: [s] stock price
  *         [k] strike price
  *         [r] interest rate, example .12
  *         [tau] time to maturity in years, example .25
  *         [c]  call price

  genr starts_=abs(log([s]/[k])+[r]*[tau])*(2/[tau])
  genr sig_=sqrt(starts_)

  ?do #=1,20
    genr d1_=(log([s]/[k])+([r]+sig_*sig_/2)*[tau])/(sig_*sqrt([tau]))
    genr fn_=exp(-d1_*d1_/2)/sqrt(2*3.14159)
    genr fprime_=[s]*sqrt([tau])*fn_
    genr d2_=d1_-sig_*sqrt([tau])
    genr c_=[s]*ncdf(d1_)-[k]*exp(-[r]*[tau])*ncdf(d2_)
    genr sig_=sig_-(c_-[c])/fprime_
    ?endif(abs(c_-[c]).lt.0.00001)
  endo

  print sig_ c_

procend
```

To run the PROC use the SHAZAM commands:

```
* An Example
sample 1 1
read stock strike call / list
     100   125     2
s:stock
k:strike
c:call
r:.12
tau:.25
exec impvol

* Another Example
read stock strike call / list
     122   125     3
s:stock
k:strike
c:call
r:.1
tau:.25
exec impvol
```

The output is:

```
|_* An Example
|_SAMPLE 1 1
|_READ STOCK STRIKE CALL / LIST
   3 VARIABLES AND        1 OBSERVATIONS STARTING AT OBS       1
      STOCK          STRIKE         CALL
   100.0000       125.0000       2.000000
|_S:STOCK
|_K:STRIKE
|_C:CALL
|_R:.12
|_TAU:.25
|_EXEC IMPVOL
 _PROC IMPVOL
 _        SET NODOECHO
      SIG_           C_
  0.4034792       2.000000
|_* Another Example
|_READ STOCK STRIKE CALL / LIST
   3 VARIABLES AND        1 OBSERVATIONS STARTING AT OBS       1

      STOCK          STRIKE         CALL
   122.0000       125.0000       3.000000
|_S:STOCK
|_K:STRIKE
|_C:CALL
|_R:.1
|_TAU:.25
|_EXEC IMPVOL
      SIG_           C_
  0.1215579       3.000000
```

```
| STOP
```

Generating Multivariate Random Numbers

This example shows how to generate random numbers from a bivariate normal distribution for a given covariance matrix. The method is explained in Judge et al. [1988, Chapter 11A] and the corresponding chapter in the *Judge Handbook*. The *MULTI* PROC is included directly in the command file in this example.

```
set noecho

proc multi

  * INPUTS: [SIGMA] DESIRED COVARIANCE MATRIX
  *         [N] SAMPLE SIZE
  * OUTPUT: [MRAN] MATRIX OF RANDOM NUMBERS
  * Do a Cholesky factorization of the Sigma Matrix
  matrix p_=chol([sigma])
  ?gen1 k_=$rows
  print p_

  * Check it out
  matrix check_=p_*iden(k_)*p_'
  print check_

  * Now use the P_ matrix to generate MRAN
  sample 1 [n]
  matrix e_=nor([n],k_)
  matrix [mran]=e_*p_'

  * Check the sample covariance of the orignal independent
  * random numbers
  stat e_ / pcov

  * Now check the sample of correlated random numbers
  stat [mran] / pcov
  delete / all_

procend

set echo

* End of Proc
* Beginning of Calling Program
set ranfix
```

```
* READ IN A SIGMA MATRIX AND GENERATE 100 RANDOM VECTORS
read s / rows=2 cols=2 list
5 2
2 8

* Specify the Inputs to the MULTI PROC
sigma:s
n:100
mran:u
exec multi

* Print the matrix
print u
```

The output is:

```
|_SET NOECHO
|_* End of Proc
|_* Beginning of Calling Program
|_* READ IN A SIGMA MATRIX AND GENERATE 100 RANDOM VECTORS
|_READ S / ROWS=2 COLS=2 LIST
      2 ROWS AND           2 COLUMNS, BEGINNING AT ROW       1
...SAMPLE RANGE IS NOW SET TO:         1         2
    S
    2 BY     2 MATRIX
   5.000000       2.000000
   2.000000       8.000000
|_* Specify the Inputs to the MULTI PROC
|_SIGMA:S
|_N:100
|_MRAN:U
|_EXEC MULTI
 _PROC MULTI
 _         MATRIX P_=CHOL(S)
 _         ?GEN1 K_=$ROWS
 _         PRINT P_
    P_
   2.236068
  0.8944272       2.683282
 _         MATRIX CHECK_=P_*IDEN(K_)*P_'
 _         PRINT CHECK_
    CHECK_
    2 BY     2 MATRIX
   5.000000       2.000000
   2.000000       8.000000
 _         SAMPLE 1 100
 _         MATRIX E_=NOR(100,K_)
 _         MATRIX U=E_*P_'
 _         STAT E_ / PCOV
NAME         N    MEAN         ST. DEV      VARIANCE     MINIMUM      MAXIMUM
...NOTE...TREATING COLUMNS OF E_       AS VECTORS
E_           100  0.94483E-01  0.96477      0.93078      -2.9865       2.4730
E_           100  0.11050      0.99368      0.98740      -2.0836       2.6220

 COVARIANCE MATRIX OF VARIABLES -      100 OBSERVATIONS
```

```
E_         0.93078
E_        -0.75042E-01  0.98740
             E_            E_
 _         STAT U / PCOV
NAME         N   MEAN         ST. DEV       VARIANCE      MINIMUM       MAXIMUM
...NOTE...TREATING COLUMNS OF U       AS VECTORS
U           100  0.21127       2.1573        4.6539       -6.6781        5.5299
U           100  0.38100       2.7375        7.4937       -6.3243        6.4619

 COVARIANCE MATRIX OF VARIABLES -      100 OBSERVATIONS
U           4.6539
U           1.4113       7.4937
              U            U
 _         DELETE / ALL_
 _         PROCEND
```

```
|_* Print the matrix
|_PRINT U
    U
  100 BY     2 MATRIX
 -0.6990535       2.138877
  -2.902652      0.6931440
 -0.6460146       3.504951
  0.4849795      -1.069619
   2.435809       5.103048
   etc.
```

43. SUMMARY OF COMMANDS

"The Sun never sets on the SHAZAM empire."
Kenneth J. White
1980

The following is a list of all the available SHAZAM commands and their available options. The underlined letters of each command and option are the acceptable abbreviations.

ARIMA *var*	ALL, DN, GRAPHAC, GRAPHDATA, GRAPHFORC, GRAPHPAC, GRAPHRES, IAC, LOG, NOCONSTANT, NOWIDE, PITER, PLOTAC, PLOTDATA, PLOTFORC, PLOTPAC, PLOTRES, RESTRICT, START, WIDE, ACF=, BEG=, END=, COEF=, COV=, FBEG=, FCSE=, FEND=, ITER=, NAR=, NDIFF=, NLAG=, NLAGP=, NMA=, NSAR=, NSDIFF=, NSMA=, NSPAN=, PACF=, PREDICT=, PSI=, RESID=, SIGMA=, START=, STDERR=, STEPSIZE=, TESTSTAT=, TRATIO=
AUTO *depvar indeps*	ANOVA, DLAG, DN, DROP, DUMP, GF, GS, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, MAX, MISS, ML, NOCONSTANT, NOPITER, NOWIDE, PAGAN, PCOR, PCOV, RESTRICT, RSTAT, WIDE, BEG=, END=, COEF=, CONV=, COV=, GAP=, ITER=, NMISS=, NUMARMA=, ORDER=, PREDICT=, RESID=, RHO=, SRHO=, STDERR=, TRATIO=.
AXISFMT *statement*	No options.
BAYES *options*	NOANTITHET, NORMAL, PSIGMA, DF=, NSAMP=, OUTUNIT=.
BOX *depvar indeps*	ACCUR, ALL, ANOVA, AUTO, DN, DUMP, FULL, GF, LIST, MAX, NOCONSTANT, PCOR, PCOV, RESTRICT, RSTAT, TIDWELL, UT, BEG=, END=, COEF=, COV=, LAMBDA=, LAME=, LAMI=, LAMS=, PREDICT=, RESID=, RHO=.

CALL *vars*	AMERICAN, BLACK, EQUAL, IMPVOL, BARRIER=, BEG=, END=, DIVIDEND=, NUMTIME=, OPTIONP=, PREDICTP=, PREDICTV=, RISKFREE=, SIGMA=, STRIKEPRICE=, TIME=, UP=, DOWN=
CHECKOUT	No options.
COEF *names values*	No options.
COINT *vars*	DN, DUMP, LOG, MAX, BEG=, END=, NDIFF=, NLAG=, RESID=, SIGLEVEL=, TESTSTAT=, TYPE=
COMPRESS	No options.
CONFID *var1 var2*	APPEND, AXIS, GRAPH, HOLD, NOAXIS, NOBLANK, NOFPLOT, NOMID, NORMAL, NOTPLOT, NOWIDE, SYMBOL, WIDE, COEF1=, COEF2=, COMMFILE=, COVAR12=, DEVICE=, DF=, FCRIT=, OUTPUT=, POINTS=, PORT=, TCRIT=, VAR1=, VAR2=, XMAX=, XMIN=, YMAX=, YMIN=.
COPY *fromvar(s) tovar*	FCOL=, FROW=, TCOL=, TROW=.
DELETE *vars*	ALL, ALLDATA.
DEMO	START.
DERIV *var res=equation*	No options.
DIAGNOS *options*	ACF, BACKWARD, BOOTLIST, CHOWTEST, CTEST, GRAPH, HANSEN, HET, JACKKNIFE, LIST, MAX, NORECEST, NORECRESID, NOWHITE, NOWIDE, RECEST, RECRESID, RECUR, RESET, WHITE, WIDE, BOOTSAMP=, BOOTUNIT=, CHOWONE=, GQOBS=, MHET=, RECUNIT=, SIGLEVEL=.
DIM *var size*	No options.
DISPLAY *option*	Options same as for the **SET** command.

DISTRIB *vars*	INVERSE, LLF, NOLIST, ACCURACY=, BEG=, END=, BIGN=, BIGX=, C=, CDF =, CRITICAL=, DF=, DFVEC=, DF1=, DF2=, EIGENVAL=, H=, K=, KURTOSIS=, LAMBDA=, LIMIT=, MEAN=, N=, NEIGEN=, NONCEN=, P=, PDF=, Q=, S=, SKEWNESS=, TYPE=, V=, VAR=, X=.
DO *dovar=beg,end,inc*	No options.
DUMP *option*	ADDCOM, ASCII, DATA, DATCOM, FCOM, GENCOM, INPCOM, IOCOM, KADD, LODCOM, MACOM, NLCOM, OCOM, OLSCOM, OPTCOM, OSCOM, RANCOM, SCNCOM, SYSCOM, TEMCOM, VCOM, VLCOM, VNAME, VPLCOM, VTCOM, VTECOM, VTICOM.
END	No options.
ENDIF *(equation)*	No options.
ENDO	No options.
EQ *equation*	No options.
EXEC *proc_name*	No options.
FC *depvar indeps*	AFCSE, BLUP, IBLUP, DYNAMIC, FIXED, GF, LIST, MAX, MEANPRED, NOCONSTANT, PERCENT, UPPER, BEG=, END=, COEF=, CSNUM=, ESTEND=, FCSE=, LIMIT=, MODEL=, NCROSS=, ORDER=, POOLSE=, PREDICT=, RESID=, RHO=, SRHO=.
FILE *option filename*	*Option* can be unit from 11-49 or the keywords: CD, CLOSE, DELETE, HELPDEMO, INPUT, KEYBOARD, LIST, OUTPUT, PATH, PLOTPATH, PRINT, PROCPATH, PWD, SCREEN or TEMP.
FLS *depvar indeps*	GRAPH, MAX, NOCONSTANT, PCOEF, BEG=, END=, COEF=, DELTA=, PREDICT=, RESID=.

FORMAT *statement*	No options.
FUZZY *vars*	DUMP, GRAPHDATA, GRAPHRULE, MEDIAN, NOLIST, NOPMATRIX, NOSTANDARD, PASSOC, PBREAK, BEG=, END=, CMA=, DEGREES=, PREDICT=, RMA=, RULES=, WEIGHT=.
GENR *newvar=equation*	No options.
GEN1 *equation*	No options.
GLS *depvar indeps*	ANOVA, BLUP, DLAG, DN, DUMP, FULLMAT, GF, HETCOV, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, MAX, NOCONSTANT, NOMULSIGSQ, PCOR, PCOV, RSTAT, UT, BEG=, END=, COEF=, COV=, OMEGA=, OMINV=, PMATRIX=, PREDICT=, RESID=, STDERR=, TRATIO=.
GME *depvar indeps*	DEVIATION, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, NOCONSTANT, PCOV, RSTAT, BEG=, END=, COEF=, CONV=, COV=, ITER=, LOGEPS=, PITER=, PREDICT=, QPRIOR=, RESID=, START=, STDERR=, TRATIO=, UPRIOR=, VENTROPY=, ZENTROPY=.
GRAPH *depvars indep*	APPEND, AXIS, AXISFMT, HISTO, KEY, LINE, LINEONLY, NOAXIS, NOKEY, RANGE, TIME, TIMEFMT, WIDE, BEG=, END=, COMMFILE=, DATAFILE=, DEVICE=, GROUPS=, OUTPUT=, PORT=.
HELP *command*	ARIMA, AUTO, AXISFMT, BAYES, BOX, CALL, CHECKOUT, COEF, COINT, COMPRESS, CONFID, COPY, DELETE, DEMO, DERIV, DIAGNOS, DIM, DISPLAY, DISTRIB, DO, DUMP, END, ENDIF, ENDO, EQ, ERROR, FC, FILE, FLS, FORMAT, FUZZY, GENR, GEN1, GLS, GME, GRAPH, HELP, HET, IF, IF1, INDEX, INST, INTEG, LAMBDA, LOGIT, LP, MATRIX, MENU, MLE, NAMES, NL, NONPAR, OLS, PAR, PAUSE, PC, PLOT, POOL, PORTFOLIO, PRINT, PROBIT, PUT,

	READ, RENAME, RESTRICT, REWIND, ROBUST, SAMPLE, SET, SIZE, SKIPIF, SMOOTH, SORT, STAT, STOCKGRAPH, STOP, SYSTEM, TEST, TIME, TIMEFMT, TITLE, TOBIT, WRITE, 2SLS.
HET *depvar indeps (exogs)*	DUMP, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, MAX, NOCONSTANT, NOWIDE, NUMERIC, OPGCOV, PCOR, PCOV, PRESAMP, RSTAT, WIDE, ARCH=, ARCHM=, BEG=, END=, COEF=, CONV=, COV=, GARCH=, GMATRIX=, ITER=, MACH=, METHOD=, MODEL=, PITER=, PREDICT=, RESID=, START=, STDERR=, STDRESID=, STEPSIZE=, TRATIO=.
IF *(expression)*	No options.
IF1 *(expression)*	No options.
INDEX *p1 q1...pn qn*	ALTERN, CHAIN, EXPEND, NOALTERN, NOLIST, BASE=, BEG=, END=, DIVISIA=, FISHER=, LASPEYRES=, PAASCHE=, QDIVISIA=, QFISHER=, QLASPEYRES=, QPAASCHE=.
INST *dep ind (inst)*	Options same as for the **2SLS** command.
INTEG *var lo up res=equ*	No options.
LAMBDA *var=value*	No options.
LOGIT *depvar indeps*	DUMP, LIST, LOG, MAX, NOCONSTANT, NONORM, PCOR, PCOV, RSTAT, BEG=, END=, COEF=, CONV=, COV=, INDEX=, ITER=, PITER=, PREDICT=, STDERR=, TRATIO=, WEIGHT=.
LP *c A b*	DUMP, MIN, DSLACK=, DUAL=, ITER=, PRIMAL=, PSLACK=.
MATRIX *newmat=equation*	No options.
MENU	No options.

MLE *depvar indeps*	ANOVA, DUMP, GF, LININV, LINLOG, LIST, LM, LOGINV, LOGLIN, LOGLOG, MAX, NOCONSTANT, NONORM, PCOR, PCOV, RSTAT, BEG=, END=, COEF=, CONV=, COV=, IN=, OUT=, ITER=, METHOD=, PREDICT=, PITER=, RESID=, STDERR=, TRATIO=, TYPE=, WEIGHT=.
NAMEFMT *statement*	No options.
NAMES *options*	LIST
NL *neq (exogs)*	ACROSS, AUTO, DRHO, DUMP, EVAL, GENRVAR, LIST, LOGDEN, MINFUNC, MAXFUNC, NOCONEXOG, NOPSIGMA, NUMCOV, NUMERIC, OPGCOV, PCOV, RSTAT, SAME, SOLVE, AUTCOV= BEG=, END=, COEF=, CONV=, COV=, GMM=, GMMOUT=, IN=, ITER=, METHOD=, DN, HYBRID SAITER= SACONV= SAUPPER= SALOWER= SANEPS= SANS= SANT= SATRF= SAUPFAC= SALOWFAC=, NCOEF=, ORDER=, OUT=, PITER=, PREDICT=, RESID=, SIGMA=, START=, STDERR=, STEPSIZE=, TRATIO=, ZMATRIX=.
NONPAR *depvar indeps*	DENSITY, GRAPH, LIST, PCOEF, BEG=, END=, BRHO=, COEF=, DELTA=, FCSE=, HATDIAG=, INCOVAR=, ITER=, METHOD=, PREDICT=, RESID=, RWEIGHTS=, SIGMA=, SMATRIX=, SMOOTH=.
OLS *depvar indeps*	ANOVA, AUXRSQR, DFBETAS, DLAG, DN, DUMP, DWPVALUE, GF, GRAPH, HETCOV, INFLUENCE, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, MAX, NOCONSTANT, NOMULSIGSQ, NONORM, PCOR, PCOV, PIL, PLUSH, REPLICATE, RESTRICT, RSTAT, UT, WIDE, AUTCOV=, BEG=, END=, COEF=, COV=, FE=, FX=, HATDIAG=, IDVAR=, INCOEF=, INCOVAR=, INDW=, INSIG2=, METHOD=, NPOP=, PCINFO=, PCOMP =, PE=, PX=, PREDICT=, RESID=, RIDGE=, STDERR=, TRATIO=, WEIGHT=.

PAR *number*	No options.
PAUSE	No options.
PC *vars*	COR, LIST, MAX, PCOLLIN, PEVEC, PFM, PRM, RAW, SCALE, BEG=, END=, EVAL=, EVEC=, MAXFACT=, MINEIG=, NC=, PCINFO=, PCOMP=.
PLOT *depvars indep*	ALTERNATE, HISTO, HOLD, NOBLANK, NOPRETTY, NOSAME, NOWIDE, PAUSE, RANGE, SAME, TIME, WIDE, BEG=, END=, GROUPS=, SYMBOL=, XMAX=, XMIN=, YMAX=, YMIN=.
POOL *depvar indeps*	ANOVA, AR1, CORCOEF, DLAG, DN, DUMP, FIXED, FULL, GF, HETCOV, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, MAX, MULSIGSQ, NOCONSTANT, OLS, PCOR, PCOV, RESTRICT, RSTAT, SAME, UT, BEG=, END=, COEF=, CONV=, COV=, CSINDEX=, ITER=, NC=, NCROSS=, NTIME=, PREDICT=, RESID=, RHO=, STDERR=, TRATIO=.
PORTFOLIO *vars*	EQUALWEIGHT, GRAPHDATA, GRAPHFRONT, GRAPHLINE, INRATES, LIST, PFRONTIER, SHARES, WEIGHTS, WIDE, BEG=, END=, INDEX=, RETURNS=, RISKFREE=, RISKS=.
PRINT *vars*	BYVAR, FORMAT, NEWLINE, NEWPAGE, NEWSHEET, NONAMES, NOWIDE, WIDE, BEG=, END=.
PROBIT *depv indeps*	DUMP, LIST, LOG, MAX, NOCONSTANT, NONORM, PCOR, PCOV, RSTAT, BEG=, END=, COEF=, CONV=, COV=, IMR=, INDEX=, ITER=, PITER=, PREDICT=, STDERR=, TRATIO=, WEIGHT=.
PROC *proc_name*	No options.
PROCEND	No options.

PUT *vars*	Options same as for the **CALL** command.
QP *c Q A b*	CHOLSPEC DUMP MIN NEGDEF PDUAL UNCONSTR CONV= DIAGR= DUAL= IFACT= ITER= LAGRANGE= LOWER= LOWSCAL= MEQ= METHOD= PRIMAL= UPPER= UPSCAL= ZEROTOL=
READ *(unit) vars*	BINARY, BYVAR, CLOSE, DB, DIF, EOF, FORMAT, LIST, NAMES, NOREWIND, REWIND, BEG=, END=, CHARVARS=, COLS=, ROWS=, SKIPLINES=.
RENAME *old new*	No options.
RESTRICT *equation*	No options.
REWIND *unit*	No options.
ROBUST *depvar indeps*	FIVEQUAN, GASTWIRT, GRAPH, LAE, LININV, LINLOG, LIST, LOGINV, LOGLIN, LOGLOG, MAX, NOCONSTANT, PCOR, PCOV, RSTAT, TUKEY, UNCOR, BEG=, END=, COEF=, CONV, COV=, DIFF=, ITER=, MULTIT=, PREDICT=, RESID=, STDERR=, THETA=, THETAB=, THETAE=, THETAI=, TRATIO=, TRIM=.
SAMPLE *beg end beg end*	No options.
SET *option*	BATCH, BYVAR, CC, CONTINUE, CPUTIME, DELETE, DOECHO, DUMP, ECHO, GRAPH, LASTCOM, LCUC, MAX, NOBYVAR, NOCC, NOCONTINUE, NODELETE, NOGRAPH, NOLCUC, NOSKIP, NOOUTPUT, NOSAMPLE, NOSCREEN, NOSTATUS, NOWARN, NOWARNMISS, NOWARNSKIP, NOWIDE, OPTIONS, OUTPUT, PAUSE, RANFIX, SAMPLE, SCREEN, SKIP, SKIPMISS STATUS, TALK, TERMINAL, TIMER, TRACE, WARN, WARNSKIP, WARNMISS, WIDE, COMLEN=, MAXCOL=, MISSVALU=, OUTUNIT=, RANSEED=.

SIZE *maximum*	No options.
SKIPIF*(expression)*	No options.
SMOOTH	ADDITIVE, ARITH, CENTRAL, NOCENTRAL, HPFILTER, BEG=, END=, EMAVE=, LAMBDA=, MAVE=, NMA=, NSPAN=, SAMAVE=, SFAC=, WEIGHT=.
SORT *sortvar vars*	DESC, LIST, BEG=, END=.
STAT *vars*	ALL, ANOVA, BARTLETT, DN, MATRIX, MAX, PCOR, PCOV, PCP, PCPDEV, PFREQ, PMEDIAN, PRANKCOR, REPLICATE, SAMEOBS, SAMPSIZE=, WIDE, BEG=, END=, COR=, COV=, CP=, CPDEV=, MAXIM=, MEAN=, MEDIANS=, MINIM=, MODES=, NPOP=, RANKCOR=, STDEV=, STEMPLOT=, SUMS=, VAR=, WEIGHT=.
STOCKGRAPH *vars*	AXISFMT, EMA, GRAPHDATA, GRAPHMA, GRAPHMACD, GRAPHVOL, LIST, SOMA, WIDE, BEG=, END=, BOLLINGER=, MALONG=, MAMACD=, MASHORT=.
STOP	No options.
SYSTEM *neq exogs*	DN, DUMP, FULL, GF, LIST, MAX, NOCONSTANT, NOCONEXOG PCOR, PCOV, PINVEV, PSIGMA, RESTRICT, RSTAT, COEF=, COEFMAT=, CONV=, COV=, IN=, OUT=, ITER=, PITER=, PREDICT=, RESID=, SIGMA=, STDERR=, TRATIO=.
TEST *equation*	No options.
TIME *beg freq var*	No options.
TIMEFMT *statement*	No options.
TITLE *title*	No options.

TOBIT *depvar indeps*	DUMP, LIST, MAX, NEGATIVE, NOCONSTANT, NONORM, PCOR, PCOV, UPPER, BEG=, END=, COEF=, CONV=, COV=, INDEX=, ITER=, LIMIT=, PITER=, PREDICT=, STDERR=, TRATIO=, WEIGHT=.
WRITE*(unit) vars*	APPEND, BINARY, BYVAR, CLOSE, DB, DIF, FORMAT, NAMES, NONAMES, NOREWIND, REWIND, WIDE, BEG=, END=.
2SLS *depvar rhsvars (exogs)*	DN, DUMP, GF, LIST, MAX, NOCONSTANT, NOCONEXOG, PCOR, PCOV, RESTRICT, RSTAT, BEG=, END=, COEF=, COV=, PREDICT=, RESID=, STDERR=, TRATIO=.

44. NEW FEATURES IN SHAZAM

"Everything that can be invented has been invented."
Charles H. Duell
U.S. Patent Office, 1899

There are many differences between SHAZAM Versions. This appendix itemizes a few of these differences. SHAZAM is a continuously expanding program with many new capabilities.

VERSION 5.0

Before Version 5.0 was released, SHAZAM was completely rewritten to make it a modern and flexible package. Users of Versions 1.0-4.6 found that they had entered a new world of econometric computing. A matrix programming language had been added and all procedures had been rewritten to provide the flexibility that researchers require. Versions prior to Version 5.0 are called ***OLD-OLD* SHAZAM** and should no longer be used by anyone except Econometric Historians.

VERSION 5.1

The following are some of the major changes and additions to Version 5.1 of SHAZAM:

COMMAND	*NEW AVAILABLE OPTIONS*
DISTRIB	**LLF**, **S=** and **V=**
GENR	**MOD(X,Y)**
MLE	**LM**
OLS	**DFBETAS**, **INFLUENCE**, **REPLICATE**, **HATDIAG=** and **UT=**
PROBIT /LOGIT	**NONORM**
SET	**NOWIDE**, **WIDE** and **RANSEED=**
STAT	**WEIGHT=**
TOBIT	**NONORM**

VERSION 6.0

A large number of changes were made to SHAZAM between Versions 5.1 and 6.0. Version 6.0 SHAZAM is capable of computing Box-Jenkins Models (ARIMA) Time-Series models, Bayesian Inequality Restrictions, Confidence Intervals and Ellipses, and Robust Regressions. It can also perform a variety of Regression Diagnostic Tests including many Heteroskedasticity Tests, Recursive Residuals, CUSUM tests and Specification Error Tests as well as Jackknife and Bootstrap estimates. Furthermore, many new options were developed for existing commands. The following are some of the major new commands and options found in Version 6.0 of SHAZAM:

COMMAND	*NEW AVAILABLE OPTIONS*
ARIMA	New Command: *Identification Phase:* **ALL , NOWIDE**, **PLOTAC**, **PLOTDATA**, **PLOTPAC**, **WIDE**, **BEG= END=**, **NDIFF=**, **NLAG=**, **NLAGP=**, **NSDIFF=** and **NSPAN=** *Estimation Phase:* **DN**, **NOCONSTANT**, **PITER**, **PLOTRES**, **START**, **BEG= END=**, **COEF=**, **ITER= , NAR=**, **NDIFF=**, **NMA=**, **NSAR=**, **NSDIFF=**, **NSMA= , NSPAN=** and **PREDICT=** *Forecasting Phase:* **LOG**, **NOCONSTANT**, **PLOTFORC**, **COEF=**, **FBEG= FEND=**, **NAR=**, **NDIFF=**, **NMA=**, **NSAR=**, **NSDIFF=**, **NSMA=**, **NSPAN=**, **PREDICT=** and **RESID=**
AUTO	**NOWIDE**, **PAGAN** and **WIDE**
BAYES	New Command: **NOANTHITHET**, **NORMAL**, **NSAMP=** and **OUTUNIT=**
CONFID	New Command: **EGA**, **GRAPHICS**, **HERCULES**, **HOLD**, **NOBLANK**, **NOWIDE**, **SYMBOL**, **TOSHIBA** and **WIDE**
DIAGNOS	New Command: **ACF**, **BACKWARD**, **CHOWTEST**, **HET**, **JACKKNIFE**, **LIST**, **MAX**, **NOWIDE**, **RECEST**, **RECRESID**, **RECUR**, **RESET**, **WIDE**, **GQOBS=**, **RECUNIT=** and **SIGLEVEL=**
DISTRIB	**LLF**, **H=**, **P= N=**, **S= V=** and **TYPE=BINOMIAL**
FC	**FCSE=**

MATRIX	**Concatenation (|), SYM(***matrix***), TRI(***matrix* **), VEC(***matrix***)** and **VEC(***matrix,nrows***)**
MENU	Gives the list of available commands in SHAZAM while in interactive mode.
NL	**LOGDEN**, **NUMCOV**, **START=**, **STDERR=**, **STEPSIZE=** and **TRATIO=**
OLS	**GF** and **INCOVAR=**
PLOT	**ALTERNATE** , **EGA**, **GRAPHICS**, **HERCULES** and **TOSHIBA**
POOL	**MULSIGSQ**, **CORCOEF=** and **RHO=**
PC	**PCOLLIN**, **RAW** and **SCALE**
PROBIT / LOGIT	**IMR=** and **WEIGHT=**
ROBUST	New command: **FIVEQUAN**, **GASTWIRT**, **LAE**, **LINLOG**, **LIST**, **LOGLIN**, **LOGLOG**, **MAX**, **PCOR**, **PCOV**, **RSTAT**, **TUKEY** , **UNCOR**, **BEG= END=**, **COEF=**, **CONV=**, **COV=**, **DIFF=** , **ITER=**, **MULTIT=**, **RESID=**, **STDERR=**, **THETA=**, **THETAB=**, **THETAE=**, **THETAI=**, **TRATIO=** and **TRIM=**
SAMPLE	Formerly called **SMPL**
SYSTEM	**COEF=**, **COEFMAT=**, **COV=**, **PITER=** and **SIGMA=**
TOBIT	**NONORM**
2SLS	**COV=**

VERSION 6.1

Further additions were made after Version 6.0 to make SHAZAM Version 6.1 a more powerful program than ever before. The Macintosh version of SHAZAM now supports the Macintosh interface and graphics, and many other substantial changes have been made. Some of the major new options are:

COMMAND	*NEW AVAILABLE OPTIONS*
AUTO	**DLAG**
CONFID	**GOAWAY, PAUSE, PRINT** and Macintosh **GRAPHICS**
DIAGNOS	**BOOTLIST**, **NORECEST**, **NORECRESID**, **BOOTSAMP=** and **MHET=**

DISPLAY	**CPUTIME**, **SCREEN** and **WARNSKIP**
DLAG	Command is no longer needed. Distributed lags are now specified on **AUTO**, **BOX**, **GLS**, **OLS** and **POOL** commands directly. The command is redesigned to make Almon Lag estimation even easier.
FC	**AFCSE**, **CSNUM=**, **ESTEND=**, **LIMIT=** and **POOLSE=**
FILE	Available for many machines: **CLOSE**, **INPUT**, **LIST**, **OUTPUT** and **SCREEN** Available for the Macintosh only: **PRINT**
GLS	**DLAG**
MATRIX	More flexible definition of matrix multiplication added.
NL	**GENRVAR** and **OPGCOV**
OLS	**DLAG**
PLOT	**GOAWAY, PAUSE, PRINT** and Macintosh **GRAPHICS**
POOL	**DLAG**, **MULSIGSQ** and **PCOV**
READ	**CLOSE** and **SKIPLINES=**
SET	**NOSCREEN**, **NOWARNSKIP** and **WARNSKIP**
STAT	**SUMS=**
WRITE	**CLOSE**

VERSION 6.2

Version 6.2 includes several new options on existing commands and a new command to estimate Heteroskedastic and ARCH models as well as new commands to compute derivatives and integrals. A new Macintosh interface has been developed. The OS/2 Version has been released and includes Presentation Manager Scrollable Windows and Graphics. The OS/2 Version allows massive amounts of virtual memory not available in the DOS version and allows multitasking and many other features. New options are:

COMMAND	*NEW AVAILABLE OPTIONS*
AUTO	**NUMARMA=**
CONFID	**VGA**
DERIV	New command: *var resultvar = equation*

DISTRIB	**BIGN=**, **BIGX=** and **TYPE=HYPERGEO**
FILE	**KEYBOARD** and **PATH**
HELP	**HET**, **DERIV** and **INTEG**
HET	New command: **DUMP**, **LIST**, **MAX**, **NOCONSTANT**, **NOWIDE**, **NUMERIC**, **OPGCOV**, **PCOR**, **PCOV**, **PRESAMP**, **RSTAT**, **WIDE**, **ARCH=**, **ARCHM=**, **BEG= END=**, **COEF=**, **CONV=**, **COV= , GARCH=**, **ITER=**, **MACH=**, **METHOD=**, **MODEL=**, **PITER=**, **PREDICT=**, **RESID=**, **START=**, **STDERR=**, **STDRESID= , STEPSIZE=** and **TRATIO=**
INTEG	New command: *var lower upper resultvar = equation*
MATRIX	**FACT(***matrix***)**
NL	**MINFUNC**, **MAXFUNC** , **SOLVE** and **ZMATRIX=**
OLS	**AUXRSQR** and **AUTCOV=**
PLOT	**VGA**
READ	**DIF** and **TSP**
SET	**COLOR**, **CPUTIME**, **DELETE**, **LCUC**, **NOCOLOR**, **NODELETE**, **NOLCUC**, **NOOUTPUT** , **OUTPUT** and **PAUSE**
WRITE	**DIF**

VERSION 7.0

Version 7.0 offers a variety of new econometric techniques including a new command for Unit Roots and Cointegration testing and new options for model estimation by Nonlinear Two Stage Least Squares and Nonlinear Three Stage Least Squares and Generalized Method of Moments. *$ERR* is a new Temporary Variable to give Error Codes. An important new feature is the capability to program SHAZAM procedures. An interface to the GNUPLOT package provides high quality graphics. In addition, system commands can be entered from a SHAZAM command prompt on some versions. A large number of minor improvements have also been made.

SHAZAM is maintained as a machine portable system and versions are now available for the following operating systems: DOS Extended Memory Version for 80386 and 80486 PC Computers, the NeXT computer, WINDOWS, OS/2 Version 2.0 and the SUN SPARCstation (in addition to operating systems supported by previous SHAZAM versions).

Some of the new options are :

COMMAND	*NEW AVAILABLE OPTIONS*
ARIMA	*Identification phase :* **GNU**, **IAC**, **ACF=**, **PACF=** and **TESTSTAT=** *Estimation phase:* **GNU**, **RESTRICT**, **ACF=**, **COV=**, **START=**, **STDERR=**, **STEPSIZE=**, **TESTSTAT=** and **TRATIO=** *Forecasting phase:* **GNU**, **FCSE=** and **SIGMA=** An interface to the GNUPLOT package is available to provide high quality graphics. The options are: **GNU**, **COMMFILE=**, **DEVICE=**, **OUTPUT=** and **PORT=** Program revisions were made including changes to some calculations to give consistency with other SHAZAM commands. Also, the estimation algorithm was improved to provide greater accuracy. As a result, output from the **ARIMA** command may give slightly different answers than previous SHAZAM versions.
AUTO	**NOPITER** Tests for autocorrelation after correcting for autocorrelation.
COINT	New command for Cointegration and Unit Roots: **DN**, **DUMP**, **LOG**, **MAX**, **BEG=**, **END=**, **NDIFF=**, **NLAG=**, **TESTSTAT=** and **TYPE=.**
COMPRESS	Improved algorithm.
CONFID	**GNU**
DELETE	**ALL_**
DISTRIB	**TYPE=(BURRII, BURRIII, BURRXII, DAVIES), DFVEC=, K=, LAMBDA=, LIMIT=**
DO	Up to 8 levels are now allowed.
EXEC	A new command to execute SHAZAM procedures.
FILE	**PROCPATH, HELPDEMO, PROC**
GEN1	Calculator Mode

HET	**GMATRIX=** Exogenous variables may be included in the ARCH variance equation. Some changes were made to the estimation algorithm so that results may be different from previous versions.
IF1	**IF** command for the first observation only.
LP	New command for Linear Programming: **DUMP, MIN, PRIMAL=, DUAL=, PSLACK=, DSLACK=, ITER=**
MAP	This command has been removed.
MATRIX	Temporary variables are *$ROWS* and *$COLS*.
NL	**NOCONEXOG, AUTCOV=, GMM=, PREDICT=, RESID=** Nonlinear Two Stage Least Squares, Nonlinear Three Least Squares and Generalized Method of Moments are available.
OLS	**AUTCOV=** gives the Newey-West Autocorrelation Consistent Covariance Matrix.
PLOT	An interface to the GNUPLOT package is available to provide high quality graphics. The options are: **GNU**, **KEY/NOKEY, COMMFILE=**, **DEVICE=**, **OUTPUT=** and **PORT=.**
PROC	Beginning of SHAZAM procedures.
PROCEND	End of SHAZAM Procedure.
SET	**SKIPMISS, MISSVAL=** options for missing values.
SYSTEM	**NOCONEXOG**
2SLS	**NOCONEXOG**

VERSION 8.0

A SHAZAM World Wide Web page has been developed to offer a variety of useful information and services related to SHAZAM.

Version 8.0 introduces the following new commands: **FLS** for the estimation of equations with time-varying coefficients; **NONPAR** for nonparametric density estimation and regression smoothing methods; and **GME** for equation estimation by generalized entropy methods.

Program improvements include the following. New temporary variables are *$PI* that contains the value of π and *$PVAL* that contains the p-value from some tests. On the **OLS**

command new temporary variables to save model selection test statistics reported with the **ANOVA** option are: *$FPE, $LAIC, $LSC, $GCV, $HQ, $RICE, $SHIB, $SC,* and *$AIC*; the **DLAG** option returns *$DURH*, and the **LM** option returns *$JB*. New options are:

COMMAND	*NEW AVAILABLE OPTIONS*
COINT	**RESID=, SIGLEVEL=**
DIAGNOS	**CTEST** **GNU** can be used with the **RECUR** option to get a plot of the CUSUM and CUSUMSQ of the recursive residuals.
FILE	**DELETE, TEMP**
FLS	New command: **GNU, MAX, NOCONSTANT, PCOEF, BEG=, END=, COEF=, DELTA=, PREDICT=** and **RESID=**
GME	New command: **DEVIATION**, **LINLOG, LIST, LOGLIN, LOGLOG, NOCONSTANT, PCOV, RSTAT, BEG=, END=, COEF=, CONV=, COV=, ITER=, LOGEPS=, PITER=, PREDICT=, QPRIOR=, RESID=, START=, STDERR=, TRATIO=, UPRIOR=, VENTROPY=** and **ZENTROPY=**
HELP	**ERROR, FLS, GME, NONPAR**
NL	**NOPSIGMA, SIGMA=**
NONPAR	New command: **DENSITY, GNU, LIST, PCOEF, BEG=, END=, BRHO=, COEF=, DELTA=, FCSE=, HATDIAG=, INCOVAR=, ITER=, METHOD=, PREDICT=, RESID=, RWEIGHTS=, SIGMA=, SMATRIX=** and **SMOOTH=**
OLS	**GNU**, **INDW=** The **EXACTDW** option has been renamed to **DWPVALUE** and the 200 observation limit has been removed.
PLOT	**GNU** can be used with the **HISTO** option. **APPEND** is available with the **GNU** option.
POOL	**AR1**, **HETCOV** and **OLS**
PROBIT /LOGIT	**LOG**
READ	The **TSP** option has been renamed to **DB**.
ROBUST	**GNU**

SET SKIPMISS	Improved implementation. **GENR** commands set missing value codes when computations involve a missing observation.
STAT	**REPLICATE, MEDIANS=** and **MODES=**
WRITE	**DB**

VERSION 9.0

SHAZAM Professional Edition is a Windows version of SHAZAM available for computers running Microsoft Windows 95/98/2000 and NT. The Professional edition of SHAZAM is a high performance, multi-threaded, native 32 bit version of SHAZAM. It provides users many features including a Data editor, Plot editor, Command Editor, and Project Viewer.

The maximum number of characters for a command line has been increased to 8192 characters. The **LININV** and **LOGINV** options are available with the **OLS, AUTO**, **GLS**, **GME**, **HET**, **MLE**, **POOL** and **ROBUST** commands.

The **GRAPH** command provides an interface to the GNUPLOT program for preparing graphs and histograms. This replaces the **PLOT** command available in previous versions. The **PLOT** command can be used to obtain plots that are printed with plain text characters on the SHAZAM output. The **GRAPH** option replaces the **GNU** option on the **CONFID**, **DIAGNOS**, **FLS**, **NONPAR**, **OLS** and **ROBUST** commands. On the **ARIMA** command the **GNU** option is replaced by the options **GRAPHAC**, **GRAPHDATA**, **GRAPHPAC**, **GRAPHRES** and **GRAPHFORC**.

Other new options are:

COMMAND	*NEW AVAILABLE OPTIONS*
AXISFMT	New command.
CALL	New command: **AMERICAN, BLACK, EQUAL, IMPVOL, BARRIER=, BEG=, END=, DIVIDEND=, NUMTIME=, OPTIONP=, PREDICTP=, PREDICTV=, RISKFREE=, SIGMA=, STRIKEPRICE=, TIME=, UP=, DOWN=**
CONFID	The name *$SIG2* can be specified to get a confidence interval for the error variance. The **CONFID** command can follow a **STAT** command for the calculation of interval estimates for the population mean.

DELETE	**ALLDATA**
DIAGNOS	**HANSEN** (for Hansen tests for parameter instability), **HET** (new test statistics for White's heteroskedasticity tests), **RESET** (new test statistics for the DeBenedictis and Giles FRESET tests).
DISTRIB	**TYPE=(BERNOULLI, CAUCHY, ERLANG, EXPONENTIAL, EXTREME, GEOMETRIC, LOGISTIC, LOGNORMAL, NEGBIN, PARETO, PASCAL, POISSON, POWER, WEIBULL)** and **X=**
DUMP	**ASCII**
FC	**MEANPRED**
FILE	**CD, PLOTPATH** and **PWD**
FUZZY	New command: **DUMP, GRAPHDATA, GRAPHRULE, MEDIAN, NOLIST, NOPMATRIX, NOSTANDARD, PASSOC, PBREAK, BEG=, END=, CMA=, DEGREES=, PREDICT=, RMA=, RULES=, WEIGHT=**
GLS	**HETCOV**
GRAPH	New command: **APPEND, AXIS, AXISFMT, HISTO, KEY, LINE, LINEONLY, NOAXIS, NOKEY, RANGE, TIME, TIMEFMT, WIDE, BEG=, END=, COMMFILE=, DATAFILE=, DEVICE=, GROUPS=, OUTPUT=, PORT=**
NL	**GMMOUT=**
MATRIX	**SEAS**(*nob,–ncross*)
MLE	**TYPE=(POISSON, EPOISSON)**
OLS	**PIL**
POOL	**CSINDEX=** Estimation with unbalanced panels is available.
PORTFOLIO	New command: **EQUALWEIGHT, GRAPHDATA, GRAPHFRONT, GRAPHLINE, INRATES, LIST, PFRONTIER, SHARES, WEIGHTS, WIDE, BEG=, END=, INDEX=, RETURNS=, RISKFREE=, RISKS=**
PUT	New command: Options same as for the **CALL** command.
READ	**NAMES, XLS**

SET	**BYVAR, CONTINUE, GRAPH / NOGRAPH, WARNMISS / NOWARNMISS**
SMOOTH	New command: **ADDITIVE, ARITH, CENTRAL, NOCENTRAL, BEG=, END=, EMAVE=, MAVE=, NMA=, NSPAN=, SAMAVE=, SFAC=, WEIGHT=**
STAT	**ANOVA** lists a two-way ANOVA table when the variables have equal observations. When only 2 variables are specified additional test statistics for equality of mean and variance are listed.
STOCKGRAPH	New command: **AXISFMT, EMA, GRAPHDATA, GRAPHMA, GRAPHMACD, GRAPHVOL, LIST, SOMA, WIDE, BEG=, END=, BOLLINGER=, MALONG=, MAMACD=, MASHORT=**
TIMEFMT	New command.
TOBIT	**NEGATIVE**
WRITE	**APPEND, XLS**

Incompatibilities with previous versions

In previous versions, the **READ** command set the **NOREWIND** option as the default. In Version 9.0, the **READ** command loads the data set starting at the first record of the data file. If multiple **READ** commands are used to load data sequentially from a data file, then the **NOREWIND** option must be specified. In previous versions, the **WRITE** command set the **NOREWIND** option as the default when a **FILE** command was used to assign a file to a unit number. The **REWIND** option is the default in Version 9.0. For the **GENR** and **GEN1** commands the expression X**–A is not permitted. This should be entered as X**(–A). The expression –X**A is evaluated as –(X**A). That is, –X**A is interpreted as 0–X**A. In previous versions the expression –X**A was evaluated as (–X)**A. The **SET NOUMINUS** command can be used to obtain the priority of operations that was effective in previous SHAZAM versions.

VERSION 10

A large number of changes have been made to the Professional Edition between the original Version 9 release and Version 10. Along with an updated Integrated Development Environment (IDE), changes include the following. The existing command editor has been replaced with one designed specifically to support the new SHAZAM debugger as well as

other features. Large file performance has been greatly enhanced, new editing features and improved printing have been added. This version introduces the ability to debug SHAZAM from the command editor. The debugger provides menus, windows, and dialog boxes to access its tools. SHAZAM wizards allow commands to be created or procedures to be executed immediately by selecting variables and procedure options using a mouse.

In previous versions of SHAZAM Professional Edition output was presented as raw unformatted text. In this version, options are available to produce output in neatly arranged sections and tables to produce formatted output that is transportable to most modern word processors. More features are provided for data access and graphing. Changes to graphs are enabled through the properties dialog once graphs are added to the current project.

A quad precision version of SHAZAM is available for high precision work.

For programming in SHAZAM, the number of **DO**-loop levels is increased to 18. The maximum command length is increased to 16384 characters (from 8192). Other new options are:

COMMAND	*NEW AVAILABLE OPTIONS*
ARIMA	**PSI=**
DIAGNOS	**NOWHITE**
FC	**FIXED**
MLE	**TYPE=(EBETA, MBETA)**
NAMEFMT	New command.
OLS	**NPOP=**
POOL	**FIXED**
PROBIT/LOGIT	Marginal effects are reported.
READ	**CHARVARS=**
SMOOTH	**HPFILTER, LAMBDA=**
STAT	**SAMEOBS, SAMPSIZE, NPOP=, STEMPLOT=**

VERSION 11

Version 11 incorporates numerous updates and architectural changes including:

- 32 Bit and 64 Bit Processor Support.

- Automatic Vectorization and Parallelization across all Processor Cores.
- A new Environment, including integrated graph, data and command editors.
- Workspace Management with Automatic READ features.
- Support for Excel Formats *.xlsx.
- Integrated Resources.
- Electronic version of this manual.
- SHAZAM Community Integration.
- Embedded licensing and Product Activation.
- Batch and Interactive command execution including: Console (interactive), Talk (interactive) and Immediate (interactive) Input capabilities.
- Graphing enhancements including new styling, options and smooth rendering.
- Hundreds of updated Textbook examples from recent textbooks.
- Menu and dialog driven Analytical techniques (Professional Edition)
- Improvements to the Data Connector (Professional Edition)
- Improvements to the debugger (Professional Edition)
- Formatted output enhancements (Professional Edition)

COMMAND	*NEW AVAILABLE OPTIONS*
QP	New Command to perform Bounded or Unbounded solutions to Quadratic Programming Problems, with or without equality or inequality constraints
	CHOLSPEC DUMP MIN NEGDEF PDUAL UNCONSTR CONV= DIAGR= DUAL= IFACT= ITER= LAGRANGE= LOWER= LOWSCAL= MEQ= METHOD= PRIMAL= UPPER= UPSCAL= ZEROTOL=
NL	Support for Simulated Annealing and automatic starting point detection for single and multiple equation models.
	Small sample adjustment with the **NODN**.
	DN NODN HYBRID SAITER= SACONV= SAUPPER= SALOWER= SANEPS= SANS= SANT= SATRF= SAUPFAC= SALOWFAC=

Incompatibilities with previous versions

In SHAZAM 10 the XLS option on READ would read data directly from Microsoft Excel Workbooks. This capability has been removed in this version. Instead Microsoft Excel Workbook formats both old and new (*.xls and *.xlsx) can be opened directly from the Open button within the SHAZAM Environment. Click 'Add to Workspace' and select a filename to automatically convert these to SHAZAM Data. To write data to a Microsoft Excel Workbook use the WRITE statement to output SHAZAM Data. The file will appear within your SHAZAM Workspace after which you may open it within the SHAZAM Data Editor and then save it as a new Workbook. Please note that it is no longer required to have Microsoft Excel on the computer for SHAZAM to read and write Excel spreadsheets.

The SHAZAM READ and WRITE statements provide support for Fixed Format and Free Format and DIF Format data only.

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